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Evolution of the Fine Structure Constant Driven by Dark Matter and the Cosmological Constant

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Abstract

Bekenstein's model of a scalar field, ϕ , that affects the electromagnetic permeability (usually identified with "changing α ") predicts tiny variations of the effective fine structure constant up to very high red-shifts, $|\alpha(z = 3.5)/\alpha_0 - 1| < 10^{-10}$, when the constraints from Eötvös-Dicke-Braginsky types of experiments are imposed. We generalize this model by allowing additional couplings of ϕ to both a dark matter candidate and to the cosmological constant. We show that in a supersymmetric generalization of Bekenstein's model, the coupling to the LSP, which is assumed to contribute significantly to the dark matter density, can be up to six orders of magnitude stronger than the coupling to the baryon energy density. This allows one to evade the present limits on the non-universality of the gravitational attraction due to ϕ -exchange and at the same time accommodate the effective shift in α at the level of $\alpha(z = 3.5)/\alpha_0 - 1 \sim 10^{-5}$, reported recently from observations of quasar absorption spectra.

1 Introduction

Speculations that fundamental constants may vary in time and/or space go back to the original idea of Dirac [1]. Despite the reputable origin, this idea has not received much attention during the last fifty years for the two following reasons. First, there exist various sensitive experimental checks that coupling constants do not change (See, e.g. [2]). Second, for a long time there has not been any credible theoretical framework which would predict such changes.

Our theoretical mindset, however, has changed since the advent of the string theory. One of the most interesting low-energy features of string theory is the possible presence of a massless scalar particle, the dilaton, whose vacuum expectation value defines the size of the effective gauge coupling constants. A change in the dilaton v.e.v. induces a change in the fine structure constant as well as the other gauge and Yukawa couplings. The stabilization of the dilaton v.e.v., which usually renders the dilaton massive, represents one of the fundamental challenges to be addressed before string theory can aspire to describe the observable world. Besides the dilaton, string theory often predicts the presence of other massless or nearly massless moduli fields, whose existence may influence particle physics and cosmology and may also change the effective values of the coupling constants as well.

Independent of the framework of string theory, Bekenstein [3] formulated a dynamical model of “changing α ”. The model consists of a massless scalar field which has a linear coupling to the F^2 term of the $U(1)$ gauge field, $M_*^{-1}\phi F_{\mu\nu}F^{\mu\nu}$, where M_* is an associated mass scale and thought to be of order the Planck scale. A change in the background value of ϕ , can be interpreted as a change of the effective coupling constant. Bekenstein noticed that F^2 has a non-vanishing matrix element over protons and neutrons, of order $(10^{-3} - 10^{-2})m_N$. This matrix element acts as a source in the ϕ equation of motion and naturally leads to the cosmological evolution of the ϕ field driven by the baryon energy density. Thus, the change in ϕ translates into a change in α on a characteristic time scale comparable to the lifetime of the Universe or larger. However, the presence of a massless scalar field ϕ in the theory leads to the existence of an additional attractive force which does not respect Einstein’s weak universality principle. The extremely accurate checks of the latter [4] lead to a firm lower limit on M_* , $M_*/M_{\text{Pl}} > 10^3$ that confines possible changes of α to the range $\Delta\alpha < 10^{-10} - 10^{-9}$ for $0 < z < 5$ [3, 5].

This range is five orders of magnitude tighter than the change $\Delta\alpha/\alpha \simeq 10^{-5}$ indicated in the observations of quasar absorption spectra at $z = 0.5 - 3.5$ and recently reported by Webb et al. [6]. Given the potential fundamental importance of such a result, one should remain cautious until this result is independently verified. Nevertheless, leaving aside the issue regarding the reliability of the conclusions reached by Webb et al. [6], it is interesting to explore the possibility of constructing a dynamical model, including

modifications of Bekenstein’s model, which could produce a large change in α in the redshift range $z = 0.5 - 3.5$ and still be consistent with the constraints on $\Delta\alpha/\alpha$ from the results of high-precision limits on the violation of equivalence principle by a fifth force. It is also interesting to study whether the range $\Delta\alpha/\alpha \simeq 10^{-5}$ could be made consistent with the limits on $\Delta\alpha/\alpha$ [7]-[10], extracted from the analysis of element abundances from the Oklo phenomenon, a natural nuclear fission reactor that occurred about 1.8 billion years ago. We note that while big bang nucleosynthesis provides limits on much longer timescales, these limits are typically quite weak, $\Delta\alpha/\alpha \sim 10^{-2}$ [11].

The gap of five orders of magnitude between the desirable range of 10^{-5} and the bounds of order 10^{-10} appear to be insurmountable for any sensible modification of Bekenstein’s theory¹. In this paper, we propose a modification of Bekenstein’s idea consistent with experimental constraints, but relies on a large coupling between the non-baryonic dark matter energy density and the ϕ field.

At first, such a coupling may appear strange. Indeed, why should dark matter interact with the ϕ field when it is known that dark matter particles are not charged [13] and their electromagnetic form-factors are also tightly constrained [14]? It turns out that in certain classes of models for dark matter, and in supersymmetric models in particular, it is natural to expect that ϕ would couple more strongly to dark matter particles than to baryons. It is easy to demonstrate this idea by a simple supersymmetrization of Bekenstein’s interaction. In addition to the coupling of ϕ to the kinetic term, F^2 , of the gauge boson, ϕ will acquire an additional coupling to the kinetic term of the gaugino, $M_*^{-1}\phi\bar{\chi}\not{\partial}\chi$. If this gaugino constitutes a significant fraction of the stable LSP neutralino, as is often the case, the source of ϕ due to the energy density of dark matter turns out to be dramatically enhanced compared to the baryonic source,

$$\frac{\text{Dark matter source}}{\text{baryonic source}} \sim (10^2 - 10^3) \frac{\Omega_{\text{matter}}}{\Omega_{\text{baryon}}} \sim 10^3 - 10^4. \quad (1.1)$$

Such an enhancement factor compensates, although not entirely, for the tremendous suppression of $\Delta\alpha$ once the Eötvös-Dicke-Braginsky (EDB) limits on M_* are imposed. It is then reasonable to study this class of models in further detail as they are numerically much more promising than the original Bekenstein framework.

We note that there is another possible “strategy” to avoid the EDB constraint. One can assume the existence of some extremal value ϕ_{ext} , in the vicinity of which only $(\phi - \phi_{\text{ext}})^2$ couples to F^2 . This type of coupling was advocated in Ref. [15]. If the cosmological evolution drives ϕ close to ϕ_{ext} now [15], i.e. at $z = 0$, the EDB constraints will be relaxed.

We organize this paper as follows. In the next section we generalize the original Bekenstein model. In section 3, we solve the field equation for the scalar field ϕ and obtain the

¹A recent publication claiming that the 10^{-5} change in α is realistic in this framework [12] does not impose the limits from Eötvös-Dicke-Braginsky experiments.

predictions for the change of α . In the same section, we impose experimental constraints and compare the results for $\Delta\alpha$ with the range suggested by Webb et al. [6]. In section 4, we consider predictions for $\Delta\alpha$ in some specific models and demonstrate one model that passes all constraints. In section 5, we analyze the class of models with quadratic couplings to F^2 . Our conclusions are presented in section 6.

2 Generalization of Bekenstein's model

We start our analysis by formulating a generic action that includes spin-2 gravity, kinetic and potential terms of a modulus ϕ , kinetic terms for the electromagnetic field and baryons as well as the dark matter action,

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_{\text{Pl}}^2 R + \frac{1}{2} M_*^2 \partial_\mu \phi \partial^\mu \phi - M_{\text{Pl}}^2 \Lambda_0 B_\Lambda(\phi) - \frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu} \right. \\ \left. + \sum_{i=p,n} \bar{N}_i (i \not{D} - m_i B_{N_i}(\phi)) N_i + \frac{1}{2} \bar{\chi} \not{\partial} \chi - \frac{1}{2} M_\chi B_\chi(\phi) \chi^T \chi \right] + \dots \quad (2.1)$$

Throughout this paper we assume a $+ - - -$ signature for the metric tensor. In (2.1), $M_{\text{Pl}} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18} \text{GeV}$ is the Planck mass and M_* is its analogue in the ϕ sector. Defined this way, ϕ is dimensionless. N_i stands for neutrons and protons, and $\not{D} = \gamma^\mu (\partial_\mu - ie_0 A_\mu)$ for protons and $\not{D} = \gamma^\mu \partial_\mu$ for neutrons. Here e_0 is the *bare* charge which remains constant throughout the cosmological evolution (modulo the standard RG evolution of e_0 which can be neglected in our analysis). For definiteness, we assume that the dark matter is predominantly the non-relativistic Majorana fermion χ . While it is clear that one can associate χ with a neutralino, our approach can be easily generalized to other forms of cold dark matter. Ellipses stand for the omitted electron and neutrino terms, as well as for a number of possible interaction terms (i.e. baryon anomalous magnetic moments, nucleon-nucleon interactions etc.). All mass and kinetic terms are supplied with ϕ -dependent factors denoted $B_i(\phi)$. In this sense, the cosmological constant term acts as a potential for ϕ .

We shall further assume that the change of ϕ over cosmological scales is small, $|\Delta\phi| \equiv |\phi(t = t_0) - \phi(t)| \ll 1$, where t_0 is the present age of the universe. As such, we can expand all couplings around the current value of ϕ , which we choose to be zero, $\phi(t = t_0) = 0$,

$$B_\Lambda(\phi) = 1 + \zeta_\Lambda \phi + \frac{1}{2} \xi_\Lambda \phi^2 \\ B_F(\phi) = 1 + \zeta_F \phi + \frac{1}{2} \xi_F \phi^2 \\ B_{N_i}(\phi) = 1 + \zeta_i \phi + \frac{1}{2} \xi_i \phi^2 \quad (2.2)$$

$$B_\chi(\phi) = 1 + \zeta_\chi \phi + \frac{1}{2} \xi_\chi \phi^2.$$

The effective fine structure constant depends on the value of ϕ . As such, $\phi(t)$ and $\Delta\alpha/\alpha$ are directly related,

$$\begin{aligned} \alpha(\phi) &= \frac{e_0^2}{4\pi B_F(\phi)} \\ \frac{\Delta\alpha}{\alpha} &= \zeta_F \phi + \frac{1}{2} (\xi_F - 2\zeta_F^2) \phi^2, \end{aligned} \quad (2.3)$$

and we have defined $\Delta\alpha/\alpha$ as $(\alpha_0 - \alpha(t))/\alpha_0$.

The cosmological evolution of ϕ follows from the scalar field equation

$$M_*^2 \square \phi = -M_{\text{Pl}}^2 \Lambda_0 B'_\Lambda - B'_F \frac{1}{4} \langle F_{\mu\nu} F^{\mu\nu} \rangle - \langle B'_n m_n \bar{n} n + B'_p m_p \bar{p} p \rangle - \frac{1}{2} B'_\chi M_\chi \langle \chi^T \chi \rangle. \quad (2.4)$$

In this formula, primes denote $d/d\phi$, and the average $\langle \dots \rangle$ denotes a statistical average over a current state of the Universe. The term with $F_{\mu\nu} F^{\mu\nu}$ can be neglected to a good approximation as its average is zero for photons, and its contribution mediated by the baryon density, $\sum_{n,p} n_i \langle i | F_{\mu\nu} F^{\mu\nu} | i \rangle$, is already included in the terms proportional to $B'_{n,p}$. We further note that for a Dirac fermion ψ , the mass term $m_\psi \bar{\psi} \psi$ (and the analogous combination for a Majorana fermion) coincides with the trace of the ψ -contribution to stress-energy tensor, or $\rho_\psi - 3p_\psi$. Thus, the only term, that drives ϕ in the radiation domination epoch when $\rho = 3p$ is $\Lambda_0 B'_\Lambda$ (see e.g. [16, 17]). One can easily check that the change of ϕ induced by this term during radiation domination will be small compared to the $\Delta\phi$ developed in the subsequent matter domination epoch. Restricting eq. (2.4) to matter domination, and assuming a linearized regime (2.2), we derive the following equation of motion in a Robertson-Walker spacetime with scale factor $a(t)$:

$$M_*^2 (\ddot{\phi} + 3H\dot{\phi}) = -\rho_m (\zeta_m + \xi_m \phi) - M_{\text{Pl}}^2 \Lambda (\zeta_\Lambda + \xi_\Lambda \phi), \quad (2.5)$$

where $H = \dot{a}/a$ and ζ_m is defined as

$$\rho_m \zeta_m \equiv \rho_\chi \zeta_\chi + \rho_b (Y_p \zeta_p + Y_n \zeta_n). \quad (2.6)$$

Here, Y_p and Y_n are the abundances of neutrons and protons in the Universe, including those bound in nuclei. We also assume that $\rho_m = \rho_\chi + \rho_b$. In a more sophisticated treatment, one may include the contributions of electrons, the Coulomb energy stored in nuclei and other minor effects. As discussed in Refs. [3, 5], to good accuracy, ζ_m remains constant during the matter dominated epoch.

If the ϕ -dependent energy density becomes comparable to ρ_m or $\rho_\Lambda \equiv M_{\text{Pl}}^2 \Lambda$, eq. (2.5) must be solved along with Einstein's equations and energy conservation as a coupled set of equations. However, the small ϕ solutions that we are interested in imply that ρ_ϕ is small and (2.5) can be treated separately, with $a(t)$ used as an input function.

3 Cosmological evolution of the fine structure constant and the EDB constraint

The cosmological evolution of ϕ can be determined by the ζ_i terms in eq. (2.5) which becomes

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{1}{M_*^2} [\zeta_m \rho_m + \zeta_\Lambda \rho_\Lambda] = -\frac{\rho_c}{M_*^2} \left[\zeta_m \Omega_m \left(\frac{a_0}{a} \right)^3 + \zeta_\Lambda \Omega_\Lambda \right], \quad (3.1)$$

Here $\rho_c = 3H_0^2 M_{\text{Pl}}^2$ is the critical density of the Universe at $t = t_0$ and $\Omega_i = \rho_i / \rho_c$. The solution to this equation can be easily found [5, 10, 12]. Throughout this paper we shall assume that the Universe is flat and is presently dominated by non-relativistic matter and a cosmological constant, $\Omega_m + \Omega_\Lambda = 1$. In this case, the time dependence of the scale factor is given by

$$a(t)^3 = a_0^3 \frac{\Omega_m}{\Omega_\Lambda} \left[\sinh\left(\frac{3}{2}\Omega_\Lambda^{1/2} H_0 t\right) \right]^2 \quad (3.2)$$

and eq. (3.1) can be integrated analytically. The first integral is given by

$$\dot{\phi} = -3\Omega_m H_0^2 \frac{M_{\text{Pl}}^2 a_0^3}{M_*^2 a^3} \left[\zeta_m t + \frac{\zeta_\Lambda}{4b} (\sinh(2bt) - 2bt) - t_c \right], \quad (3.3)$$

where $b = \frac{3}{2}\Omega_\Lambda^{1/2} H_0$. In principle, the constant of integration t_c could be kept arbitrary. There is, however, only one natural way of fixing it by imposing initial conditions for $\dot{\phi}$ deep inside the radiation domination epoch, i.e. at t close to 0. As discussed in the previous section, during radiation domination, the r.h.s of (3.1) is effectively zero. This leads to a $\dot{\phi} \sim a^{-3}$ scaling behavior and means that any initial value of $\dot{\phi}$ will be efficiently damped by the Hubble expansion over a few Hubble times. Thus, for the solution in the matter dominated epoch we can safely take $\dot{\phi}(t=0) = 0$ or equivalently $t_c = 0$.

Integrating (3.3) gives ϕ as a function of time,

$$\phi(t) = \frac{4}{3} \frac{M_{\text{Pl}}^2}{M_*^2} \left[\left(\frac{\zeta_\Lambda}{2} - \zeta_m \right) (bt_0 \coth(bt_0) - bt \coth(bt)) - \zeta_m \ln \frac{\sinh(bt)}{\sinh(bt_0)} \right]. \quad (3.4)$$

Figure 1 shows three different types of solutions for $\Delta\alpha/\alpha$ as a function of the red-shift z , where $1+z = a_0/a$. In this plot, we have chosen $\zeta_F = 10^{-5}$, $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. Comparing the three curves, one can see that the variation of α at high red-shifts is mostly determined by ζ_m . If ζ_F is negative, one would need to choose negative ζ_m in order to get smaller values of α in the past. Opposite signs of ζ_F and ζ_m lead to the larger values of α in the past.

Given the large parameter space, $(M_*, \zeta_F, \zeta_m, \zeta_\Lambda)$, one could expect that it is easy to get $\Delta\alpha(z = 0.5 - 3.5)/\alpha \sim 10^{-5}$ as suggested by the analysis of the quasar absorption

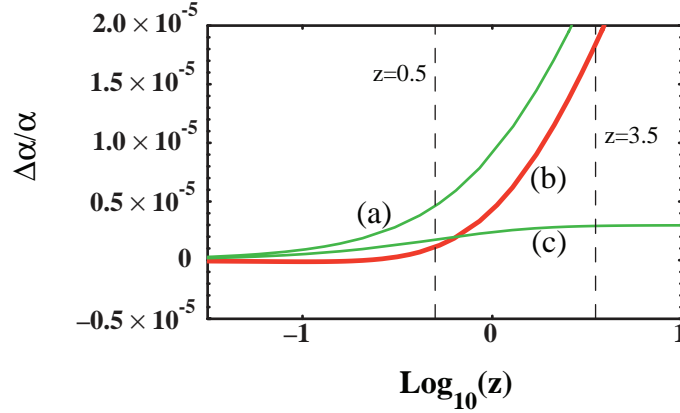


Figure 1: Three qualitatively different types of solutions for $\Delta\alpha(z)/\alpha_0$ that give smaller values of α in the past for positive ζ_F . They correspond to the choice of $\zeta_F = 10^{-5}$ and (a) $\zeta_m = 1$, $\zeta_\Lambda = 0$ (b) $\zeta_m = 1$, $\zeta_\Lambda = -2$ and (c) $\zeta_m = 0$, $\zeta_\Lambda = 1$. The interval of z , considered by Webb et al., $0.5 \leq z \leq 3.5$ is shown by two vertical dashed lines.

spectra by Webb et al. [6]. On the other hand, it is clear that the EDB constraints should severely restrict the parameter space of our model. The differential acceleration of two elements with different $A_{1,2}$ and $Z_{1,2}$ towards a common attractor can be expressed in terms of ζ_i and $\omega = M_*^2/2M_{\text{Pl}}^2$ (See, e.g. [18, 3]),

$$\frac{\Delta g}{\bar{g}} = 2 \frac{g(A_1, Z_1) - g(A_2, Z_2)}{g(A_1, Z_1) + g(A_2, Z_2)} = \frac{1}{\omega} \left(7 \times 10^{-4} \zeta_F \frac{\overline{Z^2/A^{1/3}}}{\bar{A}} + \frac{\bar{A} - \bar{Z}}{\bar{A}} \zeta_n + \frac{\bar{Z}}{\bar{A}} \zeta_p \right) \times \left[(\zeta_n - \zeta_p) \left(\frac{Z_1}{A_1} - \frac{Z_2}{A_2} \right) + 7 \times 10^{-4} \zeta_F \left(\frac{Z_2^2}{A_2^{4/3}} - \frac{Z_1^2}{A_1^{4/3}} \right) \right]. \quad (3.5)$$

where \bar{Z} and \bar{A} represent average Z and A of the common attractor, $\bar{Z} = \sum n_i M_i Z_i / \sum n_i M_i$. The terms proportional to ζ_F correspond to the electromagnetic contribution to the total energy of nuclei. The best constraints on long-range forces are extracted from $\Delta g/\bar{g}$ measured in experiments that compare the acceleration of light and heavy elements. The differential acceleration of platinum and aluminium is $\leq 2 \times 10^{-12}$ at the 2σ level (last reference in [4] as quoted in [3]), and the differential acceleration of the Moon (silica-dominated) and the Earth (iron-dominated) towards the Sun is $\leq 0.92 \times 10^{-12}$ [19]. Choosing the appropriate values of Z and A and retaining only the hydrogen contribution to the mass of the Sun, we get

$$\begin{aligned} \frac{1}{\omega} \left| \zeta_p (\zeta_n - \zeta_p + 2.9 \times 10^{-2} \zeta_F) \right| &< 2.5 \times 10^{-11} && \text{Al/Pt system} \\ \frac{1}{\omega} \left| \zeta_p (\zeta_n - \zeta_p + 1.8 \times 10^{-2} \zeta_F) \right| &< 2.5 \times 10^{-11} && \text{Si/Fe system} \end{aligned} \quad (3.6)$$

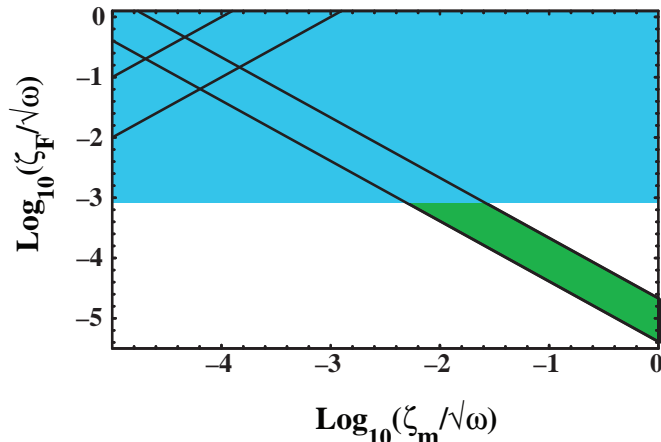


Figure 2: The $(\zeta_m/\sqrt{\omega}, \zeta_F/\sqrt{\omega})$ parameter space. The dark-shaded (green) region is consistent with both the EDB constraints and with a possible relative change of α at the 10^{-5} level, as suggested by Webb et al. [6]. The light shaded (blue) region is excluded by EDB constraints. ζ_Λ is set to zero in this plot.

These limits were also considered in a recent paper [20]. $\zeta_n - \zeta_p$ and ζ_F enter in eqs. (3.6) in different linear combinations. Thus, it is possible to extract *separate* limits on $\omega^{-1}\zeta_p\zeta_F$ and $\omega^{-1}\zeta_p(\zeta_n - \zeta_p)$. Models that have non-zero ζ_F also have non-vanishing $\zeta_{p,n}$ unless some intricate conspiracy of quark, gluon and photon contributions occur. Barring such possible cancellations, one obtains $|\zeta_{n,p}| \gtrsim |\zeta_n - \zeta_p| \gtrsim 10^{-3}|\zeta_F|$. Using these relations, we can combine the preferred range of Ref. [6] with the constraints, imposed by eqs. (3.6).

The region excluded by the EDB constraints in the $(\zeta_m/\sqrt{\omega}, \zeta_F/\sqrt{\omega})$ parameter space is shown by the light shaded (blue) region in Figure 2. Here we have set $\zeta_\Lambda = 0$. The long negative-sloped band that connects the upper-left and lower-right hand corners is the range that reproduces $\Delta\alpha/\alpha = 10^{-5}$ in the interval $0.5 \leq z \leq 3.5$. In the original Bekenstein model, $\zeta_m = (10^{-4} \text{ to } 10^{-3})\zeta_F$ and corresponds to the positive sloped band close to the upper-left corner ². As one can see, the diamond-shaped intersection is deep inside the range *excluded* by the EDB experiments. Of course, this is in agreement with conclusions of [3, 5]. Finally, the dark-shaded (green) area represents the choice of parameters that can reproduce $\Delta\alpha/\alpha = 10^{-5}$ [6] and still be in agreement with the EDB constraints. For this region, $\zeta_m/\sqrt{\omega} \gtrsim 3 \times 10^{-3}$ and $\zeta_F/\sqrt{\omega} < 10^{-3}$, which points towards models in which ϕ couples to dark matter and the couplings to baryons and ζ_F are suppressed.

In addition, we must check whether or not these choices of parameters which satisfy

² $\zeta_m = 10^{-3}\zeta_F$ would require rather “generous” assumptions concerning nucleon matrix elements and/or Ω_b .

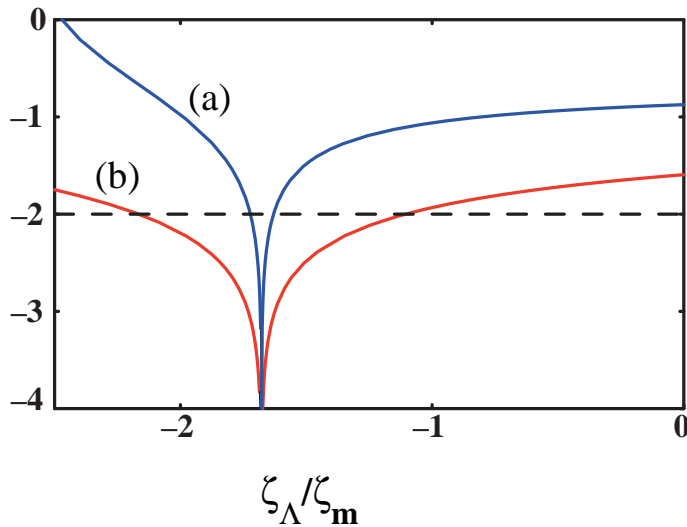


Figure 3: A plot of $\log_{10}(|\Delta\alpha(z = 0.07)/\Delta\alpha(z = 0.5)|)$ (a) and $\log_{10}(|\Delta\alpha(z = 0.07)/\Delta\alpha(z = 3.5)|)$ (b) as a function of ζ_Λ/ζ_m for the choice of $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. The portion of the curves below the horizontal dashed line are consistent with the Oklo limits [8] and the Webb et al. suggested change in α .

the EDB constraints are also in agreement with limits on $\Delta\alpha/\alpha$, derived from isotope abundances in the Oklo natural reactor. Typically, these limits are strong, $|\Delta\alpha/\alpha| < 1.2 \times 10^{-7}$ [8] and go back to $z \simeq 0.14$ ³. This seems to be dramatically smaller than the range suggested by [6]. Moreover, there is no way of suppressing $\Delta\alpha(z < 0.14)/\Delta\alpha(0.5 < z < 3.5)$ below the 10^{-2} level using our freedom in ζ_F or ω , as these parameters cancel in the ratio.

There is, however, an extra free parameter which may be used in an attempt to reconcile a change of 10^{-5} at $0.5 \leq z \leq 3.5$ and the Oklo limit. The behavior of curve (b) in Figure 1 suggests that ζ_Λ can be used to make $\Delta\alpha$ almost flat at $z < 0.2$. In order to determine the requirements on ζ_Λ , we quantify the comparison between “Oklo change” and “quasar change” as follows. In the case of the Oklo constraints, in principle, one needs to average $\alpha(t)$ over the interval $0 < t_0 - t \lesssim 2 \times 10^9$ yr. Since the exact timing of Oklo event is known only approximately, we choose to quantify it by simply taking α at the half of the Oklo redshift, $\Delta\alpha(z = 0.07)/\alpha$. This value must be approximately two orders of magnitude smaller than $\Delta\alpha/\alpha$, suggested by Webb et al. Thus, we consider the ratio, $|\Delta\alpha(z = 0.07)/\Delta\alpha(z = 0.5)|$ and $|\Delta\alpha(z = 0.07)/\Delta\alpha(z = 3.5)|$ as a function of ζ_Λ/ζ_m . The logarithms of these ratios are plotted in Figure 3. As one can see, these ratios are two funnel-

³The redshift, $z = 0.014$, corresponds to the choice $\Omega_\Lambda = 0.7, \Omega_m = 0.3, h_0 = 0.65$, and we have assumed that the Oklo event took place 1.8 Gyr ago.

like curves and it is possible to choose ζ_Λ/ζ_m in such a way that $\Delta\alpha_{\text{Oklo}}/\Delta\alpha_{\text{quasar}} < 10^{-2}$. For $z = 3.5$ this can be done rather easily in the range $-2.2 < \zeta_\Lambda/\zeta_m < -1.2$. For $z = 0.5$ one has to choose this ratio rather carefully, $\zeta_\Lambda/\zeta_m \simeq -1.7$, and requires a 5% -10% fine-tuning. We also note that this specific value of ζ_Λ/ζ_m is very sensitive to the choice of Ω_Λ and Ω_m and varies significantly when Ω_Λ and Ω_m are varied within their current error bars. The use of more restrictive bounds from Oklo by [9] would only worsen the fine tuning.

The above exercise allows us to conclude that in principle a generalized Bekenstein-like model can yield $\Delta\alpha/\alpha \sim 10^{-5}$ at $0.5 \leq z \leq 3.5$ and still be in agreement with the EDB and Oklo constraints. The limits from non-universality of a fifth force could be evaded in models with large couplings to dark matter and small couplings to baryons and $F_{\mu\nu}F^{\mu\nu}$. Also, the Oklo bounds could be avoided or softened if the dark matter provides a negative push to ϕ at later epoch. Although such a suppression of $\Delta\alpha_{\text{Oklo}}$ may happen, it would appear to be highly accidental.

4 Model realizations

It is important to note that neither the original Bekenstein model [3] nor its modifications discussed here are fully defined at the quantum level. Indeed, the $B_F(\phi)F_{\mu\nu}F_{\mu\nu}$ term contains not only the bare QED Lagrangian term but also higher dimensional operators such as $\phi^n F_{\mu\nu}F_{\mu\nu}$. It is clear then that at the loop level this will create all other possible interactions such as $\phi^n m_e \bar{e}e$, $\phi^n m_q \bar{q}q$, etc., generally, with divergent coefficients which cannot be fixed from first principles. While these terms are not expected to drastically change the model if one makes some plausible assumptions about the cutoff in the theory, there is, however, the set of operators contained in $B_\Lambda(\phi)$ which are very sensitive to the cutoff and are very important as they can give rise to the mass of the ϕ field, an effective cosmological constant, etc. Unfortunately, the present status of the underlying theory does not allow for a meaningful calculation of $B_\Lambda(\phi)$. This problem is, of course, tightly related to the cosmological constant problem [21], and/or to the smallness of the mass term for the quintessence field. As we have nothing to add to these issues, we must assume that ζ_Λ and ξ_Λ are basically incalculable input parameters and fix Λ to its value implied by the observation of high z supernovae, the anisotropy of the cosmic microwave background and large scale structure formation. In what follows, we compile a list of models which predict certain values for the ζ_i couplings and/or ω and confront them with the phenomenological constraints, discussed in the previous section.

1 The Bekenstein model

In this model, one initially introduces the coupling of ϕ to F^2 , $B_F(\phi) = \exp(-2\phi)$ or $\zeta_F = -2$. B_Λ can be set to a constant so that $\zeta_\Lambda = 0$. In the original model, $\omega = 1$, however, we will keep it arbitrary for now. The change of ϕ is driven by the electromagnetic

fraction of the baryon energy density. The coupling of ϕ to nucleons is given by the same matrix elements, $\zeta_N = m_N^{-1} \langle N | \frac{\zeta_F}{4} F_{\mu\nu} F^{\mu\nu} | N \rangle \simeq -m_N^{-1} \langle N | \frac{\zeta_F}{2} (E^2 - B^2) | N \rangle$, that determine the contribution of a “photon cloud” to the nucleon mass. Both the Naive quark model and dispersion approaches give consistent estimates of these matrix elements [22]. Using the results of [22], presumably valid to 50% accuracy, we find that $\zeta_p \simeq -0.0007\zeta_F$ and $\zeta_n \simeq 0.00015\zeta_F$. Incidentally, these values almost coincide with simple extrapolations of the nuclear mass formula to $Z = 1, 0$: $\zeta_p \simeq -0.0007$, $\zeta_n = 0$. Since ζ_b is determined mostly by ζ_p , $\zeta_m = \zeta_b(\Omega_b/\Omega_m) \sim -10^{-4}\zeta_F$. As we have discussed earlier, the constraints from EDB experiments, as exemplified in Figure 2, do not allow α to change by more than 1 part per billion at red-shifts $z < 3.5$.

Restricting our attention to small variations in α , we see from eq. (2.3) that

$$\frac{\Delta\alpha}{\alpha} = \zeta_F\phi \quad (4.1)$$

Then evaluating eq. (3.4) at $z = 3.5$, we find that

$$\frac{\Delta\alpha}{\alpha} = \frac{1.2}{\omega} \zeta_F \zeta_m \simeq -10^{-4} \omega^{-1} \zeta_F^2 \quad (4.2)$$

Note that in this model ζ_F and ζ_m are of opposite sign and the final result does not depend on the sign of ζ_F . Thus from eq. (4.2), we see that this model leads to *larger* values of α in the past, which is *opposite* the trend reported by Webb et al. Moreover, from Figure 2, we see that the EDB constraint requires that $\zeta_F^2/\omega < 10^{-6}$ and thus we see again that $|\Delta\alpha/\alpha|$ is limited to $\mathcal{O}(10^{-10})$, in agreement with the results of [5]. These results however, differ from those of Ref. [12] in both the allowed magnitude and sign of $\Delta\alpha(z < 3.5)/\alpha$.

2 A String-dilaton-type model

The starting point for this class of models is the action

$$\int d^4x \sqrt{-g} \exp(-\sqrt{2}\phi) \left(R + (\partial_\mu\phi)^2 + \Lambda + \mathcal{L}_{\text{matter}} \right). \quad (4.3)$$

The functions $B_i(\phi)$ are easily obtained by making a conformal transformation to the Einstein frame. We find that $\zeta_F = -\sqrt{2}$, $\zeta_\Lambda = \sqrt{2}$, and $\zeta_m = \sqrt{2}/2$. Furthermore, since there is only one scale in the theory, the Planck scale, we have $\omega = 1/2$. Therefore, we are able to obtain a definite value, $\Delta\alpha/\alpha \simeq -3$, (over the redshift range $z = 3.5$ to 0). Clearly this is not realistic and is related to the well known problem of a massless runaway dilaton in string theory. Moreover, such a model is ruled out by the EDB constraints, as it predicts $\zeta_{n,p} \simeq 1$ and $\zeta_n - \zeta_p \sim 10^{-3}$. Until more can be said about the function $B_\Lambda(\phi)$, there is no useful way to use string theory to predict changes in α .

3 A Brans-Dicke model

In a Brans-Dicke model ϕ is initially coupled only to the gravitational sector of the theory

$$S_{BD} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[\phi R + \frac{\omega}{\phi} \partial_\mu \phi \partial^\mu \phi \right] + S_{\text{matter}} \quad (4.4)$$

As in the previous example, it is easy to show that after the conformal rescaling of metric to the standard Einstein frame and a field redefinition for ϕ , the “new” field ϕ acquires a universal coupling ζ to the mass sector of the matter fields which is given in terms of ω : $\zeta_m = -1/\sqrt{4\omega + 6}$. One also obtains $\zeta_\Lambda = -2\sqrt{2/(2\omega + 3)}$. As in the Bekenstein model, all physical results depend only on $\zeta/\sqrt{\omega}$.

Due to conformal invariance of the action for the gauge field, a tree level coupling of ϕ to F^2 is absent yielding $\zeta_F = 0$. However, the conformal symmetry is anomalous, and at the one loop level a ϕF^2 term can be generated. In some sense, the couplings of ϕ to the quarks and leptons will be similar (apart from their magnitude) to those of the Higgs boson. It is then clear that a non-zero value of ζ_F will be generated through the loops of charged particles in the same way that the Higgs- γ - γ coupling is generated. For example, the coupling of the Higgs boson, h , to F^2 due to the top quark loop can be obtained by differentiating the top quark contribution to the *QED* β -function, $F^2 \ln(\Lambda_{UV}/(m_t(1 + h/v))) \rightarrow -F^2 h/v$, where v is the Higgs vacuum expectation value. This assumes that the ultra-violet cutoff Λ_{UV} is h -independent. In principle, there could be different possibilities with regard to the ϕ -dependence of Λ_{UV} . If one assumes that the regulator mass depends on ϕ in exactly the same way as an ordinary mass, then ϕ drops out of the loop amplitude, ζ_F is not generated and the Brans-Dicke scalar respects the weak equivalence principle even at the one-loop level. A different result would arise if we postulate a ϕ -independent regularization Λ_{UV} . In this case, a non-zero value for ζ_F is generated and one would typically have

$$\zeta_F = -\frac{2\alpha}{\pi} \zeta_m \left(-\frac{7}{4} + 1 + 2 \times \frac{5}{9} \right) \simeq -1.5 \times 10^{-3} \zeta_m. \quad (4.5)$$

The three terms in parenthesis correspond to the contributions from W -bosons, charged leptons and quarks from second and third generations. (u and d quarks require a separate and quite complicated treatment. However, their main contribution is given by the charged pion loop and turns out to be numerically small compared to the contributions of heavy quarks.) The couplings of ϕ to baryons will be simply ζ_m , and its non-universality appears at the $\zeta_n - \zeta_p \sim 10^{-3} \zeta_m$ level. Thus, the bounds (3.6) push the constraints on $\zeta_m^2 \omega^{-1}$ to the level of 10^{-8} or so and leave no room for a $O(10^{-5})$ relative change of α at $0.5 \leq z \leq 3.5$. The maximum allowed change is not expected to be larger than 10^{-11} . The result of Webb et al. cannot be accommodated in a Brans-Dicke model.

4 A Supersymmetrized Bekenstein model

While there are many different possible supersymmetric generalizations of the original Bekenstein model, we consider the simplest version which begins by promoting $B_F(\phi)(FF)$ to the rank of a superpotential:

$$- \int d^4x \frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu} \rightarrow \int d^4x d^2\theta \frac{1}{4} B_F(\hat{\phi}) W_\mu W^\mu + (\text{h.c.}).. \quad (4.6)$$

Here $\hat{\phi}$ denotes a chiral superfield, which has ϕ as its bosonic component and W is the supersymmetric field strength. In component notation this interaction can be rewritten as

$$\int d^4x \left(B_F(\phi) \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\chi} \partial_\mu \bar{\gamma}_\mu \chi \right] - \frac{1}{2} B'_F(\phi) F_\phi \chi^T \chi \right) \dots \quad (4.7)$$

F_ϕ denotes the F -component of $\hat{\phi}$ and ellipses stand for other terms not relevant for the present discussion. We see that in addition to the interaction with the gauge boson, ϕ acquires a coupling to the gauge fermion or gaugino, χ . F_ϕ may acquire a v.e.v. which contributes to the supersymmetry-breaking gaugino mass. There may also be additional soft-breaking contributions leading to a mass term of the form $\frac{1}{2} M \chi^T \chi$. Performing the rescaling $\chi \rightarrow \chi / \sqrt{B_F(\phi)}$, we arrive at the following Lagrangian in the $\phi - \chi$ sector,

$$\mathcal{L}_{\phi\chi} = \frac{1}{2} \bar{\chi} \partial_\mu \bar{\gamma}_\mu \chi - \frac{1}{2} \frac{B'_F(\phi) \langle F_\phi \rangle + M}{B_F(\phi)} \chi^T \chi \quad (4.8)$$

In the linearized version of the theory given by eqs. (2.2), we arrive at the following expression for ζ_χ ,

$$\zeta_\chi = \frac{(\xi_F - \zeta_F^2) \langle F_\phi \rangle - M \zeta_F}{M_\chi}, \quad (4.9)$$

where $M_\chi = M + \zeta_F \langle F_\phi \rangle$.

Clearly, ζ_χ can be $O(1)$, if $\zeta_F \sim O(1)$, however its sign is not uniquely defined unless we make some specific assumptions about ζ_F , ξ_F , $\langle F_\phi \rangle$ and M . For example, let us take B_F as in the original Bekenstein model so that $\zeta_F = -2$. Let us further assume that supersymmetry breaking occurs outside the ϕ sector so that $F_\phi = 0$. In this case, $M_\chi = M$ and $\zeta_\chi = 2$. Since the dark matter dominates the energy density of non-relativistic matter, we have $\zeta_m \simeq \zeta_\chi$. Indeed, it is quite reasonable to expect that in general $|\zeta_m| \sim |\zeta_F|$ which leads to the relation between the dark matter and baryonic sources of ϕ advertized in (1.1).

The final parameter which must be specified in the model is ω . In order to obtain consistency with the combination of EDB constraints, we must have $|\zeta_F / \sqrt{\omega}| < 10^{-3}$ or $\omega > 4 \times 10^6$. If we again assume $\zeta_\Lambda = 0$, we can compute the change in the fine structure constant (from $z = 3.5$ to $z = 0$)

$$\frac{\Delta\alpha}{\alpha} = \frac{1.2}{\omega} \zeta_F \zeta_m \simeq -5/\omega \quad (4.10)$$

| Type of model | ζ_F | ζ_b | ζ_m | $\frac{\Delta g}{g}$ | $ \frac{\Delta\alpha}{\alpha} _{\max}$ at $0.5 \leq z \leq 3.5$ |
|---------------------|-------------|-------------------------|-------------------------|----------------------------|--|
| Bekenstein model | -2 | 10^{-3} | 10^{-4} | $\frac{10^{-6}}{\omega}$ | 10^{-10} |
| “String dilaton”* | $-\sqrt{2}$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | 10^{-3} | 1 |
| Brans-Dicke model** | 0 | $-1/\sqrt{4\omega + 6}$ | $-1/\sqrt{4\omega + 6}$ | $\frac{10^{-3}}{\omega^2}$ | 10^{-11} |
| SUSY BM | 1 | 10^{-3} | 1 | $\frac{10^{-6}}{\omega}$ | 10^{-6} |
| M_χ -driven | 10^{-4} | $10^{-7} - 10^{-5}$ | 0.1 | $\frac{10^{-12}}{\omega}$ | $10^{-5} - 10^{-4}$ |

Table 1: Order of magnitude model predictions for the set of relevant couplings, non-universality of ϕ -exchange, and maximum allowed $|\Delta\alpha/\alpha|$ at $0.5 \leq z \leq 3.5$. (*) The tree-level form of $B_i(\phi)$ is assumed. (**) A ϕ -dependent cutoff is assumed.

Note again the sign of $\Delta\alpha$ predicts that α was larger in the past, although this conclusion could be modified if ϕ contributes to supersymmetry breaking (so that $F_\phi \neq 0$). Also, because of the EDB constraint, the relative change of $|\Delta\alpha/\alpha|$ at z in the interval $0.5 - 3.5$ would typically be at the level of 10^{-6} , unless some additional fine-tuning is introduced. (For example, if a partial cancellation between M and $\zeta_F \langle F_\phi \rangle$ in M_χ occurs, one can get $|\zeta_m| > |\zeta_F|$ and thus satisfy the constraints shown in Figure 2.)

In contrast with the non-supersymmetric version of the Bekenstein model, the change in ϕ from the time of the radiation domination/matter domination transition to the present epoch can be of order 1 or even larger. In this case, obviously, the linearized approach to $B_i(\phi)$ may fail for $z \gg 1$. Therefore, it is impossible to determine the total change of α from the BBN epoch, without specifying the complete functional form for both $B_\chi(\phi)$ and $B_F(\phi)$. Nevertheless, it can be shown that if the $B_i(\phi)$ are dominated by the few first terms in the Taylor expansion up to $z \sim 10^5$, the change of α is within the BBN bounds. Large changes in ϕ may also entail a non-negligible backreaction of the ϕ -dependent stress-energy tensor on Freidman’s equations. In this case, one could get interesting effects in the expansion of the Universe due to the $B_\chi(\phi)M_\chi\bar{\chi}\chi$ term, which can be interpreted at the same time as varying mass dark matter [23] or the potential term for ϕ , that has an overall factor of ρ_m .

An interesting consequence of models where ϕ couples to dark matter is the non-universality of the free fall towards an attractor dominated by dark matter, e.g. the center

of a galactic halo. The magnitude of differential acceleration of a system of heavy/light elements towards a dark matter attractor is enhanced compared to the acceleration towards the Sun by the factor ζ_m/ζ_p :

$$\frac{\Delta g}{\bar{g}} = \frac{\zeta_m}{\omega} \left[(\zeta_n - \zeta_p) \left(\frac{Z_1}{A_1} - \frac{Z_2}{A_2} \right) + 7 \times 10^{-4} \zeta_F \left(\frac{Z_2^2}{A_2^{4/3}} - \frac{Z_1^2}{A_1^{4/3}} \right) \right] \simeq 3 \times 10^{-2} \frac{\zeta_m \zeta_F}{\omega} \quad (4.11)$$

Depending on the ratio of ζ_m/ζ_p , this effect may be as large as $10^{-7} - 10^{-9}$ when the usual EDB constraints are satisfied. Unfortunately, this is much lower than the current experimental sensitivity achieved, $\sim 0(10^{-3})$, in tests of differential acceleration towards galactic center [24].

5 A gaugino driven modulus

It may happen that the modulus field that changes α is coupled primarily to the soft breaking parameters. Then the coupling to F^2 and baryons may only appear at the loop level. Let us suppose for simplicity that initially ϕ couples only to gaugino masses,

$$\mathcal{L} = \sum_{i=1,2,3} \left[\frac{1}{2} \bar{\lambda}_i \partial_\mu \gamma^\mu \lambda_i - \frac{1}{2} M_i (1 + \zeta_{M_i} \phi) \lambda^T \lambda \right], \quad (4.12)$$

where the summation is over the three Standard Model gauge groups (color and weak indices are suppressed). We assume that the lightest supersymmetric particle is the neutralino χ which is predominantly the bino. Therefore, $M_\chi \simeq M_1$ and $\zeta_m \simeq \zeta_\chi \simeq \zeta_{M_1}$.

We now consider the possibility that all couplings of ϕ to standard model fields are induced radiatively. At the one loop level, the couplings with $SU(2)$ and $SU(3)$ gauge bosons will be generated,

$$\zeta_W = -\frac{2\alpha_W}{3\pi} \zeta_{M_2} \quad (4.13)$$

$$\zeta_G = -\frac{\alpha_s}{\pi} \zeta_{M_3} \quad (4.14)$$

The calculation of these couplings is trivial: they are obtained by differentiating gluino and wino contributions to the corresponding beta functions over M_i . In the derivation of these couplings we assumed that the cutoff scale is ϕ -independent.

After the breaking of the $SU(2) \times U(1)$ gauge symmetry, ζ_W induces a contribution to ζ_F ,

$$\zeta_F = \sin^2 \theta_W \zeta_W \simeq -1.5 \times 10^{-3} \zeta_{M_2}. \quad (4.15)$$

The coupling to baryons is mediated by ζ_F as before or by ζ_G or by the $\phi m_q \bar{q} q$ operators, induced at the supersymmetric threshold. Typically, ζ_G induces too large a coupling to baryons, $\zeta_{n,p} \sim (0.02 - 0.06) \zeta_{M_3}$, to be consistent with EDB limits and $\omega = O(1)$. Therefore,

one must require $\zeta_{M_3} \ll \zeta_{M_1}, \zeta_{M_2}$. Wilson coefficients in front of supersymmetric threshold-induced $\phi m_q \bar{q} q$ operators are expected to be at the level of $10^{-5} - 10^{-3}$ from ζ_{M_1} and ζ_{M_2} . This creates a coupling of ϕ to nucleons at the level $\zeta_{n,p} \sim (10^{-6} - 10^{-4})\zeta_{M_2}$, which is mostly due to a large matrix element of the $m_s \bar{s} s$ operator and/or the two-loop induced $G_{\mu\nu}^a G^{a\mu\nu}$ operators. $m_u \bar{u} u$ and $m_d \bar{d} d$ generate the difference between the couplings to neutron and proton at the level of $\zeta_n - \zeta_p \sim (10^{-7} - 10^{-5})\zeta_{M_2}$. Due to the small values of $\zeta_{p,n}, \zeta_p - \zeta_n$, and ζ_F , the constraints based on the violation of the equivalence principle (3.6) do not lead to very restrictive bounds,

$$\zeta_{M_2}^2/\omega \lesssim 10^{-4} - 10^{-2}. \quad (4.16)$$

The possibility of “choosing” ζ_{M_1}, ζ_{M_2} and ω creates sufficient freedom to satisfy EDB constraints and at the same time have $\Delta\alpha/\alpha$ at $0.5 \leq z \leq 3.5$ compatible with the Webb et al. result. Recall that

$$\frac{\Delta\alpha}{\alpha} = \frac{1.2}{\omega} \zeta_F \zeta_m \simeq -\frac{1.8 \times 10^{-3}}{\omega} \zeta_{M_1} \zeta_{M_2} \quad (4.17)$$

Indeed, to satisfy both it is sufficient to take

$$\frac{\zeta_{M_1}}{\sqrt{\omega}} = -\frac{\zeta_{M_2}}{\sqrt{\omega}} \simeq \pm 0.1; \quad \text{and} \quad \zeta_{M_3} \ll \zeta_{M_1}, \zeta_{M_2}. \quad (4.18)$$

To conclude this section, we combine all model predictions in Table 1.

5 Models of an oscillating fine structure constant

Finally, we turn to the case when all of the functions $B_i(\phi)$ have a common extremum point ϕ_{ext} . As was shown by Damour and Nordtvedt and Damour and Polyakov [15], the matter energy density may serve as a cosmological attractor for ϕ , so that today its value is close to ϕ_{ext} . In our approach, without losing generality, we choose $\phi_{\text{ext}} = 0$. The requirement of a common extremum is equivalent to condition that *all* linear couplings $\zeta_i = 0$.

The cosmological evolution of ϕ is now given by the ξ_i couplings. There are two distinct regimes to consider: $\phi(t) = \text{const}$ at early times and an oscillating or runaway regime at late times. These two regimes are common for cosmological evolution of any quasi-modulus field, e.g. axion. The transition occurs when the Hubble rate drops below the effective (time-dependent) mass of ϕ ,

$$m_\phi^2 = \frac{2}{\omega} \left[\Lambda_0 \xi_\Lambda + \frac{\rho_m \xi_m}{M_{\text{Pl}}^2} \right] = \frac{6H_0^2}{\omega} \left[\Omega_\Lambda \xi_\Lambda + \Omega_m \xi_m \left(\frac{a_0}{a} \right)^3 \right] \quad (5.1)$$

The sign of m_ϕ^2 determines if it is a runaway or oscillatory evolution. Here, we are interested in the oscillatory regime, and thus assume that m_ϕ^2 is positive. The amplitude of these

oscillations red-shifts as $a^{-\kappa}$, where $3/4 < \kappa < 3/2$. $\kappa = 3/2$ occurs if (5.1) is dominated by the first term, i.e. rigid mass, and $\kappa = 3/4$ if the second matter-induced term is dominant [15]. Thus, the effective value of α also oscillates, at twice the frequency, $2m_\phi$, and with an amplitude decreasing as $a^{-2\kappa}$. In this regime, is it possible to satisfy the EDB and Oklo bounds and have $\Delta\alpha/\alpha \sim 10^{-5}$ at $0.5 \leq z \leq 3.5$?

If ξ_F is the dominant source of the couplings to baryons, the expected level of the violation of the equivalence principle is

$$\frac{\Delta g}{\bar{g}} \simeq 10^{-6} \frac{\xi_F^2 \phi_{\text{now}}^2}{\omega} \quad (5.2)$$

Of course, it is possible that the value of ϕ today is close to zero, simply because in the oscillatory regime, $\phi = 0$ occurs regularly. This is, however, an accidental situation, and one would naturally expect ϕ_{now} to be on the order of the amplitude of oscillations. On the other hand, the relative change of α is given by

$$\frac{\Delta\alpha}{\alpha} = \frac{1}{2} \xi_F (\phi^2(z) - \phi_{\text{now}}^2) \simeq \frac{1}{2} \xi_F \phi^2(z) \quad (5.3)$$

Using the relation between $\phi(z)$ and ϕ_{now} , and plugging in the constraint from (5.2), we get

$$\frac{\Delta\alpha}{\alpha} \lesssim 10^{-5} (1+z)^{2\kappa} \frac{\omega}{\xi_F} \quad (5.4)$$

It is then clear that this can be consistent with 10^{-5} at $0.5 \leq z \leq 3.5$ naturally without a fine-tuning of parameters when $\omega/\xi_F \sim 1$. The Oklo bounds can be made marginally consistent with [6] in this scenario only for large z (close to 3.5 rather than 0.5) and for large κ , $\kappa = 3/2$. This favors models where the oscillations of ϕ at $z < 3.5$ are driven by the rigid ξ_Λ -proportional mass term of ϕ .

6 Conclusions

We have shown that generalized versions of the Bekenstein's model may be consistent with the strong limits imposed by Eötvös-Dicke-Braginsky type of experiments and at the same time provide a relative change of α at the 10^{-5} -level, claimed recently in Webb et al. [6]. The necessary flexibility in our models is achieved by the coupling of the modulus field ϕ to the dark matter energy density and to the cosmological constant. We argue that it is natural to expect that the cosmological evolution of ϕ will be mostly driven by these sources rather than by the baryon energy density. This can be seen explicitly in the simplest SUSY-version of the Bekenstein model, where the supersymmetric partner of the $U(1)$ gauge field is the dominant non-baryonic component of dark matter.

In practice, it turns out that among various models where ϕ couples to F^2 , baryons and dark matter, only a few survive the EDB constraints and provide the $O(10^{-5})$ relative change in α over the redshift range $0.5 \leq z \leq 3.5$. In particular, we find that the models where ϕ is coupled initially only to $U(1)$ and $SU(2)$ gaugino mass terms can easily satisfy both criteria.

The bounds on $\Delta\alpha$ from the Oklo phenomenon are less dependent on the details of the coupling of ϕ to the matter field. Generally, they are strong enough to rule out the change of the fine structure constant, implied by Webb et al. In the context of the generalized models discussed here, the negative coupling of ϕ to the cosmological constant may be used to slow down its evolution and make Oklo bounds consistent with [6]. This possibility, however, looks accidental and very fine-tuned for $\Delta\alpha/\alpha \sim 10^{-5}$ at $z = 0.5$. Of course, our treatment of all models at the loop level is plagued by the usual problem of the cosmological constant and the near-masslessness of the moduli field ϕ . This prevents us from making any prediction for the size of the ζ_Λ coupling constant. We also find that $\zeta_i = 0$ [15] is easier to reconcile with EDB constraints and Webb et al., as in this case there is an additional suppression of the ϕ -mediated force.

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