A Model of Varying Fine Structure Constant and Varying Speed of Light

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Abstract

The recent evidence for a cosmological evolution of the fine structure constant $\alpha = e^2/\hbar c$ found from an analysis of absorption systems in the spectra of distant quasars, is modelled by a cosmological scenario in which it is assumed that only the speed of light varies. The model fits the spectral line data and can also lead to a solution of the initial value problems in cosmology.

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1 Introduction

The possibility that a large increase in the speed of light in the early universe can solve the initial value problems in cosmology [1, 2], and present an alternative to the standard inflationary models [3], has received mounting attention [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Higher-dimensional models lead to a varying speed of light (VSL) when the radion field in e.g. five-dimensional models varies [14]. The recent increasing evidence for a cosmological evolution of the fine structure constant in a red shift range $0.5 \leq z \leq 3.5$ is therefore of considerable interest [15]. The idea that the fine structure constant is varying over the history of the universe has a long history [16, 17, 18, 19, 20, 21], and if this discovery is confirmed by further observations and analyses, then it will have a profound impact on the future of physics. Models of varying α have been proposed recently [22, 23, 24, 25] and a study has been made of the possibility that the cosmic microwave background (CMB) data could be used to detect a variation in α [26]. It is an old argument [18, 27] that observations cannot measure directly fundamental dimensional constants. Only dimensionless constants such as the fine structure constant can be measured, which involves the dimensional constants e, c and $\hbar$. However, we can form theoretical prejudices about which dimensional constants are responsible for a variation of dimensionless fundamental constants and this entails different models that describe the variation of these constants. Moreover, independent observational evidence can be obtained that can rule out one or another of these models.

In the following, we shall study a simple model that incorporates a VSL behaviour in a cosmological setting, assuming that the electric charge e and Planck's constant $\hbar$ are truly constants of nature. A detailed analysis of a possible variation

in the electric charge e was given by Bekenstein [21], in which the local gauge invariance of the electromagnetic field was preserved, although conservation of charge was broken. More recently, Bekenstein's model has been reanalyzed in a cosmological setting by Sandvik, Barrow and Magueijo [23]. One of Bekenstein's conclusions, based on reasonable physical assumptions, was that spatial gradients of the electric charge would cause a large discrepancy with the weak equivalence principle experiments [28]. If such a violation of the weak equivalence principle is extrapolated back to red shifts in the early matter dominated era of the universe, when the density of matter was greater, then such spatial gradient violations could be so large as to imply a significant violation of the weak equivalence principle in that era. Variations of Planck's constant $\hbar$ at red shifts of order $z \sim 2 - 3$ could significantly affect atomic and molecular spectral line observations and other quantum phenomena. In view of this, we find that the possibility of a varying speed of light is more attractive, even though our understanding of special relativity, general relativity and spacetime will be significantly altered. We should also emphasize that a VSL explanation of a varying fine structure constant has the important theoretical consequence of being able to resolve the initial value problems of cosmology, whereas at this stage of our theoretical understanding, it is not clear what advantages there could be for a varying e or $\hbar$.

In Maxwell's theory, the speed of light is predicted from the equation

$$c = \frac{1}{\sqrt{\epsilon\mu}}, \tag{1}$$

where ϵ and μ denote the electric permittivity and the magnetic permeability of the vacuum, respectively. If we have a varying speed of light, then we can write

$$c(x) = \frac{1}{\sqrt{\chi(x)}}, \tag{2}$$

where χ is a function of the spacetime coordinates. This implies that we picture the vacuum as a variable medium and the velocity of electromagnetic waves depends on the magnitude of χ. In particular, the increase in the value of c in the early universe would be traced to a phase transition in the function χ, associated with a spontaneous symmetry breaking of Lorentz invariance of the vacuum [1].

Once χ is treated as a function of the spacetime coordinates, then we can no longer simply change units such that $\chi = 1$.

2 Varying Fine Structure Constant and Speed of Light Model

We shall use a simple minimal scheme to illustrate physical consequences of a VSL, and defer the investigation of a more geometrically rigorous theory of VSL, such as

the bimetric theory [11, 12] to a future publication. In a minimally coupled VSL theory, one replaces c by a field in a preferred frame of reference, $c(t) = c_0\phi(t)$, where c_0 denotes the present value of the speed of light. The dynamical variables in the action are the metric $g_{\mu\nu}$, matter variables contained in the matter action, and the scalar field ϕ which is assumed not to couple to the metric explicitly [4, 5]. In the preferred frame the curvature tensor is to be calculated from $g_{\mu\nu}$ at constant ϕ in the normal manner. Varying the action with respect to the metric gives the field equations

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{c_0^4 \phi^4} T_{\mu\nu}, \tag{3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, Λ is the cosmological constant and $T_{\mu\nu}$ denotes the energy-momentum tensor. This theory is not locally Lorentz invariant. Choosing a specific time to be the comoving proper time, and assuming that the universe is spatially homogeneous and isotropic, so that c only depends on time $c = c(t)$, then the FRW metric can still be written as

$$ds^2 = c^2 dt^2 - a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{4}$$

where $k = 0, +1, -1$ for spatially flat, closed and open universes, respectively. The Einstein equations are still of the form

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho_m - \frac{kc^2}{a^2} + \frac{c^2\Lambda}{3}, \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + 3\frac{p_m}{c^2} \right) + \frac{c^2\Lambda}{3}. \tag{6}$$

The conservation equations are modified due to the time dependence of c (we assume that G is constant):

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho_m + \frac{p_m}{c^2} \right) = \frac{3kc^2}{4\pi Ga^2}\frac{\dot{c}}{c}, \tag{7}$$

where $\rho = \rho_m + \rho_\Lambda$ and $\rho_\Lambda = c^2\Lambda/8\pi G$. We can fit the present cosmological data by choosing $\Omega_{0m} = 0.3$ and $\Omega_{0\Lambda} = 0.7$, where $\Omega_{0m} = 8\pi G\rho_m/3H_0^2$ and $\Omega_{0\Lambda} = 8\pi G\rho_\Lambda/3H_0^2$ [29].

As shown in ref. [1], and subsequently in refs. [4, 5, 11], VSL theories can solve the horizon, flatness and particle relic problems of early universe cosmology, when $\phi(t)$ takes on large values in the very early universe. The basic problem of the existence of cosmological horizons in the standard big bang model, leads to the puzzle that regions that have not be in causal contact have the same physical properties. This puzzle is solved in VSL theories by considering the proper distance

$$d_H = a(t) \int_{t_0}^t \frac{dt' c(t')}{a(t')}. \tag{8}$$

For a large increase in the value of $c(t)$ corresponding to light travelling faster in the early universe, it is possible for the horizon to be much larger, so that all regions in our past have been in causal contact. The flatness problem is also explained, for if the speed of light undergoes a sharp change in a phase transition, then it can be shown that $\Omega - 1 \sim 0$ is an attractor solution for $|\dot{c}/c| < 0$, i.e. the speed of light decreases as the universe expands.

We model $\phi(t)$ by

$$\phi(t) = \frac{1}{1 + A(t)\left[\left(\frac{t}{t_0}\right)^b - 1\right]}, \tag{9}$$

where t_0 denotes the present age of the universe and b is a positive constant. The speed of light has the form

$$c(t) = \frac{c_0}{1 + A(t)\left[\left(\frac{t}{t_0}\right)^b - 1\right]}, \tag{10}$$

where $c(t_0) = c_0$ and for $t \to 0$, we have

$$c(t) \to \frac{c_0}{1 - A(t)}. \tag{11}$$

The change in the speed of light c is

$$\frac{\Delta c}{c} \equiv \frac{c - c_0}{c_0} = \frac{A\left[1 - \left(\frac{t}{t_0}\right)^b\right]}{1 + A\left[\left(\frac{t}{t_0}\right)^b - 1\right]}. \tag{12}$$

We shall assume that $A(t)$ is a slowly varying function of t as $t \to 0$ until some critical time $t = t_c$, when $A(t)$ undergoes a sharp increase to $A(t_c) \sim 1$, resulting in a sudden increase in $c(t)$. This sharp increase in $c(t)$ corresponds to a phase transition in the function $\chi(t)$, in Eq.(2), such that $\chi(t_c) \sim 0$.

Let us write

$$\alpha(t) \equiv \frac{e^2}{\hbar c_0 \phi(t)} = \frac{\alpha_0}{\phi(t)}, \tag{13}$$

where we have assumed that e and $\hbar$ are strictly constants of nature, α_0 denotes the present value of α and $\alpha(t_0) = \alpha_0$. This yields the fractional varying value for the fine structure constant

$$\frac{\Delta \alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = A(t)\left[\left(\frac{t}{t_0}\right)^b - 1\right]. \tag{14}$$

In order not to spoil the agreement of the standard cosmological model with the CMB data at the red shift $z \sim 1000$ and big bang nucleosynthesis results, we assume that $t_c \ll t_{NS}$ where t_{NS} denotes the time of nucleosynthesis at the red shift

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$z \sim 10^9$. Moreover, we assume that $A(t_{\text{CMB}}) \sim A(t_{NS}) < 10^{-3}$ where t_{CMB} denotes the time of CMB.

Assuming that $A \sim$ const., the time variation of α is given by

$$\frac{\dot{\alpha}}{\alpha} = \frac{bA\left(\frac{t}{t_0}\right)^{b-1}}{t_0\left\{1 + A\left[\left(\frac{t}{t_0}\right)^b - 1\right]\right\}}. \tag{15}$$

For $b = 1.5, A = 10^{-5}$, $t/t_0 = 0.125$ corresponding to $z \sim 3$ and $t_0 = 13.9$ Gyr we get

$$\frac{\dot{\alpha}}{\alpha} = 3.8 \times 10^{-16}\,\text{yr}^{-1}. \tag{16}$$

We observe that $\Delta\alpha/\alpha \sim -\Delta c/c$ for $A \ll 1$ and at the phase transition $t = t_c$ $\Delta\alpha/\alpha \sim -1$ so that $\alpha(t_c) \sim 0$.

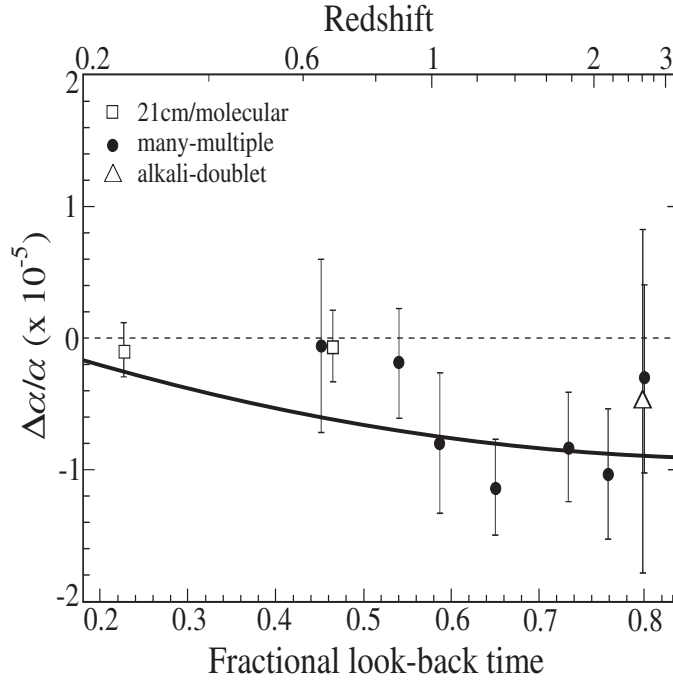In Fig. 1, we display a fit to the quasar spectral line data of Webb et al. [15] for $b = 1.5$ and $A = 10^{-5}$.

Fig 1.

$\Delta\alpha/\alpha$ vs. fractional look-back time to the big bang and red shift z. The data points are from ref [15].

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For $b = 1.5$, $A = 10^{-5}$ and $t/t_0 = 0.125$ ($z \sim 3$), we get

$$\frac{\Delta c}{c} = 0.9558 \times 10^{-5}, \tag{17}$$

which is equivalent to a 1 part in 10^5 increase in the presently measured speed of light $c_0 = 299792458 \, m \, s^{-1}$ [30].

3 Conclusions

We have shown that a varying speed of light can explain the reported variations in the fine structure constant, while satisfying all the observational bounds. In particular, we can maintain the good agreement with the CMB data and the big bang nucleosynthesis calculations.

Our simple model for a varying fine structure constant is phenomenological in nature and serves the purpose of showing that a VSL model can be consistent with the data, while resolving the initial value problems in cosmology. A more fundamental model can be based on a covariant bimetric formulation of speed of light and gravitational wave speed, in which two light cones expand or contract in the early universe [11]. More research will be needed to resolve the issue as to which fundamental constants are varying and causing the reported variation in the fine structure constant.

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