

Varying couplings in electroweak theory

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We extend the theory of Kimberly and Magueijo for the spacetime variation of the electroweak couplings in the unified Glashow-Salam-Weinberg model of the electroweak interaction to include quantum corrections. We derive the effective quantum-corrected dilaton evolution equations in the presence of a background cosmological matter density that is composed of weakly interacting and non-weakly-interacting nonrelativistic dark-matter components.

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I. INTRODUCTION

The studies of relativistic fine structure in the absorption lines of dust clouds around quasars by Webb *et al.* [1–3] have led to widespread theoretical interest in the question of whether the fine structure constant, $\alpha_{em} = e^2/\hbar c$, has varied in time and, if so, how to accommodate such a variation by a minimal perturbation of existing theories of electromagnetism. These astronomical studies have proven to be more sensitive than laboratory probes of the constancy of the fine structure “constant,” which currently give bounds on the time variation of $\dot{\alpha}_{em}/\alpha_{em} \equiv -0.4 \pm 16 \times 10^{-16} \text{ yr}^{-1}$ [4], $|\dot{\alpha}_{em}/\alpha_{em}| < 1.2 \times 10^{-15} \text{ yr}^{-1}$ [5], $\dot{\alpha}_{em}/\alpha_{em} \equiv -0.9 \pm 2.9 \times 10^{-16} \text{ yr}^{-1}$ [6] by comparing atomic clock standards based on different sensitive hyperfine transition frequencies, and $\dot{\alpha}_{em}/\alpha_{em} \equiv -0.3 \pm 2.0 \times 10^{-15} \text{ yr}^{-1}$ from comparing two standards derived from 1S-2S transitions in atomic hydrogen after an interval of 2.8 years [7]. The quasar data analyzed in Refs. [1–3] consist of three separate samples of Keck-Hires observations which combine to give a data set of 128 objects at redshifts $0.5 < z < 3$. The many-multiplet technique finds that their absorption spectra are consistent with a shift in the value of the fine structure constant between these redshifts and the present of $\Delta\alpha_{em}/\alpha_{em} \equiv [\alpha_{em}(z) - \alpha_{em}]/\alpha_{em} = -0.57 \pm 0.10 \times 10^{-5}$, where $\alpha_{em} \equiv \alpha_{em}(0)$ is the present value of the fine structure constant [1–3]. Extensive analysis has yet to find a selection effect that can explain the sense and magnitude of the relativistic line shifts underpinning these deductions. Further observational studies have been published in Refs. [8,9] using a different but smaller data set of 23 absorption systems in front of 23 VLT-UVES quasars at $0.4 \leq z \leq 2.3$ and have been analyzed using an approximate form of the many-multiplet analysis techniques introduced in Refs. [1–3]. They obtained $\Delta\alpha_{em}/\alpha_{em} \equiv -0.6 \pm 0.6 \times 10^{-6}$, a figure that disagrees with the results of Refs. [1–3]. However, reanalysis is needed in order to understand the accuracy being claimed. Other observational studies of lower sensitivity have also been made using OIII emission lines of galaxies and quasars. The analysis of data sets of 42 and 165 quasars from the Sloan Digital Sky Survey gave the constraints $\Delta\alpha_{em}/\alpha_{em} \equiv 0.51 \pm 1.26 \times 10^{-4}$

and $\Delta\alpha_{em}/\alpha_{em} \equiv 1.2 \pm 0.7 \times 10^{-4}$, respectively, for objects in the redshift range $0.16 \leq z \leq 0.8$ [10]. Observations of a single quasar absorption system at $z = 1.15$ by Quast *et al.* [11] gave $\Delta\alpha_{em}/\alpha_{em} \equiv -0.1 \pm 1.7 \times 10^{-6}$, and observations of an absorption system at $z = 1.839$ by Levshakov *et al.* [12] gave $\Delta\alpha_{em}/\alpha_{em} \equiv 4.3 \pm 7.8 \times 10^{-6}$. A preliminary analysis of constraints derived from the study of the OH microwave transition from a quasar at $z = 0.2467$, a method proposed by Darling [13], has given $\Delta\alpha_{em}/\alpha_{em} \equiv 0.51 \pm 1.26 \times 10^{-4}$ [14]. A comparison of redshifts measured using molecules and atomic hydrogen in two cloud systems by Drinkwater *et al.* [15] at $z = 0.25$ and $z = 0.68$ gave a bound of $\Delta\alpha_{em}/\alpha_{em} < 5 \times 10^{-6}$ and an upper bound on spatial variations of $\delta\alpha_{em}/\alpha_{em} < 3 \times 10^{-6}$ over 3 Gpc at these redshifts. A new study comparing UV absorption redshifted into the optical with redshifted 21 cm absorption lines from the same cloud in a sample of 8 quasars by Tzanavaris *et al.* [16]. This comparison probes the constancy of $\alpha^2 g_p m_e/m_p$ and gives $\Delta\alpha_{em}/\alpha_{em} \equiv 0.18 \pm 0.55 \times 10^{-5}$ if we assume that the electron-proton mass ratio and proton g -factor, g_p , are both constant.

Observational bounds derived from the microwave background radiation structure [17] and big bang nucleosynthesis [18] are not competitive at present (giving $\Delta\alpha_{em}/\alpha_{em} \lesssim 10^{-2}$ at best at $z \sim 10^3$ and $z \sim 10^9\text{--}10^{10}$) with those derived from quasar studies, although they probe much higher redshifts.

Other bounds on the possible variation of the fine structure constant have been derived from geochemical studies, although they are subject to awkward environmental uncertainties. The resonant capture cross section for thermal neutrons by samarium-149 about two billion years ago ($z \approx 0.15$) in the Oklo natural nuclear reactor has created a samarium-149:samarium-147 ratio at the reactor site that is depleted by the capture process $^{149}\text{Sm} + n \rightarrow ^{150}\text{Sm} + \gamma$ to an observed value of only about 0.02 compared to the value of about 0.9 found in normal samples of samarium. The need for this capture resonance to be in place two billion years ago at an energy level within about 90 meV of its current value leads to very strong bounds on all interaction coupling constants that contribute to the energy

level, as first noticed by Shlyakhter [19,20]. The latest analyses by Fujii *et al.* [21] allow two solutions (one consistent with no variation, the other with a variation) because of the double-valued form of the capture cross section's response to small changes in the resonance energy over the range of possible reactor temperatures: $\Delta\alpha_{em}/\alpha_{em} \equiv -0.8 \pm 1.0 \times 10^{-8}$ or $\Delta\alpha_{em}/\alpha_{em} \equiv 8.8 \pm 0.7 \times 10^{-8}$. The latter possibility does not include zero but might be excluded by further studies of other reactor abundances. Subsequently, Lamoureux [22] has argued that a better (non-Maxwellian) assumption about the thermal neutron spectrum in the reactor leads to a 6σ lower bound on the variation of $\Delta\alpha_{em}/\alpha_{em} > 4.5 \times 10^{-8}$ at $z \approx 0.15$.

Studies of the effects of varying a fine structure constant on the β -decay lifetime was first considered by Peebles and Dicke [23] as a means of constraining allowed variations in α_{em} by studying the ratio of rhenium to osmium in meteorites. The β -decay $^{187}_{75}\text{Re} \rightarrow ^{187}_{76}\text{Os} + \bar{\nu}_e + e^-$ is very sensitive to α_{em} and the analysis of new meteoritic data together with new laboratory measurements of the decay rates of long-lived beta isotopes has led to a time-averaged limit of $\Delta\alpha_{em}/\alpha_{em} = 8 \pm 16 \times 10^{-7}$ [24] for a sample that spans the age of the solar system ($z \leq 0.45$). Both the Oklo and meteoritic bounds are complicated by the possibility of simultaneous variations of other constants which contribute to the energy levels and decay rates (for reviews, see Refs. [25,26]). They also apply to environments within virialized structures that do not take part in the Hubble expansion of the universe and so it is not advisable to use them in conjunction with astronomical information from quasars without a theory that links the values of α_{em} in the two different environments that differ in density by a factor of $O(10^{30})$.

In order to interpret these investigations it is essential to be in possession of a self-consistent theory of the space-time variation of α_{em} , analogous to the Brans-Dicke theory [27] for the variation of G , from which to derive further observational consequences of any inferred variation. In the past, in the absence of any such theory, there has been a tendency to produce limits on variations of couplings other than α_{em} by simply "writing in" a time variation of the coupling into the equations that hold when it is constant. Many of the quoted experimental bounds on the variation of nongravitational "constants" that appear in the literature have been arrived at in this way. Also questionable is the use of laboratory bounds to limit possible variations of constants on extragalactic scales without any theory of the link between the two domains of variation. Detailed discussions of this problem when G and α vary have been made in Refs. [28,29]. A further problem is the likelihood that if one coupling constant, like α_{em} , varies then others will vary also, especially in the presence of any pattern of unification [15,30]. Most deductions of bounds on constants assume that only one constant is varying.

In order to deal with these problems of simultaneous variation and spatial variation consistently, it is necessary to have self-consistent theories in which constants vary through the dynamics of scalar fields, which gravitate and conserve energy and momentum. Sandvik, Barrow and Magueijo (SBM) [31] have extended Bekenstein's (B) [32] original generalization of Maxwell's theory to include general relativity. This resulting BSBM theory provides a framework for studying varying α_{em} , and can be easily generalized to study the simultaneous variation of α_{em} and G , as in Ref. [33]. A number of detailed cosmological studies of the behavior of this theory have been made in Refs. [31,34–38]. A further step has recently been taken by Kimberly and Magueijo [39] who have extended the BSBM theory to create a generalization of the Glashow-Salam-Weinberg theory to allow variation of the electromagnetic and weak couplings. This allows the consequences of electroweak unification to be investigated self-consistently for the first time. This approach has also been applied to the strong interaction alone by Chamoun *et al.* [40] and it is likely that a further step could be taken to investigate the consequences of the simultaneous variation of all gauge couplings in a grand unified theory. In this paper we determine the quantum corrections to the Kimberly-Magueijo unified electroweak models and formulate the propagation equations for the two scalar fields that carry the allowed time variations in the electromagnetic and weak couplings in a universe populated by the particle spectrum of the standard model.

II. THE SIMPLEST MODEL

There has been strong interest in a class of scalar theories where $\alpha_{em} = e^2/\hbar c$ can vary [31,32,34–38,41–44]. These theories reduce to Maxwell's equations and general relativity in the limiting case of no variation in the fine structure constant. In order to evaluate the constraints introduced by a program of unification, it is important to extend these simple models to include electroweak and grand unification. Kimberly and Magueijo [39] have proposed an extension to the Glashow-Salam-Weinberg (GSW) electroweak theory in which the weak couplings can also vary in space and time and which reduces to the standard GSW theory in the limit of no variation.

A. A single-dilaton theory

The first model (KM-I) contains a single dilaton and allows both $\alpha_W := g_W^2$ and α_{em} to vary but their ratio, and hence the mixing angle θ_W , are true constants. In this model the electroweak sector of the standard model is described by the following Lagrangian density:

$$\begin{aligned} \mathcal{L}_{\text{KM-I}} = & -\frac{1}{4}e^{-2\varphi}[\mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} + y_{\mu\nu}y^{\mu\nu}] + (D_\mu\Phi)^\dagger \\ & \times (D^\mu\Phi) + \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2 - \frac{\omega}{2}\varphi_{,\mu}\varphi^{,\mu}, \end{aligned} \quad (1)$$

where

$$\mathbf{w}_{\mu\nu} = 2\mathbf{w}_{[\nu,\mu]} - \bar{g}_W \mathbf{w}_\mu \wedge \mathbf{w}_\nu, \quad (2)$$

$$y_{\mu\nu} = 2y_{[\nu,\mu]}, \quad (3)$$

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} \bar{g}_W \mathbf{t} \cdot \mathbf{w}_\mu - \frac{i}{2} \bar{g}_Y y_\mu \right) \Phi \quad (4)$$

and Φ is the Higgs field. The dilaton field is φ and ω is a dimensional parameter with units of $(\text{mass})^2$. Since this theory is perturbatively nonrenormalizable, we would like $\omega = \mathcal{O}(M_{pl}^2)$ so that the dilaton enters at the same level as gravity, and we are justified in ignoring any quantum fluctuations of the dilaton field when quantizing with respect to the other fields. The *auxiliary* gauge fields, \mathbf{w}_μ and y_μ , take values in the adjoint representations of $\mathfrak{su}(2)$ and $\mathfrak{u}(1)$ respectively. They are not the *physical* gauge fields, which will be denoted by capital letters, but are related to them by the transformations

$$\bar{g}_W \mathbf{w}_\mu = g_W \mathbf{W}_\mu, \quad (5)$$

$$\bar{g}_Y y_\mu = g_Y Y_\mu, \quad (6)$$

$$g_W = \bar{g}_W e^\varphi, \quad (7)$$

$$g_Y = \bar{g}_Y e^\varphi. \quad (8)$$

The distinction between the *physical* and *auxiliary* fields will only become important at the quantum level (see Sec. III below). When written in terms of these *auxiliary* gauge fields the covariant derivatives which act upon matter species are independent of φ . This makes it simpler to derive the classical field equations. The *physical* gauge couplings, g_W and g_Y , are dynamical whereas the *auxiliary* couplings, \bar{g}_W and \bar{g}_Y , are *arbitrary* constants. At tree-level the Fermi constant G_F and the fermion masses do not vary, whereas the W and Z boson masses do.

We will also define the physical field strength tensors by

$$\mathbf{W}_{\mu\nu} := \frac{\bar{g}_W}{g_W} \mathbf{w}_{\mu\nu}, \quad (9)$$

$$Y_{\mu\nu} := \frac{\bar{g}_Y}{g_Y} y_{\mu\nu}. \quad (10)$$

These field strengths reduce to the standard definitions of the weak and hypercharge field strengths with gauge couplings g_W and g_Y , respectively, whenever the dilaton field φ is constant.

B. A two-dilaton theory

In the second model (KM-II) proposed in Ref. [39] a second dilaton field is added. This results in the weak mixing angle θ_W becoming a dynamical quantity. The Lagrangian density of the electroweak sector in this model

is

$$\begin{aligned} \mathcal{L}_{\text{KM-II}} = & -\frac{1}{4} e^{-2\varphi} \mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} - \frac{1}{4} e^{-2\chi} y_{\mu\nu} y^{\mu\nu} \\ & + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{\lambda}{4} (\Phi^\dagger \Phi - v^2)^2 \\ & - \frac{\omega_1}{2} \varphi_{,\mu} \varphi^{,\mu} - \frac{\omega_2}{2} \chi_{,\mu} \chi^{,\mu}. \end{aligned} \quad (11)$$

The definitions (2)–(4) still hold, as do relationships (5) and (6), but the *physical* coupling constants are now related to their *auxiliary* values by the transformations

$$g_W = \bar{g}_W e^\varphi, \quad (12)$$

$$g_Y = \bar{g}_Y e^\chi, \quad (13)$$

$$\tan\theta_W = \left[\frac{\bar{g}_Y}{\bar{g}_W} := \tan\bar{\theta}_W \right] e^{\chi-\varphi}. \quad (14)$$

The dilaton fields are φ and χ and their respective dimensional scales are ω_1 and ω_2 . As remarked above, we would like $\omega_i \sim \mathcal{O}(M_{pl}^2)$. In accord with KM-I, G_F and the fermion masses are constant at tree-level whereas the boson masses are dynamical.

C. Symmetry breaking

At low energies and temperatures, the most important feature of the GSW electroweak model is that the $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian is spontaneously broken to $U(1)_{em}$ via the Higgs doublet Φ , assuming a vacuum expectation value Φ_0 , which minimizes its potential. At tree-level this value is

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (15)$$

A perturbative expansion about this vacuum can be written as

$$\Phi = \begin{pmatrix} 0 \\ v + \frac{H(x^\mu)}{\sqrt{2}} \end{pmatrix}. \quad (16)$$

Expanding out the kinetic Higgs term gives

$$\begin{aligned} (D_\mu \Phi)^\dagger (D^\mu \Phi) = & \frac{1}{2} H_{,\mu} H^{,\mu} + \frac{v^2 (\bar{g}_W)^2}{4} [(\mathbf{w}_\mu^1)^2 + (\mathbf{w}_\mu^2)^2] \\ & + \frac{v^2}{4} (\bar{g}_W \mathbf{w}_\mu^3 - \bar{g}_Y y_\mu)^2. \end{aligned} \quad (17)$$

When written in terms of the *physical* gauge fields, the broken-phase boson fields of both KM-I and KM-II are given by the usual formulas:

$$W_\mu^\pm := \frac{1}{\sqrt{2}} (\mathbf{W}_\mu^1 \pm i \mathbf{W}_\mu^2), \quad (18)$$

$$Z_\mu := \frac{g_W \mathbf{W}_\mu^3 - g_Y Y_\mu}{\sqrt{g_W^2 + g_Y^2}}, \quad (19)$$

$$A_\mu := \frac{g_Y \mathbf{W}_\mu^3 + g_W Y_\mu}{\sqrt{g_W^2 + g_Y^2}}. \quad (20)$$

Hence, the tree-level boson masses and their dilaton field dependence can read off from (17):

$$M_W = \frac{v}{\sqrt{2}} g_W, \quad (21)$$

$$M_Z = \frac{v}{\sqrt{2}} \sqrt{g_W^2 + g_Y^2}, \quad (22)$$

$$M_A = 0. \quad (23)$$

D. Classical field equations

In order to make and test the predictions of these theories we need to know the field equations. In both KM-I and KM-II, the Einstein and Yang-Mills equations are

$$G_{\mu\nu} = 8\pi G \left\{ \frac{\bar{g}_W^2}{g_W^2} T_{\mu\nu}^w + \frac{\bar{g}_Y^2}{g_Y^2} T_{\mu\nu}^y + T_{\mu\nu}^\phi + T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{dilaton}} \right\}, \quad (24)$$

$$\mathbf{D}^\mu \left(\frac{\bar{g}_W^2}{g_W^2} \mathbf{w}_{\mu\nu} \right) = - \frac{\delta \mathcal{L}_{\text{matter}}}{\delta w^\nu}, \quad (25)$$

$$\partial^\mu \left(\frac{\bar{g}_Y^2}{g_Y^2} y_{\mu\nu} \right) = - \frac{\delta \mathcal{L}_{\text{matter}}}{\delta y^\nu}, \quad (26)$$

where $T_{\mu\nu}^w$ and $T_{\mu\nu}^y$ are the standard Yang-Mills energy-momentum tensors written in terms of the *auxiliary* fields and couplings. In KM-I the dilaton conservation equation is

$$\begin{aligned} \square \varphi &= - \frac{1}{2\omega} e^{-2\varphi} (\mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} + y_{\mu\nu} y^{\mu\nu}) \\ &= - \frac{1}{2\omega} (\mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} + Y_{\mu\nu} Y^{\mu\nu}). \end{aligned} \quad (27)$$

and in KM-II we have conservation equations for the two fields:

$$\square \varphi = - \frac{1}{2\omega_1} e^{-2\varphi} \mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} = - \frac{1}{2\omega_1} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu}, \quad (28)$$

$$\square \chi = - \frac{1}{2\omega_2} e^{-2\chi} y_{\mu\nu} \cdot y^{\mu\nu} = - \frac{1}{2\omega_2} Y_{\mu\nu} Y^{\mu\nu}. \quad (29)$$

The conservation equations for other matter fields, like perfect fluids, are unchanged. Although this system represents the full classical field equations, their current form is

not very useful. In order to do cosmology, and make experimentally testable predictions, we need to understand how the right-hand sides of (27)–(29) depend on macroscopic quantities such as the background matter density and the value of the dilaton field. It is this question that we address in the next section.

III. THE DILATON-TO-MATTER COUPLING

Most tests which we might wish to apply to these, and other, varying-constant theories require knowledge of how the dilaton fields will evolve in the presence of some background matter density ρ and pressure P :

$$\rho := \langle T_0^{0(\text{matter})} \rangle, \quad P := \frac{1}{3} \langle T_i^{i(\text{matter})} \rangle,$$

where $\langle \cdot \rangle$ denotes the quantum expectation. In order to understand the dilaton evolution, we must therefore evaluate the terms on the right-hand sides of (27)–(29) under the $\langle \cdot \rangle$ operation. Those terms all consist of terms quadratic in the Yang-Mills field strengths, i.e. $\mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu}$ and $Y_{\mu\nu} Y^{\mu\nu}$; henceforth we refer to them collectively as $F_{\mu\nu} F^{\mu\nu}$ or F^2 terms.

When we quantize this theory (leaving φ as a classical field) we must do so with respect to the *physical* gauge fields rather than the *auxiliary* ones, since it is only the physical fields whose kinetic terms possess the correct normalization. This feature is important if we want the renormalization procedure to go through as usual when $\varphi = \text{const}$. It is for this reason that we have labeled the capitalized fields as *physical*.

While the $SU(2)_L$ sector of GSW theory is a non-Abelian Yang-Mills theory, non-Abelian effects may be ignored when finding the leading-order g dependence of the F^2 terms so long as perturbative theory holds i.e. whenever $g \ll 1$. When this happens all non-Abelian Feynman graphs enter with a higher power of g than does the Abelian single vector boson exchange graph, and so, provided that perturbative theory is appropriate, we may ignore the non-Abelian graphs when finding the leading-order behavior. The non-Abelian effects represent some of the radiative corrections to the calculated results; at energies $\ll M_Z$, these will be small. The running of g_{weak} means that GSW theory remains in this sense perturbative at all energies. This is not the whole story since the Higgs couplings also run, and when this is taken into account nonperturbative behavior will occur at high energies. The precise energy where the onset of nonperturbative behavior occurs depends on the Higgs mass, M_H . For reasonable values of $M_H < 750$ GeV, however, this energy is still >1 TeV and hence well above M_Z . Hence our results are certainly true for all particles, mass m , where $m^2 \ll M_Z^2$; the $m^2 \gtrsim M_Z^2$ is given with the caveat that perturbation theory should still hold at energies $E = m$. For energies $E \ll M_H$ we may also ignore fluctuations in

the Higgs field, since they are suppressed by factors of the order of $\frac{F^2}{M_H^2}$.

A. Derivation

We need to understand how these $\langle F^2 \rangle$ terms depend, in a functional sense, on the dilaton fields and the background matter density ρ . For simplicity, we consider the case of a single Dirac fermion ψ of mass m , coupled to a $U(1)$ gauge field A_μ with Higgs scalar Φ and dilaton φ . The effective Lagrangian, including 't Hooft's gauge-fixing term, is

$$\mathcal{L} = \mathcal{L}_{\text{gauge},0} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\psi,0} + \mathcal{L}_{\text{int}} \quad (30)$$

$$\begin{aligned} \mathcal{L}_{\text{gauge},0} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left\| \left(\partial_\mu + i\frac{M_A}{v}A_\mu \right) \Phi \right\|^2 \\ & - \frac{\lambda}{4} \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 \end{aligned} \quad (31)$$

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi} (\partial^\mu A_\mu + \xi M_A \text{Im}(\Phi))^2 \quad (32)$$

$$\mathcal{L}_{\psi,0} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad (33)$$

$$\mathcal{L}_{\text{int}} = -e(\varphi)\bar{\psi}\gamma^\mu\psi A_\mu. \quad (34)$$

The ghost fields have been excluded since in this gauge they decouple from the Abelian sector. By taking a space-

time region \mathcal{R} , of volume V and temporal extent T , that is large compared to the scale of quantum fluctuations of the matter and gauge fields, and yet small when compared to the scale over which the dilaton field varies, we can quantize this theory inside \mathcal{R} by taking $\varphi = \text{const}$ and defining the partition function in the usual manner. For energies far below the Higgs mass m_H , we can ignore quantum Higgs fluctuations in the first approximation. Doing this and integrating out the gauge fields we determine that

$$\begin{aligned} -\frac{1}{4}\langle F_{\mu\nu}F^{\mu\nu} \rangle = & \frac{e^2(\varphi)}{2VT} \iiint d^4x d^4y d^4z \langle \partial_x^2 \mathcal{D}_{\mu\nu}^{\text{gauge}}(x-y) \\ & \times (\bar{\psi}\gamma^\nu\psi)(y) \mathcal{D}_\rho^{\mu\text{ gauge}}(x-z) (\bar{\psi}\gamma^\rho\psi)(z) \\ & + \partial_x^\mu \mathcal{D}_{\mu\nu}^{\text{gauge}}(x-y) (\bar{\psi}\gamma^\nu\psi) \\ & \times (y) \partial_x^\tau \mathcal{D}_{\tau\rho}^{\text{gauge}}(x-z) (\bar{\psi}\gamma^\rho\psi)(z) \rangle_\Psi, \end{aligned} \quad (35)$$

where the gauge-field propagator is given by

$$\begin{aligned} \mathcal{D}_{\mu\nu}^{\text{gauge}}(x) = & \int \frac{d^4k}{(2\pi)^4} e^{ikx} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi M_A^2} \right] \\ & \times \frac{1}{k^2 - M_A^2}. \end{aligned}$$

The quantum expectation operator $\langle \cdot \rangle_\Psi$, used in Eq. (35), is defined thus:

$$\langle A(\psi, \bar{\psi}) \rangle_\Psi := \frac{\int [d\bar{\psi}][d\psi] A(\psi, \bar{\psi}) \exp(-i\frac{e^2}{2} \iint d^4x d^4y J^\nu(x) \mathcal{D}^{\text{gauge}}(x-y) J^\nu(y)) e^{i \int d^4x \mathcal{L}_{\psi,0}}}{\int [d\bar{\psi}][d\psi] \exp(-i\frac{e^2}{2} \iint d^4x d^4y J^\nu(x) \mathcal{D}^{\text{gauge}}(x-y) J^\nu(y)) e^{i \int d^4x \mathcal{L}_{\psi,0}}}. \quad (36)$$

The $\int [d\psi]$ represents a functional integral with respect to the ψ -field, and we have defined $J^\mu(x) := \bar{\psi}(x)\gamma^\mu\psi(x)$. The term with an exponent that is quadratic in the J 's in Eq. (36) encodes the self-energy corrections to the fermion and gauge boson propagators. By writing both the fermion and gauge boson propagators as full propagators we are justified in dropping the aforementioned exponential term because its effect is of subleading order. Finally then, by writing this expression in momentum space we reduce it to the more transparent and manifestly gauge invariant form:

$$\begin{aligned} -\frac{1}{4}\langle F_{\mu\nu}F^{\mu\nu} \rangle = & \frac{e^2(\varphi)}{2VT} \int \frac{d^4p}{(2\pi)^4} p^2 [(p^2 - M_A^2(\varphi))g^{\mu\nu} \\ & + \Pi_A^{\mu\nu}(p, e^2(\varphi))]^{-2} \langle \tilde{J}^\mu(p) \tilde{J}^\nu(-p) \rangle_{\Psi^*}, \end{aligned} \quad (37)$$

where \tilde{J}_μ is the momentum space representation of J^μ and $\Pi_A^{\mu\nu}$ is the vacuum polarization of the gauge boson. The expectation here is defined with respect to the partition function

$$Z_{\Psi^*} := \int [d\bar{\psi}][d\psi] \exp\left(i \int d^4x d^4y \bar{\psi}(x) K(x-y) \psi(y)\right)$$

with $K(x-y)$ denoting the inverse of the full fermion propagator.

B. Interpretation

By Wick's theorem it is apparent that the remaining quantum expectation in (37) contains two distinct contributions. The first contribution will always involve boson exchange between two fermionic particles and so is proportional to $(\rho - 3P)^2/m^2$ to leading order. The second contribution results from a single fermionic particle admitting and reabsorbing a gauge boson (i.e. the process that results in the fermion's self-energy). This second term will clearly be proportional to $m^2(\rho - 3P)$ to leading order. In almost all cases of experimental interest $(\rho - 3P)/m^4 \ll 1$. Only in objects whose density approaches that of nuclear matter does it fail to hold and this naive quantization procedure is not suitable for dealing with such high-density backgrounds. For this reason we drop the contribution due to photon exchange.

When the perturbation theory holds it is appropriate to expand (37) as a series in the gauge coupling $e^2(\varphi)$, giving

$$-\frac{1}{4}\langle F_{\mu\nu}F^{\mu\nu} \rangle = e^2\zeta(\varphi)(\rho - 3P)(1 + \mathcal{O}(e^2 \ln e))\left(1 + \mathcal{O}\left(\frac{\rho}{m^4}\right)\right) \quad (38)$$

where, for this model, ζ is defined by

$$\zeta(\varphi)(\rho - 3P) = \iint \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^2} \frac{p^2}{(p^2 - M_A^2(\varphi))^2} \text{tr}\left(\gamma^\nu \frac{q \cdot \gamma + m}{q^2 - m} \gamma_\nu \frac{(p - q) \cdot \gamma + m}{(p - q)^2 - m}\right). \quad (39)$$

When interfermion strong interactions are introduced, confinement may occur. If this happens the trace factor in (39) will take on a more complicated form. At leading order, however, it will remain independent of φ so long as Λ_{QCD} is. At densities much lower than the nucleon mass, the right-hand side will be proportional to the hadronic energy density.

C. The unbroken symmetry case: $M_A = 0$

It is helpful to have a physical interpretation of the ζ parameter. When dynamical symmetry breaking does not occur (and so we have $M_A^2 = 0$) the right-hand side of (39) is proportional to the 1-loop fermion field self-energy resulting from its interaction with the A_μ gauge field. Defining this self-energy to be $\delta m^2(\varphi)$ we have

$$\zeta = \frac{\delta m^2(\varphi)}{e^2(\varphi)m^2} = \frac{\delta \bar{m}^2}{\bar{e}^2 m^2} = \text{const.} \quad (40)$$

In this case ζ is φ -independent at leading order. $\delta \bar{m}$ is defined as the electromagnetic mass-correction when the electric charge is some value $\bar{e} = \text{const.}$

D. The broken symmetry case: $M_A \neq 0$

When $M_A^2 \neq 0$ the above interpretation of ζ in terms of the self-energy mass-correction will, in general, fail. The correct physical interpretation will depend heavily on the size of $\frac{m}{M_A(\varphi)}$. We consider the three cases:

- (i) $\frac{m}{M_A} \gg 1$. The dominant contribution to the ζ integral will come from momenta $p^2 \gg M_A^2$, and so we can ignore M_A at leading order and interpret ζ just as we did in the $M_A = 0$ case. Hence, ζ is dilaton independent to leading order.
- (ii) $\frac{m}{M_A} \ll 1$. The dominant contribution to the (39) will come from momenta $p^2 \approx m^2$. Hence

$$\zeta(\varphi) = \left(\frac{m}{M_A(\varphi)}\right)^4 \zeta_0, \quad (41)$$

where ζ_0 is of the order the value of ζ which we would have had if $M_A = 0$. In this case ζ will vary with φ like $\frac{1}{M_A^4}$.

- (iii) $\frac{m}{M_A} = \mathcal{O}(1)$. This is by far the most difficult case to analyze. In general, ζ will have a leading-order dependence on the dilaton field through $M_A(\varphi)$, but the precise form $\zeta(\varphi)$ will depend on the nature of the trace term in Eq. (39). This in turn rests on

the precise details of the microscopic physical model for the matter fields in question. This chain of complications means that a general prescription for the dilaton dependence of ζ , in this case, is not possible.

IV. APPLICATIONS TO THE KIMBERLY-MAGUEIJO MODELS

A. Ultrarelativistic matter

At the very high energies required to restore the broken gauge symmetry in these theories, most matter species will be ultrarelativistic and we will assume this to be the case for all species. We will also assume that there is a discrete spectrum of spin states. Matter will therefore behave like blackbody radiation with $\rho = 3P$. It is clear from Eq. (38) that these ultrarelativistic species do not contribute to the $\langle F^2 \rangle$ terms which source the dilaton fields' evolution.

The only uncharged fundamental field is the neutrino, which we will assume to be so light that it remains relativistic at experimental and cosmological temperatures. Such neutrinos make only a very small contribution to the dilaton source terms. The corollary of this is that, among the known fundamental matter species, we are justified in assuming that only those with nonzero charge contribute to the right-hand sides of Eqs. (27), (29), and (29).

The seesaw mechanism for neutrino mass-generation results in three light neutrinos, masses $m_\nu^{(i)}$ (corresponding to the currently observed particles) and three very heavy neutrinos, masses $M_N^{(i)} \approx$ a few tens of GeV. The light neutrinos are formed mostly from the weakly interacting left-handed components, while the heavy neutrinos are primarily composed of the weak singlet right-handed particles. Such heavy neutrinos would therefore interact with the weak bosons much more weakly than other massive particle species. To zeroth order in $m_\nu^{(i)}/M_N^{(i)} \ll 1$, we can assume these heavy neutrinos to be noninteracting with either electromagnetic or weakly interacting particles. Their contribution to the $\langle F^2 \rangle$ terms will be negligible compared to that of the other matter species.

B. The top quark

With a mass of about 180 GeV, the top quark is the only fermionic species present in the standard model that does

not fall into the $\frac{m}{M_W} \ll 1$ category. While in principle its contribution to the dilaton terms could be calculated, it would be a difficult procedure which would require accounting for gauge boson self-interactions, Higgs boson fluctuations, and nonperturbative effects—as well as having to do QCD calculations. However, the results would actually have very little bearing upon most cosmological tests of this theory, since the background energy density of top quarks is entirely negligible when compared to that of all the other forms of matter.

C. “Light” charged matter

We have just argued that, among the standard model particle species, the only non-negligible contributions to the dilaton source terms will come from particles with mass m and charge $Q \neq 0$, which are light compared to M_W . We must also require the particles to be relativistic, allowing us to restrict to the low-energy broken symmetric phase of the KM electroweak theories. This is appropriate for energies well below the 100 GeV level. At these energies, perturbation theory will be appropriate and non-Abelian effects will be of subleading order. In this case then, the results of Sec. II should be valid.

Writing the $\langle \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} \rangle$ and $\langle Y_{\mu\nu} \cdot Y^{\mu\nu} \rangle$ quantities in terms of the low-energy physical fields, we find

$$\begin{aligned} \langle -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} \rangle &= [\langle -\frac{1}{2} F_{W\pm}^\dagger \cdot F_{W\pm} \rangle + \sin^2 \theta_W \langle -\frac{1}{4} F_{em}^2 \rangle \\ &\quad + \cos^2 \theta_W \langle -\frac{1}{4} F_Z^2 \rangle \\ &\quad + \sin \theta_W \cos \theta_W \langle -\frac{1}{2} F_{em} \cdot F_Z \rangle] \\ &\quad \cdot [1 + \mathcal{O}(\partial_\mu \theta_W) + \mathcal{O}(g_W, g_Y)] \end{aligned} \quad (42)$$

$$\begin{aligned} \langle -\frac{1}{4} Y_{\mu\nu} \cdot Y^{\mu\nu} \rangle &= [\cos^2 \theta_W \langle -\frac{1}{4} F_{em}^2 \rangle + \sin^2 \theta_W \langle -\frac{1}{4} F_Z^2 \rangle \\ &\quad - \sin \theta_W \cos \theta_W \langle -\frac{1}{2} F_{em} \cdot F_Z \rangle] \\ &\quad \cdot [1 + \mathcal{O}(\partial_\mu \theta_W)]. \end{aligned} \quad (43)$$

The $\mathcal{O}(\partial_\mu \theta_W)$ terms produce only negligible corrections. The $\mathcal{O}(g_W, g_Y)$ symbol represents the additional terms that arise from gauge boson self-interactions; these only contribute at subleading order.

Thus for a nonrelativistic particle species ψ_i of mass $m_i \ll M_W$, charge $Q_i \neq 0$, and weak isospin t_{3i} , we see that only $\langle -\frac{1}{4} F_{em}^2 \rangle$ will contribute to (42) and (43) at leading order. The $\langle -\frac{1}{4} F_Z^2 \rangle$, $\langle -\frac{1}{4} |F_{W\pm}^2| \rangle$ and $\langle -\frac{1}{2} F_{em} \cdot F_Z \rangle$ terms are suppressed by relative factors of the order of $[g_W^2 / (Q_i^2 e^2 \cos^2 \theta_W)] (m_i^4 / M_W^4) \times (t_{3i} - 2Q_i \sin^2 \theta_W)^2$, $[g_W^2 / (Q_i^2 e^2)] (m_i^4 / M_W^4) t_{3i}^2$ and $[g_W / (Q_i e \cos \theta_W)] (m_i^2 / M_W^2) (t_{3i} - Q_i \sin^2 \theta_W)$, respectively. The leading-order contributions of such a light matter species to (42) and (43) therefore reduce to

$$\begin{aligned} \left\langle -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} \right\rangle_i &= \sin^2 \theta_W \frac{e^2}{\bar{e}^2} \frac{\delta \bar{m}_i^2}{m_i^2} \rho_i, \\ \left\langle -\frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} \right\rangle_i &= \cos^2 \theta_W \frac{e^2}{\bar{e}^2} \frac{\delta \bar{m}_i^2}{m_i^2} \rho_i, \end{aligned} \quad (44)$$

where $\delta \bar{m}$ is defined as the electromagnetic mass-correction when the electric charge is some value $\bar{e} = \text{const}$.

D. Dark matter

There is a great deal of evidence from cosmological and astronomical data for the existence of dark matter which contributes about 27% of the gravitating mass density of the universe. Such matter must be nonbaryonic, electromagnetically noninteracting and nonrelativistic (“cold”). It appears that the standard model lacks any suitable dark-matter candidates. Weakly interacting massive particles (WIMPs), such as the lightest putative supersymmetric partners in Minimally Supersymmetric Standard Model, are one possible class of candidates for dark matter. The masses of these particles tend to be of the order of a few tens of GeVs and so fall into the $m \approx M_W$ category. Locally, if dynamically virialized, they will have keV energies which may allow them to be detected in underground nuclear recoil experiments. They are necessarily uncharged ($Q \neq 0$) but they can however interact weakly and as such contribute to the $\langle -\frac{1}{4} F_Z^2 \rangle$ and $\langle -\frac{1}{4} |F_{W\pm}^2| \rangle$ terms. Thus the leading-order contribution from these WIMPs to (42) and (43) would be given by

$$\begin{aligned} \left\langle -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} \right\rangle_{\text{wimp}} &= \frac{g_W^2}{\bar{g}_W^2} \left[\mathcal{F}_W \left(\frac{m_{\text{wimp}}^2}{M_W^2} \right) \right. \\ &\quad \left. + \mathcal{F}_Z \left(\frac{m_{\text{wimp}}^2}{M_Z^2} \right) \right] \rho_{\text{wimp}}, \\ \left\langle -\frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} \right\rangle_{\text{wimp}} &= \frac{g_W^2}{\bar{g}_W^2} \tan^2 \theta_W \mathcal{F}_Z \left(\frac{m_{\text{wimp}}^2}{M_Z^2} \right) \rho_{\text{wimp}}, \end{aligned} \quad (45)$$

where we have defined \mathcal{F}_W and \mathcal{F}_Z to be WIMP-model dependent “structure” functions. These encode precisely how the WIMP’s ζ parameters depend on the gauge boson masses. We expect $|\mathcal{F}_W|$ and $|\mathcal{F}_Z|$ to be $\ll 1$. Not much more can be said about their structure as functions of the dilaton field without the aid of a microscopic model for dark matter. We shall therefore leave this discussion for a separate work. It should be noted also that, if \mathcal{F}_W and \mathcal{F}_Z are no more than an order of magnitude or so smaller than the ζ_{em} parameter of baryonic matter, then WIMP-based dark matter could well be the dominant factor in the cosmological evolution of the dilaton fields because the cosmological dark-matter density in WIMPs is an order of magnitude greater than that of baryonic matter.

V. DISCUSSION

In this paper we have derived the form of the effective dilaton field equations for an electroweak theory with varying couplings in the presence of a background matter density. It is this form that is of most use for experimental, observational and cosmological tests of this theory. We took particular care in the identification of the *physical* gauge fields. In previous work on BSBM theory, the dilaton dependence of the F^2 source term was incorrectly determined as a result of the auxiliary fields mistakenly taken to be the physically propagating modes. This error resulted in the statement:

$$\frac{\bar{e}^2}{e^2} \left\langle -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} \right\rangle = \frac{\bar{e}^2}{e^2} \frac{\delta \bar{m}^2}{m^2} = e^{-2\varphi} \zeta \rho, \quad (46)$$

where $f_{\mu\nu} = 2\partial_{[\mu} a_{\nu]}$ was the auxiliary field strength, and a_μ the auxiliary photon field. $\delta \bar{m}$ is defined as the electromagnetic mass-correction when the electric charge is some value $\bar{e} = \text{const}$. This led to the conclusion that the leading-order dilaton field dependence went like $\frac{\bar{e}^2}{e^2} = e^{-2\varphi}$. However, it is clear from Eqs. (39) and (40) that in reality the leading-order dilaton dependence should be $\frac{e^2}{\bar{e}^2} = e^{+2\varphi}$ and Eq. (46) should read

$$\left\langle -\frac{\bar{e}^2}{e^2} \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \right\rangle = \frac{e^2}{\bar{e}^2} \frac{\delta \bar{m}^2}{m^2} \rho = e^{2\varphi} \zeta \rho. \quad (47)$$

Indeed Eq. (46) is problematic because, in the limit of zero electric charge, matter decouples from the photon, and so cannot possibly contribute to the F_{em}^2 term. This situation corresponds to $\varphi \rightarrow -\infty$. However, in this limit, the F_{em}^2 term as given by the right-hand side of Eq. (46) grows infinity large. Equation (47) shows the correct behavior, and vanishes, as it should, at this limit.

We have also seen that if the gauge bosons become massive then, for particles which are much lighter than the gauge boson in question, the leading-order dilaton dependence of the F^2 term changes from g^2 (g is the physical gauge coupling) to $g^2/M_{\text{gauge}}^4 \sim \frac{1}{g^2}$. For particles that are much heavier than the gauge boson in question, provided perturbation theory is still valid at energies of the order of the particle's mass, the leading-order dilaton dependence remains as g^2 . We also briefly discussed the complication of $m_{\text{particle}} \approx M_{\text{gauge}}$, whereby the leading-order dilaton dependence will be highly susceptible to the details of the matter model in question, and noted that this effect might be important in cosmology if dark matter is weakly interacting.

In all of this analysis we assumed both that perturbation theory holds and that any non-Abelian effects are negligible. While this is true for electroweak theory at energies well below the Higgs boson mass, it will not be true for QCD. If we were to construct a BSBM-like varying $\alpha_{\text{strong}} = g_{\text{strong}}^2$ (or equivalently, varying Λ_{QCD}) theory, we would expect the leading-order dilaton dependence of

the F_{strong}^2 term to come from a complicated function of g_{strong} . Evaluating this function would, at the very least, require us to be able to predict quark and nucleon masses accurately via a QCD calculation which is not yet possible. In the absence of such calculations, varying Λ_{QCD} theories will be difficult to test accurately or make use of in the early universe except at the very highest energies. Fortunately, we do not encounter these problems in the electroweak KM theories.

We conclude with a statement of the effective dilaton field equations we have derived. In the KM-I theory these are

$$\square \varphi = \frac{2}{\omega} e^{2\varphi} \sum_i \zeta_i \rho_i + \frac{2}{\omega} e^{2\varphi} \left[\mathcal{F}_W \left(\frac{m_{\text{wimp}}^2}{M_W(\varphi)^2} \right) + \sec^2 \theta_W \mathcal{F}_Z \left(\frac{m_{\text{wimp}}^2}{M_Z(\varphi)^2} \right) \right] \rho_{\text{wimp}}, \quad (48)$$

where the sum is over all charged matter species (with $m \ll M_W$). The first term is identical to the properly evaluated source term in BSBM. Hence the effective KM-I theory differs (at low energies and densities) from BSBM only in the putative WIMP matter contribution. If we transform $\varphi \rightarrow -\varphi$ then we can read off the explicit correspondence with the studies in Refs. [28,29,31,34–38]. When $\zeta/\omega < 0$ we obtain slow logarithmic growth of the fine structure constant during the dust dominated era, as $\varphi \propto \ln[\ln(t + t_0)]$, t_0 constant but constant- φ behavior during the radiation and dark-energy dominated eras. If $\zeta/\omega > 0$ then the solutions predict a much stronger evolution of α with time that is difficult to reconcile with the observational constraints. The sign of ζ/ω is controlled by the sign of $\zeta \in [-1, 1]$ and of ω . Positive ζ corresponds to “normal” matter which is dominated by electrostatic contributions, whereas negative ζ corresponds to matter (like superconducting cosmic strings which have $\zeta = -1$) that is dominated by magnetic energy. Positive ω corresponds to a positive kinetic contribution to the energy by the φ field while negative ω indicates that it is a ghost field, as in Ref. [45].

In the KM-II theory the effective dilaton equations are

$$\square \varphi = \frac{2}{\omega_1} \sin^2 \theta_W \frac{\alpha_{em}(\varphi, \chi)}{\bar{\alpha}_{em}} \sum_i \zeta_i \rho_i + \frac{2}{\omega_1} e^{2\varphi} \left[\mathcal{F}_W \left(\frac{m_{\text{wimp}}^2}{M_W(\varphi)^2} \right) + \mathcal{F}_Z \left(\frac{m_{\text{wimp}}^2}{M_Z(\varphi, \chi)^2} \right) \right] \rho_{\text{wimp}} \quad (49)$$

$$\square \chi = \frac{2}{\omega_2} \cos^2 \theta_W(\varphi, \chi) \frac{\alpha_{em}(\varphi, \chi)}{\bar{\alpha}_{em}} \sum_i \zeta_i \rho_i + \frac{2}{\omega_2} e^{2\chi} \tan^2 \bar{\theta}_W \mathcal{F}_Z \left(\frac{m_{\text{wimp}}^2}{M_Z(\varphi, \chi)^2} \right) \rho_{\text{wimp}}. \quad (50)$$

A similar menu of possibilities exists for the sign of the leading term on the right-hand side of the $\square\varphi$ and $\square\chi$ equations as was the case for the KM-I theory discussed above. In the absence of the permitted dark-matter contributions, this two-dilaton theory will only reduce to BSBM when $\omega_2\sin^2\theta_W = \omega_1\cos^2\theta_W$. In all other cases θ_W will vary and lead to an evolution of α_{em} that is different from

that of BSBM theory. Further cosmological consequences of these results will be explored elsewhere.

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