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The stability of fundamental constants

La stabilité des constantes fondamentales

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ABSTRACT

The tests of the constancy of fundamental constants are tests of the local position invariance and thus of the equivalence principle, at the heart of general relativity. After summarising the links between fundamental constants, gravity, cosmology and metrology, a brief overview of the observational and experimental constraints on their variation is proposed.

RÉSUMÉ

Les tests de la variation des constantes fondamentales sont des tests de l'invariance de position locale et donc du principe d'équivalence, au cœur de la relativité générale. Après un résumé des liens entre constantes fondamentales, gravitation, cosmologie et métrologie, ce texte propose un état des lieux des contraintes expérimentales et observationnelles sur leur variation.

1. Introduction

Fundamental constants are not determined by the theories in which they appear. They can only be measured, which is actually their most important property. This explains why metrology has engaged in the quest of measuring physical constants, fundamental or not, to the highest precision that is deeply entangled with the improvement of the definition and realisation of the standards of units [1].

These constants play an important role in physics since they set the order of magnitude of phenomena, allow one to forge new concepts and characterise the domain of validity of the theories in which they appear. They also play a central role in cosmology and astrophysics. Their value fixes the rate of local clocks (e.g., radioactive decay rates, atomic transitions, etc.) that allow one to perform datation of geophysical and astrophysical phenomena. There are a key to the reconstruction of the history of our Universe. The phenomena that can be observed in our local Universe, from big-bang nucleosynthesis time to now, rely mostly on general relativity, electromagnetism, and nuclear physics. The fact that we can understand the Universe

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and its laws has an important implication in the structure of physical theories. At each step of their construction, we have been dealing with phenomena below a typical energy scale, for technological constraints, and it turned out (experimentally) that we have always been able to design a consistent theory valid in such a restricted regime. This is not expected a priori and is deeply rooted in the mathematical structure of the theories that describe nature. This property, called *scale decoupling principle*, refers to the fact that there exist energy scales below which effective theories are sufficient to understand a set of physical phenomena. *Effective theories* are the most fundamental concepts in the scientific approach to the understanding of nature and they always come with a domain of validity inside which they are efficient to describe all related phenomena. They are a successful explanation at a given level of complexity based on concepts of that particular level. For instance, we do not need to understand and formulate string theory to develop nuclear physics and we do not need to know anything about nuclear physics to develop chemistry or biology.

The set of theories that describe the world around us can then be split into a hierarchy of modular levels in interaction. The relation between the different levels have the following properties [2,3]. (1) Higher-level behaviours are constrained by the lower level laws from which they emerge. This is the usual bottom-up causation in which microscopic forces determine what happens at the higher levels. The more fundamental gives the space of possibilities in which a higher level can develop, by constraining, e.g., causality, the type of interactions or structures that can exist. (2) Scale separation implies that at each level of complexity, one can define fundamental concepts that are not affected by the fact that they may not be fundamental at a lower level. In that sense, much of the higher level phenomena remain quite independent of the microscopic structures, fields, and interactions. (3) At least for the lowest level, the fact that physical theories are renormalisable implies that they influence higher levels mostly through some numbers. This in particular the case of the fundamental constants of a given effective theory. While they cannot be explained within the framework of this particular level, they can however be replaced by more fundamental constants of an underlying level. For instance, the mass or the gyromagnetic factor of the proton are fundamental constants of nuclear physics. They can however be "explained", even if the actual computation maybe difficult (see [4]), in terms of constants of the lower level (such as the quark mass, binding energies). This explanation of the constants of an effective theory may reveal new phenomena that could not be dealt with before (e.g., the fundamental parameters of the effective theory may now be varying), but these phenomena have to be at the margin (or below the error bars) of the experiences that have validated this effective theory. (4) Not all the concepts of a higher level can be explained in terms of lower-level concepts. Each level may require its own concepts that do not exist, or even are not related, to lower-level concepts. These are emergent properties so that the whole may not be understood in terms of its parts. (5) The fact that there exists a lower level of complexity and thus microscopic degrees of freedom implies that these degrees of freedom can be heated up so that we expect to see entropy and dissipation arising. (6) The higher levels of complexity can back-react on the lower levels. This is the notion of top–down causation. (7) Historically, various disciplines have developed independently in almost quasi-autonomous domains, each of them having its own ontology. Sometimes two such theories collide and show inconsistency that will need, in order to be resolved, the introduction of new concepts, more fundamental, from which the concepts of each one of the theories can be derived in a limiting behaviour. For example, Maxwell electromagnetism and Galilean kinematics are incompatible, which is at the origin of special relativity with the new concept of spacetime; or in quantum mechanics, the concept of wave function has to be coined from the preexisting concepts of particle and wave. This implies that concepts that were thought to be incommensurable (such as space and time, or momentum and wave number) need to be unified, which is usually achieved by the introduction of new fundamental constants (speed of light, or Planck constant, in the two examples above) that were not considered as fundamental (or even existed) in the previous theories; see, e.g., [5].

I shall thus define a *fundamental constant* as "any parameter not determined by the theories in which it appears", which emphasises that constants and theories cannot be considered separately. Indeed, this parameters have to be assumed constant for two reasons. First, from a theoretical point of view, we have no evolution equation for them (since otherwise they would be fields) and they cannot be expressed in terms of other more fundamental quantities. Second, from an experimental point of view, in the regimes in which these theories have been validated, their constants should be constant at the accuracy of the experiments, to ensure their reproducibility. This means that testing for the constancy of these parameters is a test of the theories in which they appear and allow us to extend the knowledge of their domain of validity.

These constants raise a number of questions. First, are they really constant during the evolution of the Universe? Then, can we explained their value? The first question is related to the validity of Einstein's equivalence principle, while the second is related to the apparent fine tuning of our Universe. In the following, I will summarise in Section 1 some theoretical aspects about fundamental constants, in particular their relation to general relativity and cosmology. Section 2 will provide an overview of the constraints on their variation, focusing on the latest developments. More details can be found in reviews [5,6], as this text focuses on more recent developments.

2. Theoretical considerations

This section briefly summarises some theoretical considerations about the constants, in particular their nature (Section 2.1), their link with the equivalence principle (Section 2.2), and cosmology (Section 2.3)

2.1. Some properties of the constants

Given the previous definition, the number of fundamental constants depends on the theoretical framework we are considering to describe the laws of nature. Today, gravitation is described by general relativity, and the three other interactions and whole fundamental fields are described by the standard model of particle physics. In such a framework, one has 22 unknown constants—the Newton constant, six Yukawa couplings for quarks and three for leptons, the mass and vacuum expectation value of the Higgs field, four parameters for the Cabibbo–Kobayashi–Maskawa matrix, three coupling constants, a UV cut-of to which one must add the speed of light and the Planck constant; see, e.g., [7]. Indeed, when introducing new, more unified or more fundamental theories the number of constants may change so that the list of what we call fundamental constants is a time-dependent concept and reflects both our knowledge and our ignorance [8].

For instance, we experimentally know today that neutrinos have to be massive. This implies that the standard model of particle physics has to be extended and that it will involve at least seven more parameters (three Yukawa couplings and four CKM parameters). On the other hand, this number can decrease, e.g., if the non-gravitational interactions are unified. In such a case, the coupling constants may be related to a unique coupling constant α_U and a mass scale of unification M_U through $\alpha_i^{-1}(E) = \alpha_U^{-1} + (b_i/2\pi)\ln(M_U/E)$, where the b_i are numbers that depend on the explicit model of unification. This would also imply that the variations, if any, of various constants will be correlated.

Since any physical measurement reduces to the comparison of two physical systems, one of them often used to realise a system of units, it only gives access to dimensionless numbers. This implies that only the variation of dimensionless combinations of the fundamental constants can be measured and would actually also correspond to a genuine modification of the physical laws; see, e.g., [5,6,9]. Changing their value while letting all dimensionless numbers unchanged would correspond to a change of units. It follows that out of the 22 constants of our reference model, we can pick three of them to define a system of units (such as, e.g., c, G and h to define the Planck units) so that we are left with 19 unexplained dimensionless parameters, characterising the mass hierarchy, the relative magnitude of the various interactions, etc. This also means that in theory three fundamental constants can be used to define three of the seven base units of the international system of units (e.g., the metre, the second, and the kilogram). While the speed of light has already been fixed in 1983 so that the metre derives from the second, the possibility to fix the Planck constant, the charge of the electron, the Boltzmann constant and the Avogadro number to define the metre, the kilogram, the Candela, the Ampere, the mole and the Kelvin from the second (hence reducing all units to a single clock, the dream of all theoreticians, and in particular of relativists) is now lively debated [10].

The necessity of theoretical physics in our understanding of fundamental constants and for deriving bounds from their variation is, at least, threefold. (i) It is necessary to understand and to model the physical systems used to set the constraints (and to determine the effective parameters that can be observationally constrained to a set of fundamental constants); (ii) it is necessary to relate and compare different constraints that are obtained at different space-time positions (this requires a space-time dynamics and thus to specify a cosmological model); (iii) it is necessary to relate the variation of different fundamental constants. One has also to be aware that while *in principle* some constant parameters of a theory can be expressed in terms of the 22 above-mentioned fundamental constants, *in practice* the computation cannot be achieved with enough accuracy. This is the case, for example, of the masses of the neutron and proton, or the gyromagnetic factors [4]. This limitation has important consequences in the tests I shall discuss below, since even if one can set constraints on the variation of some of these parameters, it is often difficult and generically model-dependent to translate this to constraints on the variation of the 19 fundamental parameters. This also means that this is an active line of research.

2.2. Links with general relativity

The tests of the constancy of fundamental constants take all their importance in the realm of the tests of the equivalence principle [11].

This principle, which states the universality of free fall, the local position invariance and the local Lorentz invariance, is at the basis of all metric theories of gravity and implies that all matter fields are universally coupled with a unique metric $g_{\mu\nu}$; $S_{\rm mat}(\psi, g_{\mu\nu})$. The dynamics of the gravitational sector is dictated by the Einstein–Hilbert action $S_{\rm grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}R_*\,\mathrm{d}^4x$. General relativity [12] assumes that both metrics coincide, $g_{\mu\nu} = g^*_{\mu\nu}$.

The test of the constancy of constants is a test of the local position invariance hypothesis and thus of the equivalence principle. Let us remind that it is deeply related to the universality of free fall [13] since if any constant c_i is a space-time-dependent quantity, so will the mass of any test particle. Starting from the action of a point particle of mass m_A

$$S_{p.p.} = -\int m_A[c_j]\sqrt{-g_{\mu\nu}(x)v^\mu v^\nu}\,\mathrm{d}t$$

with $v^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}t$, its equation of motion is

$$u^\nu\nabla_\nu u^\mu = -\left(\frac{\partial\ln m_A}{\partial c_i}\nabla_\beta c_i\right)(g^{\beta\mu} + u^\beta u^\mu)$$

Hence, a test body does not enjoy a geodesic motion and experience an anomalous acceleration that depends on the sensitivity $f_{A,i} \equiv \partial\ln m_A/\partial c_i$ of the mass m_A to a variation of the fundamental constants c_i. In the Newtonian limit,

$g_{00} = -1 + 2\Phi_N/c^2$ so that $\mathbf{a} = \mathbf{g}_N + \delta\mathbf{a}_A$ with the anomalous acceleration $\delta\mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla c_i + \frac{\mathbf{v}_A}{c^2}\dot{\alpha}_i\right)$. Such deviations are strongly constrained in the Solar system and also allow one to bound the variation of the constants [14].

2.3. Links with cosmology

This property allows one to extend the tests of the equivalence principle, and thus tests of general relativity, on astrophysical scales. Such tests are central in cosmology, in which the existence of a dark sector (dark energy and dark matter) is required to explain the observations [15]. Universality classes of dark energy models have been defined [16,17], and the constants give tests on some of these classes, hence complementing other tests of general relativity on astrophysical scales [17] and of the hypothesis of the cosmological model [18].

A cosmological insight into the value of the fundamental constants is becoming more and more popular. It is based on the idea that our observable Universe is part of a larger multiverse and on the application of the anthropic principle as an observer selection bias. I shall not address this in this short note.

2.4. Theory with varying constant

In order to construct a theory with a "varying constant", one needs to replace it by a dynamical field. The action $S[\phi, A_\mu, g_{\mu\nu}, \psi, \dots; c_1, \dots c_n]$, in which the c_i are assumed constant, leads to one equation per degree of freedom. When extended to, e.g., $S[\phi, A_\mu, g_{\mu\nu}, \psi, \dots, c_1(x); c_2, \dots c_n]$, it describes a theory in which c_1 has become dynamical. This has two consequences [9]: (1) the equations derived under the assumption that this parameter is constant are modified so that one cannot just make it vary in the equations, and (2) the theory provides an equation of evolution for c_1, since it is now a degree of freedom.

The field responsible for the time variation of the "constant" c_1 is also responsible for a long-range (composition-dependent) interaction, i.e. at the origin of the deviation from general relativity, indeed depending on its mass.

Many frameworks have been proposed, from the scalar-tensor theories of gravity to theories with higher dimensions, including string theories in which all dimensionless constants are supposed to become dynamical (see [6] for a detailed review of these models). This shifts the question to "why are the constants so constant today ?", since one has to provide a stabilisation mechanism in order to explain the apparent constancy of this dynamical parameters.

3. Observational and experimental constraints

This section summarises the constraints on the variation of the fundamental constants. Sadly, this is a very active field that cannot be summarised in a short text. Hence, I shall first summarise in Section 3.1 the physical systems that have been used and the main constraints. I then focus on recent developments concerning the physical interpretation of atomic clocks constraints (Section 3.2), constraints derived from the observation of the cosmic microwave background by the Planck satellite (Section 3.3) and developments in nuclear astrophysics (Section 3.4).

3.1. Summary

3.1.1. Physical systems

The physical systems that have been considered can be classified in many ways [5,6].

First, we can classify them according to their space-time position. This is summarised in Fig. 1, which represents our past lightcone, the location of the various systems (in terms of their redshift z). These systems include atomic clocks comparisons ($z = 0$), the Oklo phenomenon ($z \sim 0.14$), meteorite dating ($z \sim 0.43$), both having a space-time position along the worldline of our Solar system, quasar absorption spectra ($z = 0.2$–4), population III stars ($z \sim 10$–15), cosmic microwave background (CMB) anisotropy ($z \sim 10^3$), and primordial nucleosynthesis (BBN, $z \sim 10^8$). Indeed, higher redshift systems offer the possibility to set constraints on a larger time scale, but at the prize of usually involving other parameters such as the cosmological parameters. This is particularly the case of CMB and BBN, the interpretation of which requires a cosmological model.

The systems can also be classified in terms of the physics they involve in order to be interpreted. For instance, atomic clocks, quasar absorption spectra and CMB require only to use quantum electrodynamics to draw the primary constraints, so that these constraints will only involve the fine-structure constant α, the ratio between the proton-to-electron mass ratio μ and the various gyromagnetic factors g_I. On the other hand, the Oklo phenomenon, meteorite dating and nucleosynthesis require nuclear physics and quantum chromodynamics to be interpreted.

3.1.2. Setting constraints

Setting constraints goes through several steps.

First, any system allows us to derive an observational or experimental constraint on an observable quantity $O(G_k, X)$ that depends on a set of primary physical parameters G_k and a set of external parameters X, which usually are physical parameters (e.g., temperature, etc.). These external parameters are related to our knowledge of the physical system and the lack of their knowledge is usually referred to as systematic uncertainty.

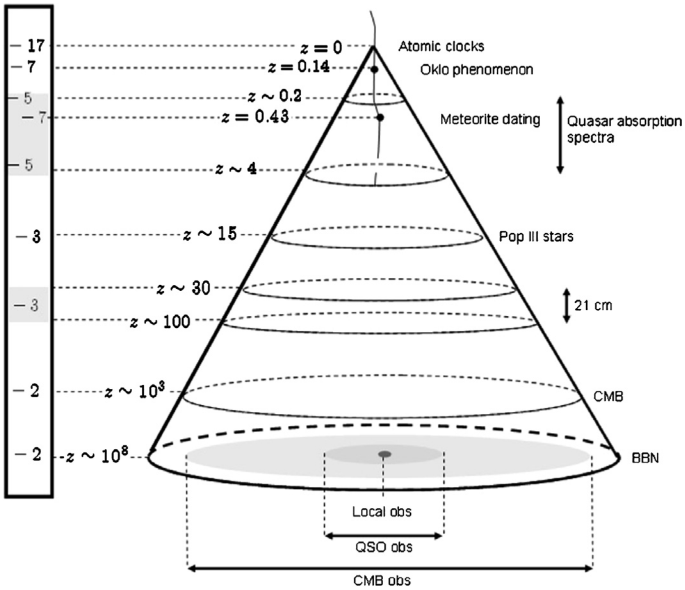

Fig. 1. Systems that have been used to probe the constancy of the fundamental constants in a space-time diagram in which the cone represents our past lightcone. The shaded areas represent the comoving space probed by different tests with respect to the largest scales probed by big-bang nucleosynthesis.

From a model of the system, one can deduce the sensitivities of the observables to an independent variation of the primary physical parameters:

$$\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k} \tag{3.1}$$

The primary parameters G_k are usually not fundamental constants (e.g., the resonance energy of the samarium E_r for the Oklo phenomenon, the deuterium binding energy B_D for BBN etc.). The second step is thus to relate the primary parameters to fundamental constants c_i. This gives a series of relations

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i \tag{3.2}$$

The determination of the parameters d_{ki} requires to choose the set of constants c_i (do we stop at the masses of the proton and neutron, or do we try to determine the dependencies on the quark masses, or on the Yukawa couplings and Higgs vacuum expectation value, etc.) and also requires to deal with nuclear physics and the intricate structure of QCD. In particular, the energy scales of QCD, Λ_QCD, is so dominant that at lowest order, all parameters scale as Λ^n_QCD so that the variation of the strong interaction would not affect dimensionless parameters and one has to take the effects of the quark masses.

An example of such a computation is described below. It also implies that the variation of the primary parameters are usually correlated, which means that drawing constraints by assuming independent variations is not a good approximation. Unfortunately, the correlations between the variation is model-dependent.

3.2. Atomic clocks and gyromagnetic factors

The comparison of two atomic clocks A and B allows one to set constraints on the local (i.e. today) time drift of various combinations of the fundamental constants. It relies on the simple fact that different transitions have different dependencies in the fundamental constants. For instance, for the hydrogen atom, the frequencies of the gross, fine and hyperfine-structures are roughly given by $2p$–$1s:\ \nu \propto cR_\infty$, $2p_{3/2}$–$2p_{1/2}:\ \nu \propto cR_\infty\alpha^2$, and $1s: \propto cR_\infty\alpha^2 g_\mathrm{p}\mu$, respectively, where the Rydberg constant set the dimension. g_p is the proton gyromagnetic factor and $\mu = m_\mathrm{e}/m_\mathrm{p}$.

In the non-relativistic approximation, the transitions of all atoms have similar dependencies, but two important effects have to be taken into account. First, the hyperfine-structures involve a gyromagnetic factor g_i (related to the nuclear magnetic moment by $\mu_i = g_i\mu_\mathrm{N}$, with $\mu_\mathrm{N} = e\hbar/2m_\mathrm{p}c$), which are different for each nuclei. Second, relativistic corrections (including the Casimir contribution), which also depend on each atom (but also on the type of the transition), can be included through a multiplicative function $F_\mathrm{rel}(\alpha)$. It has a strong dependence [29] on the atomic number Z, which can be illustrated on the case of alkali atoms, for which

$$F_\mathrm{rel}(\alpha) = \left[1 - (Z\alpha)^2\right]^{-1/2}\left[1 - \frac{4}{3}(Z\alpha)^2\right]^{-1}$$

Table 1
Summary of the constraints of the atomic clock experiments. For each experiment, one can determine the scaling of the relative frequency drift in terms of $\{g_i, \alpha, \mu\}$ (see e.g. Refs. [30,31]). The values of the coefficients $\{\lambda_{g_p}, \lambda_{g_n}, \lambda_b, \lambda_\mu, \lambda_\alpha\}$ as obtained in Ref. [4].

Clocks	ν_{AB}	λ_{g_p}	λ_{g_n}	λ_b	λ_μ	λ_α	$\dot\nu_{AB}/\nu_{AB}$ (yr^{-1})	Ref.
Cs–Rb	$\frac{g_{Cs}}{g_{Rb}}\alpha^{0.49}$	−1.383	0.325	0.714	0	0.49	$(0.5\pm5.3)\times10^{-16}$	[21]
Cs–H	$g_{Cs}\,\mu\,\alpha^{2.83}$	−0.619	0.152	0.335	1	2.83	$(32\pm63)\times10^{-16}$	[22]
Cs–^{199}Hg$^+$	$g_{Cs}\,\mu\,\alpha^{6.03}$	−0.619	0.152	0.335	1	6.03	$(-3.7\pm3.9)\times10^{-16}$	[23]
Cs–^{171}Yb$^+$	$g_{Cs}\,\mu\,\alpha^{1.93}$	−0.619	0.152	0.335	1	1.93	$(0.78\pm1.40)\times10^{-15}$	[24]
Cs–Sr	$g_{Cs}\,\mu\,\alpha^{2.77}$	−0.619	0.152	0.335	1	2.77	$(1.0\pm1.8)\times10^{-15}$	[25]
Cs–SF$_6$	$g_{Cs}\sqrt{\mu}\alpha^{2.83}$	−0.619	0.152	0.335	0.5	2.83	$(-1.9\pm0.12\pm2.7)\times10^{-14}$	[26]
Dy	α	0	0	0	0	1	$(-2.7\pm2.6)\times10^{-15}$	[27]
^{199}Hg$^+$–^{27}Al$^+$	$\alpha^{-3.208}$	0	0	0	0	−3.208	$(5.3\pm7.9)\times10^{-17}$	[28]

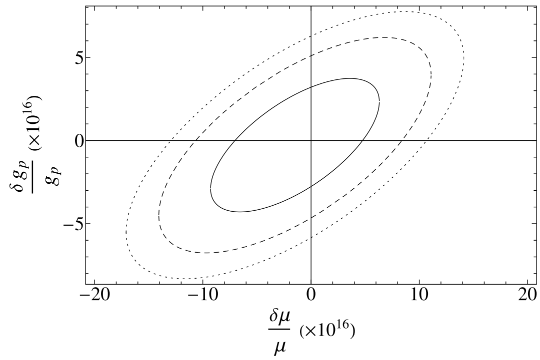

Fig. 2. (Colour online.) Constraints on the variation of $\{g_p, \mu\}$ assumed to be independent once the constraint from the variation of α from the Hg–Al clock is taken into account. Solid, dashed and dotted contours correspond to 68.27%, 95% and 99% C.L. From [4].

The development of highly accurate atomic clocks using different transitions in different atoms offer the possibility to test a variation of various combinations of the fundamental constants. The computation of the dependencies of the frequency ratios on the set of constants $\{\alpha, \mu, g_i\}$ involves quantum electrodynamics and N-body simulations. As an example, the ratio of two hyperfine-structure transitions depends only on g_I and α, while the comparison of fine-structure and hyperfine structure transitions depend on g_I, α and the electron-to-proton mass ratio, μ. For instance [19,20], $\nu_{Cs}/\nu_{Rb} \propto \frac{g_{Cs}}{g_{Rb}}\alpha^{0.49}$ for the comparison of rubidium and caesium clocks and $\nu_{Cs}/\nu_H \propto g_{Cs}\mu\alpha^{2.83}$ for the comparison of hydrogen and caesium clocks. Different set of clocks have been used and the existing constraints and the references to the experiments are summarised in Table 1; we refer the reader to other contributions in this volume for a detailed description of these experiments. Recent experiments using ^{171}Yb$^+$ and Cs clocks tend to slightly improve these constraints [32].

All the constraints involve only four quantities, μ, α and the two gyromagnetic factors g_{Cs} and g_{Rb}. In Ref. [4], the nuclear g-factors have been related to the proton and neutron g-factors as

$$\frac{\delta g_{Rb}}{g_{Rb}} = 0.764\frac{\delta g_p}{g_p} - 0.172\frac{\delta g_n}{g_n} - 0.379\frac{\delta b}{b}$$

$$\frac{\delta g_{Cs}}{g_{Cs}} = -0.619\frac{\delta g_p}{g_p} + 0.152\frac{\delta g_n}{g_n} + 0.335\frac{\delta b}{b}$$

in which b is determined by the spin–spin interaction and appears in the expressions for the spin expectation value of the valence proton. These expressions allow one to relate the observational constraints to $\{g_p, g_n, \mu, \alpha\}$ as

$$\frac{\dot\nu_{AB}}{\nu_{AB}} = \lambda_{g_p}\frac{\dot g_p}{g_p} + \lambda_{g_n}\frac{\dot g_n}{g_n} + \lambda_b\frac{\dot b}{b} + \lambda_\mu\frac{\dot\mu}{\mu} + \lambda_\alpha\frac{\dot\alpha}{\alpha} \quad (3.3)$$

with the coefficients λ summarised in Table 1. Using the constraint [28] on α, this relation leads to an independent constraint on the time variation of μ and g_p; see Fig. 2.

However, one has to be aware that g_p and μ are not independent, since the latter involves the proton mass. The goal is then to express them in terms of the masses of the quark u, d, s and Λ_{QCD}, which would then allow one to get constraints on the time variation of the Yukawa couplings h, the Higgs vacuum expectation value v, and Λ_{QCD}. This has been performed in Ref. [4] to show that, depending on the approach (non-relativistic constituent quark model, chiral perturbation theory, lattice QCD), one gets different results that can vary by two orders of magnitude.

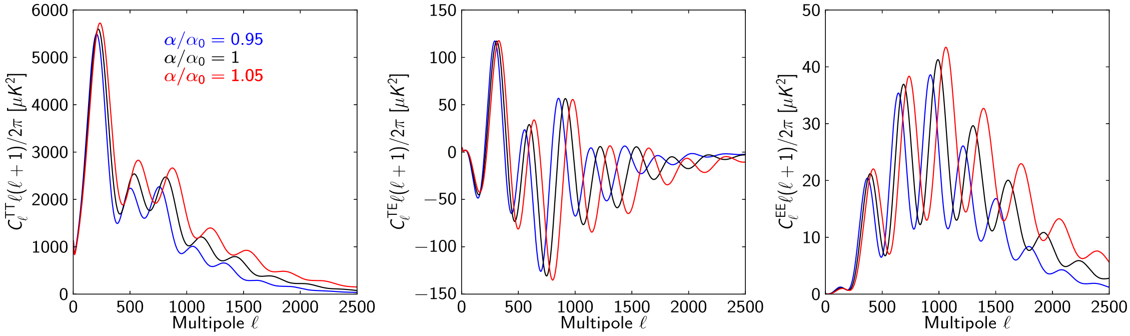

Fig. 3. (Colour online.) CMB TT (left), TE (middle) and EE (right) angular power spectra for different values of variations of −5% (blue) and −5% (red) of the fine-structure constant. From [33].

3.3. Cosmic microwave background

The cosmic microwave background radiation has been emitted roughly 300.000 yr after the big bang, when the temperature of the photon bath filling the Universe has dropped enough to allow proton (resp. helium nuclei) and electron to combine to form neutral hydrogen (resp. helium); see [15] (Chapters 4 and 6).

Any variation of the fundamental constants, and more particularly of the fine-structure constant or mass of the electron, affects the recombination history and imprints the CMB angular power spectrum. This can be taken into account by determining the influence of α and m_e on (1) the binding energies of hydrogen and helium, (2) the Thomson scattering cross-section, (3) the different photoionisation cross-sections, (4) the recombination parameters, (5) the photoionisation parameters, (6) the Einstein coefficient and (7) the 2s decay rate by emission of two photons.

All these modifications have been included in a CMB code [33]. It was shown that a larger value of any of the two constants shifts the recombination epoch to earlier times. This results in a smaller sound horizon at decoupling and a larger angular diameter distance to the last scattering surface. As a consequence, the position of the acoustic peaks shifts to higher multipoles, in a way that can be degenerated with other cosmological parameters that have analogues effects, e.g., the Hubble constant. However, a larger value of the constants also affects the amplitude of the peaks. In fact, an earlier recombination induces an increase in the amplitude at large scales through an increase in the early integrated Sachs–Wolfe effect, and at small scales through a decrease in the Silk damping. The effects of the angular power spectra for temperature and E-polarisation are depicted in Fig. 3.

Assuming a (standard) cosmological model (i.e. a flat ΛCDM model) with two additional varying constants (α or m_e) and with purely adiabatic initial conditions with an almost scale invariant power spectrum, and no primordial gravity waves, these spectra were compared to the recent Planck data [34]. These spectra thus depend on an eight-dimensional parameter space that includes the baryon and cold dark matter densities $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the Hubble constant H_0, the optical depth at reionisation τ, the scalar spectral index n_s, the overall normalisation of the spectrum A_s and two parameters for the varying constants.

It was shown [33] that the Planck data allow one to improve the constraint on the time variation of the constants by almost an order of magnitude compared to WMAP. An independent time variation of α is constrained to $\Delta\alpha/\alpha = (3.6 \pm 3.7) \times 10^{-3}$ when one includes BAO data, while an independent variation of m_e is constrained to $\Delta m_e/m_e = (4 \pm 11) \times 10^{-3}$, both at a 68% confidence level.

The Planck data also permit to set a constraint on α and m_e when they are both allowed to vary. This can be understood by the fact that the degeneracy is broken at high multipoles (see the detailed analysis in Appendix B of [33]) and that no such high resolution data existed before Planck. This is summarised in Fig. 4.

These data also constrain, for the first time with CMB, a dipolar spatial variation of α to show that it cannot exceed 6.5×10^{-4} from a redshift of $z \sim 10^3$. This shows that CMB data are now competitive with lower redshift data. In particular, recent analysis of quasar data have supported the claim that there may exist such a dipole in the fine-structure constant [35]. Combining the observations of 154 absorption systems from VLT and 161 absorption systems observed from the Keck telescope, it was concluded [35] that the variation of α was well represented by an angular dipole pointing in the direction RA $= (17.3 \pm 1.0)$ hr, dec. $= (-61 \pm 10)$ deg, with an amplitude $\Delta\alpha/\alpha = 0.97^{+0.22}_{-0.20} \times 10^{-5}$, at a 4.1$\sigma$ level. From a theoretical point of view, such a dipolar modulation can be realised in some models [36].

3.4. Big bang and stellar nucleosynthesis: nuclear physics at work

The nucleosynthesis of light nuclei took place in the early Universe (BBN) and during the stellar evolution. For the former, we have been interested in modelling the effects of a variation of the fundamental constants on the production of the light elements during BBN, while for the latter we have focused on the production of carbon-12 in population-III stars.

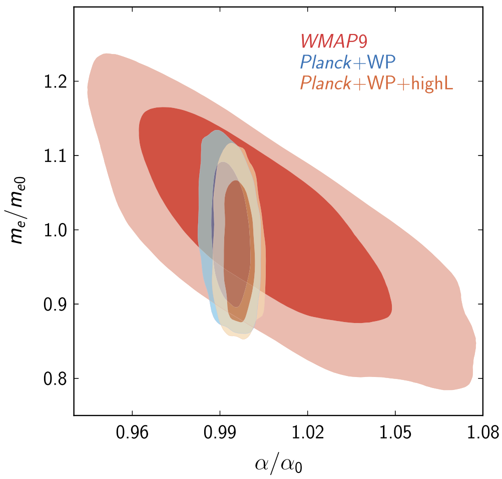

Fig. 4. (Colour online.) Two-dimensional likelihood contours (68% and 95%) in the (α, m_e) plane for Planck (blue) and Planck combined with higher multipole data (yellow). We also show the results using WMAP data in red. From [33].

The effects of the variation of fundamental constants on BBN predictions is difficult to model. However, one can proceed in a two-step approach: first by determining the dependencies of the abundances on the nuclear parameters and then by relating those parameters to the fundamental constants [38].

The abundances of the nuclei synthesised during BBN rely on the balance between the expansion of the Universe and the weak interaction rates that control the neutron-to-proton-ratio [15]. Basically, the abundance of helium-4 depends mainly on the neutron-to-proton ratio at the freeze-out time, $(n/p)_f = \exp(-Q_{np}/kT_f)$, determined (roughly) by $G_F^2(kT_f)^5 = \sqrt{GN}(kT_f)^2$, N being the number of relativistic degrees of freedom; $Q_{np} = m_n - m_p$, G_F the Fermi constant and the neutron lifetime. It also depends on t_N, the time after which the photon density becomes low enough for the photo-dissociation of deuterium to be negligible. Hence, the predictions of BBN involve a large number of parameters. In particular, t_N depends on the deuterium binding energy and on the photon-to-baryon ratio, η_{10}. Besides, one needs to include the effects of α in the Coulomb barriers. For a different analysis of the effect of varying fundamental constants on BBN, see, e.g., [37,38]. Thus, the predictions are mainly dependent on the effective parameters $G_k = (G, \alpha, m_e, \tau, Q_{np}, B_D, \sigma_i)$, while the external parameters are $X = (\eta_{10}, h, N_\nu, \Omega_i)$. It was shown [38] that the most sensitive parameter is the deuterium binding energy, B_D.

In principle, it would be desirable to know the dependence of each of the main SBBN reaction rates to fundamental quantities. This was achieved in Ref. [38], but only for the first two BBN reactions: the n $\leftrightarrow$ p weak interaction and the p(n,γ)d bottleneck. Recently [40,39], it was extended to the ^{3}H(d,n)^{4}He and ^{3}He(d,p)^{4}He reactions that proceed through the $A = 5$ compound nuclei ^{5}He and ^{5}Li, and to the ^{4}He($\alpha\alpha,\gamma$)^{12}C reaction that could bridge the $A = 8$ gap.

In a second step, the parameters G_k can be related to a smaller set of fundamental constants, namely α, the Higgs VEV v, the Yukawa couplings h_i and the QCD scale Λ_{QCD}, since $Q_{np} = m_n - m_p = \alpha a \Lambda_{QCD} + (h_d - h_u)v$, $m_e = h_e v$, $\tau_n = G_F^2 m_e^5 f(Q/m_e)$ and $G_F = 1/\sqrt{2}v$. The deuterium binding energy can be expressed in terms of h_s, v and Λ_{QCD} using a sigma nuclear model or in terms of the pion mass. Assuming that all Yukawa couplings vary similarly, the set of parameters G_k reduces to $\{\alpha, v, h, \Lambda_{QCD}\}$.

For the ^{3}H(d,n)^{4}He, ^{3}He(d,p)^{4}He and ^{4}He($\alpha\alpha,\gamma$)^{12}C reactions, we used a different approach, which has also been used for the stellar production of carbon [41]. In these three reactions, the rates are dominated by the contribution of resonances whose properties can be calculated within a microscopic cluster model. The nucleon–nucleon interaction $V(\mathbf{r})$, which depends on the relative coordinate, is written as

$$V(\mathbf{r}) = V_C(\mathbf{r}) + (1 + \delta_{NN})V_N(\mathbf{r})$$

where $V_C(\mathbf{r})$ is the Coulomb force and $V_N(\mathbf{r})$ the nuclear interaction [42]. The parameter δ_{NN} characterises the change in the nucleon–nucleon interaction. It is related to the binding energy of deuterium by $\Delta B_D/B_D = 5.7701 \times \delta_{NN}$ [41]. The next important step is to relate ΔB_D to the more fundamental parameters [38,43]. From the astrophysical measurements of the primordial abundances of D and ^{4}He, we obtained [40] $-0.0025 < \delta_{NN} < 0.0006$ at a redshift $z \sim 10^8$.

For the triple-alpha reaction two alpha-particles fuse into the ^{8}Be ground state, then another alpha capture leads to the Hoyle state in ^{12}C. In our cluster the wave functions of the ^{8}Be and ^{12}C nuclei are approximated by a cluster of respectively two and three α particle wave functions. It allows us to calculate the variation of the ^{8}Be ground state and ^{12}C Hoyle state w.r.t. the nucleon–nucleon interaction, i.e. δ_{NN}. In Ref. [41], we obtained $E_{g.s.}(^8\mathrm{Be}) = (0.09208 - 12.208 \times \delta_{NN})$ MeV, for the ^{8}Be g.s. and $E_R(^{12}\mathrm{C}) = (0.2877 - 20.412 \times \delta_{NN})$ MeV, for the Hoyle state.

This formalism was used in Ref. [41] to show that $-0.0005 < \delta_{NN} < 0.0015$ to ensure the production of carbon and oxygen by population-III stars to be high enough.

4. Discussion

This short overview stresses the importance of the study of fundamental constants and of strong constraints on their variation at different epochs, using different physical systems. These tests are particularly lively with the necessity to test the general relativity on astrophysical scales in order to better understand the dark sector.

One question remained unresolved is the mechanism that determines the values of the dimensionless constants. While the existing tests show that their values have been almost frozen since BBN time, they do not give any answer to this question. By changing the value of these parameters, we change the physics and thus the properties of nature from the nuclear matter to the dynamics of the Universe. It appears that the value of some constants has to be extremely tuned for a complex Universe to develop. These fine-tuning, to be distinguished from numerical coincidences, characterises some catastrophic boundaries in the space of fundamental constants across which some phenomena drastically change.

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