COSMOLOGY 'WITHOUT' CONSTANTS

JOÃO MAGUEIJO
The Blackett Laboratory, Imperial College, South Kensington, London SW7 2BZ, UK

Abstract. We review varying speed of light (VSL) theories, in a cosmological and astrophysical setting, and as phenomenological descriptions of quantum gravity. We first introduce two observational puzzles, and explain how they may related to VSL. We then catalogue the various theoretical efforts associated with VSL. We focus on recent VSL theories rendering the Planck energy invariant, and capable of explaining the unwarranted existence of ultra high energy cosmic rays. We conclude highlighting the main shortcoming of current VSL theories: structure formation; and present directions in which it may be addressed.

1. Introduction

One field of work in which there has been too much speculation is cosmology. There are very few hard facts to go on, but theoretical workers have been busy constructing various models for the universe, based on any assumptions that they fancy. These models are probably all wrong. It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular quantities which are considered to be constants of nature may be varying with cosmological time. Such variations would completely upset the model makers.

Paul Dirac, 'On methods in theoretical physics', June 1968, Trieste

Since Dirac wrote these words, in 1968, much has changed in our understanding of the universe. It is fair to say that cosmologists have now quite a few 'hard facts' to go on. They have mapped the cosmological expansion up to redshifts of order one. They have made high precision observations of the cosmic microwave background (CMB), a major asset to observational cosmology – and hardly an established fact in 1968. And Big Bang nucleosynthesis has become a (more or less) direct probe of the conditions in the universe 1 second after the Big Bang. No longer can cosmologists say whatever populates their imaginations. It would not be an exaggeration to say that cosmology has finally become an experimental science or – some might say – a proper science.

And yet, any statement about the universe before one second of age is necessarily a speculation. No observational technique has so far been able to penetrate into this murky past, and Dirac's views are still painfully applicable. In particular, it could well be that the constants of nature are not constant at all, but were varying

Astrophysics and Space Science **283:** 493–503, 2003.

significantly during this early phase in the life of the universe. Assuming their constancy at all times requires massive extrapolation, and has no observational basis. Could the universe come into being riding on top of wildly varying constants?

This question is far from a new consideration. Physicists have for long entertained the possibility of a varying gravitational constant (e.g. the so-called Brans-Dicke theories), or a varying electron charge (and other coupling constants too). Indeed with the advent of string theory these varying constants seem to be fashionable. In contrast, the constancy of the speed of light has remain sacred. The reason is clear: the constancy of c is the pillar of special relativity. And yet, the first varying constant theory I am aware of was Kelvin's 1874 varying speed of light proposal. Some 30 years before relativity, a varying c did not shock anyone. A few years after relativity was proposed, however, Eddington would say 'A variation in c is self-contradictory', a statement smelling of religion, and which does not do a favour to the issue of the testability (and scientific respectability) of relativity. The religious fervour of Eddington, unfortunately, is still with us.

More exciting, of course, is the possibility that residual variations in c may be within the reach of current cosmological observations. And indeed there are observational puzzles that suggest such a conclusion. Firstly, the recent high redshift mapping of the fine structure constant has provided evidence for a time dependence of this supposed constant of Nature. Yet another puzzle was the observation of rare *very* high energy cosmic rays. Standard kinematic calculations, based on special relativity, predict a cut-off well below the observed energies, so this may perhaps represent the first experimental mishap of special relativity. Could these be seen as evidence for a varying speed of light?

2. Some Observational Puzzles

2.1. A VARYING FINE STRUCTURE CONSTANT

Webb and collaborators (Murphy et al., 2001; Webb et al., 1999) have reported evidence for a redshift dependence in the fine structure constant. The trend of these results is that the value of α was *lower* in the past, with $\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}$ for $z \approx 0.5 - 3.5$. In Figure 2.1 these results are displayed. It is clear that such a result, if true, has tremendous implications. What could be the meaning of a changing α? Looking at the formula $\alpha = e^2/(\hbar c)$, one is immediately faced with a related question – if α is varying, what else must be varying: e, c, $\hbar$, or a combination thereof? Could such a matter be *directly* resolved by experiment?

A moment of thought reveals that this question doesn't make much sense. Whereas α is a dimensionless constant, the 3 constants which make it up do have dimensions or units. But discussing observational constraints on varying *dimensional* constants is dangerous, because they depend upon the way the units have been defined. If α is seen to vary, the units employed to quote physical measurements

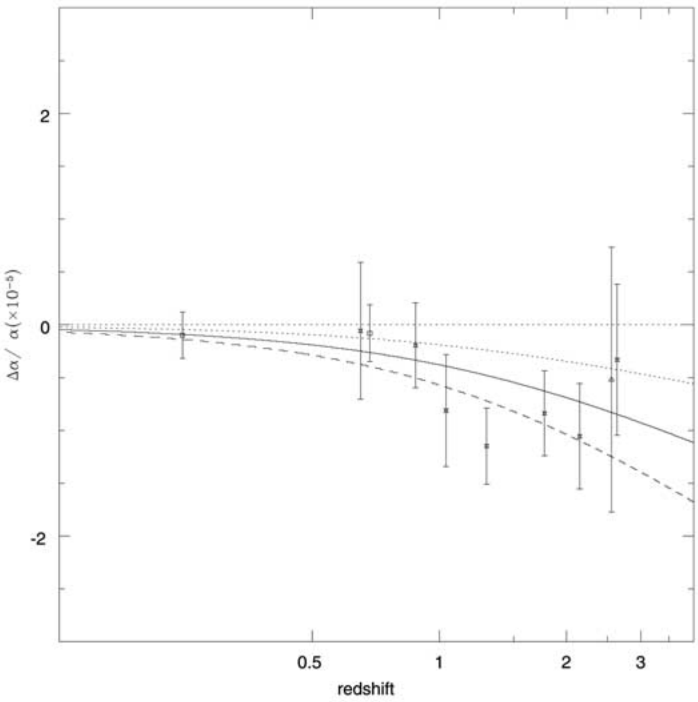

Figure 1. The data points are the QSO results for the changing α. The various lines depict theoretical prediction in several varying-α models.

may also be expected to vary. A meter stick may elongate or contract and a clock tick faster or slower. Hence under a changing α there is no a priori guarantee that units of length, time and mass are fixed, and discussing the variability or constancy of a parameter with dimensions is necessarily circular and depends on the definition of units one has employed. It is precisely to avoid such embarrassing situations that astronomers choose to discuss their observational constraints in terms of parameters, like α, which have no dimensions. They are then testing the true immutability or otherwise of physics, beyond conventions or definitions.

Theorists, however, need dimensional constants in order to set up their theories. Hence the question as to which of e, c, or $\hbar$ is varying is really a question for theorists, at least in the first instance. In order to set up a theory it may be more convenient to make one choice rather than any other. In dilaton theories, or variants thereof (Sandvik et al., 2002; Bekenstein, 1982; Olive and Pospelov, 2001), the observed variations in α are attributed to e; VSL theories (Moffat, 1993; Albrecht and Magueijo, 1999; Barrow, 1999; Barrow and Magueijo, 1999; Barrow and Magueijo, 1998; Magueijo, 2000) blame c for this variation (and in some cases

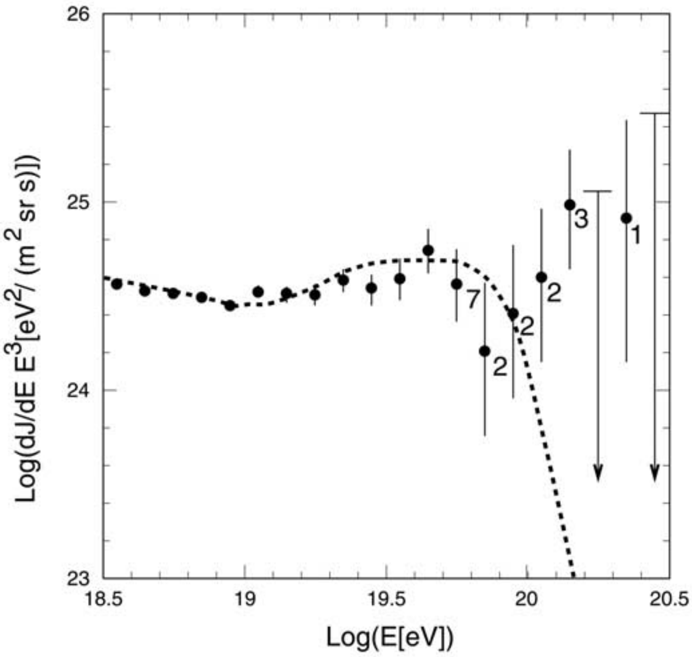

Figure 2. The flux of cosmic rays at high energies. The dashed line illustrates the GZK cut-off.

$\hbar$ too, see Magueijo, 2000). These choices are purely a matter of convenience, and one may change the units so as to convert a VSL theory into a constant c, varying e theory; however such an operation is typically very contrived, with the resulting theory looking extremely complicated. Hence the dynamics associated with each varying α theory "chooses" the units to be used, on the grounds of convenience, and this choice fixes which combination of e, c and $\hbar$ is *assumed* to vary.

The good news for experimentalists is that once this theoretical choice is made, the different theories typically lead to very different predictions. Dilaton theories, for instance, violate the weak equivalence principle, whereas VSL theories do not(Moffat, 2001). VSL theories often entail breaking Lorentz invariance, whereas dilaton theories do not. These differences have clear observational implications, for instance the STEP satellite could soon rule out the dilaton theories capable of explaining the Webb et al results (http://einstein.stanford.edu/STEP/). Violations of Lorentz invariance, as we shall see, should also soon be observed – or not.

Hence the question 'is it e or c?', although not directly an observational matter, does return to experiment, and we may hope to get an answer in this respect in the near future.

2.2. THRESHOLD ANOMALIES

Another puzzling set of observations are ultra high energy cosmic rays (UHECRs). These are rare events in which one observes showers derived from primary cosmic rays, probably protons, with energies above 10^{11} Gev. At these energies there are no known cosmic ray sources within our own galaxy, so it’s expected that in their travels the extra-galactic UHECRs should interact significantly with the cosmic microwave background (CMB). These interactions can be shown to impose a hard cut-off above about 10^{11} Gev, the cosmic ray energy at which it becomes kinematically possible to produce a pion. This is the so-called GZK cut-off, and the fascinating result is that UHECRs have been observed beyond this threshold (see Figure 2.2). Somehow the Universe is more transparent to these rays than is predicted.

Several explanations for this result have been advanced, with the most radical noting that the argument leading to the GZK cut-off relies on Lorentz transformations relating the CMB (or cosmological) frame, the proton rest frame, and the center of mass frame. The suggestion is then that the observed threshold anomaly results from quantum gravitational effects breaking Lorentz invariance, producing corrections to the Lorentz transformations (Amelino-Camelia and Piran, 2001).

UHECRs are not lonely freaks: a similar threshold anomaly results from the observation of high energy gamma rays above 10 Tev. In this case one expects a cut-off due to interactions with the infra-red background, with the cut-off energy corresponding to the kinematical condition for production of an electron-positron pair (Finkbeiner et al., 2000). For a threshold reaction, the electron-positron pair should have no momentum in the center of mass frame. Hence, the two photons should both have energies of approximately 0.5 Mev in this frame (the electron’s rest energy). In order to infer how these energies are perceived in the cosmological frame, one then performs a boost so as to redshift one of the photons to the infra-red background energy. The same boost blueshifts the other photon to the expected γ-ray threshold energy. Could the latter operation differ from the special relativistic prediction?

In both cases it is clear that threshold anomalies may imply corrections to the special relativity formula for boosts at very high energies. Conflict with special relativity leaves three options: the existence of a preferred frame, VSL, or a combination of the two. The first and last possibilities are fascinating: perhaps UHECRs are nothing but our first detection of an ‘ether wind’. But this is not necessary: VSL theories without a preferred frame do exist, and may explain the observed anomalies.

3. Varying Speed of Light Theories

There is a growing literature on VSL theories, and here I categorize the main implementations proposed so far, without trying to be exhaustive. All VSL theories conflict in one way or another with special relativity and here I shall use the type of insult directed at special relativity as my classification criterion for VSL theories.

Recall that special relativity is based upon two *independent* postulates – the relative nature of motion and the constancy of the speed of light. VSL theories do not need to violate the first of these postulates, but in practice one finds it difficult to dispense with the second without destroying the first. This leads to our first criterion for differentiating the various proposals: do they honour or insult the relative nature of motion?

Regarding the second postulate of special relativity, VSL theories behave in a variety of ways, all arising from a careful reading of the small print associated the constancy of c. Loosely this postulate means that c is a constant, but more precisely it states that the speed of all *massless* particles is the same, regardless of their color (frequency), direction of motion, place and time, and regardless of the state of motion of observer or emitter. The number of combinations in which this can be violated to accommodate a VSL is large, and explains the large number of theories which have been put forward.

Bearing this in mind we can now distinguish the following main VSL mechanisms:

– **Hard breaking of Lorentz symmetry**
  The most extreme model is that proposed by Albrecht and Magueijo (Albrecht and Magueijo, 1999), and studied further by Barrow (Barrow, 1999). In this model both postulates of special relativity are violated: there is a preferred frame in physics, identified with the cosmological frame; the speed of light varies in time, although usually only in the very early Universe; and the invariance in time of physics is broken. In spite of the extreme violence done to relativity, to a large extent this is still the best model as far as cosmological applications are concerned.

– **Deformed dispersion relations**
  This approach was pioneered by Amelino-Camelia and collaborators (Amelino-Camelia et al., 1997; Ellis et al., 2000; Ellis et al., 2001; Amelino-Camelia and Majid, 2000; Amelino-Camelia, 2001; Bruno et al., 2001). At its most sophisticated it preserves the relative nature of motion, while violating the second postulate of special relativity in the sense that the speed of light is allowed to vary with its color, for frequencies close to the Planck frequency. This is achieved by deforming the photon dispersion relations so that its group velocity acquires an energy dependence. These theories are popular mainly as phenomenological descriptions of quantum gravity. Cosmologies based on them have been constructed (Alexander and Magueijo, 2001; Alexander et al., 2001) and they could explain the threshold anomalies (Amelino-Camelia and Piran, 2001).

– **Bimetric theories of gravity**
  This approach was initially proposed by Moffat and Clayton (Clayton and Moffat, 1999), and also by Drummond (Drummond, 1999). Again, one does

not sacrifice the first principle of special relativity, and special care is taken with the damage caused to the second. In these theories the speeds of the various massless species may be different, but special relativity is still realized within each sector. Typically the speed of the graviton is taken to be different from that of all massless matter particles. This is implemented by introducing two metrics (one for gravity and one for matter), related by the gradients of a scalar field. The greatest achievement of this type of theory is that it is the only VSL theory so far which has led to a model of structure formation (Clayton and Moffat, 2002).

– **'Lorentz invariant' VSL theories**
  At the other end of the scale, it is possible to preserve the essence of Lorentz invariance in its totality and still have a varying c. One possibility is that Lorentz invariance is spontaneously broken, as proposed by Moffat in his seminal paper (Moffat, 1993). Here the full theory is endowed with exact local Lorentz symmetry; however the vacuum fails to exhibit this symmetry. For example an $O(3, 1)$ scalar field could acquire a time-like vacuum expectation value providing a preferred frame. Another example is the covariant and locally Lorentz invariant theory proposed in (Magueijo, 2000). Beautiful as these theories may be, their application to cosmology is somewhat cumbersome.
– **String/M-theory approaches**
  In the brane-world scenario we are stuck to a 4-brane which lives in 11 dimensions. Kiritis (Kiritsis, 1999) and Alexander (Alexander, 1999) found that if such a brane lives in the vicinity of a black hole it is possible to have perfect 11D Lorentz invariance (and hence a constant 11D speed of light), while realizing VSL on the brane. In this approach VSL results from a projection effect, and the Lorentz invariance of the full theory remains unaffected.

We shall now focus on one particular approach.

4. The Role of VSL in Quantum Gravity

The main idea in this approach is disconcertingly simple: the combination of gravity (G), the quantum ($\hbar$) and relativity (c) gives rise to the Planck length, $l_P = \sqrt{\hbar G/c^3}$ or its inverse, the Planck energy E_P. These scales mark thresholds beyond which the classical description of space-time breaks down and qualitatively new phenomena are expected to appear. No one knows what these new phenomena might be, but both loop quantum gravity (Rovelli, 1998; Carlip, 2001) and string theory (Polchinski, 1996; Forste, 2001) are expected to make clear predictions about them once suitably matured.

However, whatever quantum gravity may turn out to be, it is expected to agree with special relativity when the gravitational field is weak or absent, and for all experiments probing the nature of space-time at energy scales much smaller than E_P. This immediately gives rise to a simple question: *in whose reference frame are*

l_P and E_P the thresholds for new phenomena? For suppose that there is a physical length scale which measures the size of spatial structures in quantum space-times, such as the discrete area and volume predicted by loop quantum gravity. Then if this scale is l_P in one inertial reference frame, special relativity suggests it may be different in another observer's frame: a straightforward implication of Lorentz-Fitzgerald contraction.

There are several different answers to this question, the most obvious of which being that Lorentz invariance (both global and local) may only be an approximate symmetry, which is broken at the Planck scale. One may then correct the Lorentz transformations so as to leave the Planck scale invariant, and hope that the modified transformations have something to say about threshold anomalies (Amelino-Camelia, 2001; Bruno et al., 2001) (but see also (Amelino-Camelia and Piran, 2001; Amelino-Camelia et al., 1997; Ellis et al., 2000; Ellis et al., 2001; Gambini and Pullin, 1999; Alfaro et al., 2000; Adunas et al., 2000; Alexander and Magueijo, 2001) for other possible experimental implications). Another possibility is that Lorentz invariance gives way to a more subtle symmetry based on a quantum-group extension of the Poincare or Lorentz group(Bruno et al., 2001; Amelino-Camelia and Majid, 2000; Lukierski and Nowicki, 2002).

But perhaps the most conservative response is the one proposed in (Magueijo and Smolin, 2002), where it is shown that it is possible to modify the action of the Lorentz group on physical measurements so that a given energy scale, which is taken to be the Planck energy, is left invariant. Hence it is possible to have complete relativity of inertial frames, and have all observers agree that the scale on which a transition from classical to quantum spacetime occurs is the Planck scale, which is the same in every reference frame. At the same time, the familiar and well tested actions of Lorentz boosts are maintained at large distances and low energy scales.

According to this proposal one simply combines each boost with an energy dependent dilatation. The boost redshifts the energy; in turn the dilatation blueshifts it, negligibly so for small energies, but just enough to perfectly cancel the standard redshift at the Planck energy, so that this is left invariant. If the ordinary Lorentz generators act as ⋆

$$L_{ab} = p_a \frac{\partial}{\partial p^b} - p_b \frac{\partial}{\partial p^a} \tag{1}$$

then we consider the modified algebra

$$K^i \equiv L_0^{\ i} + l_P p^i D \equiv M_0^{\ i}. \tag{2}$$

where the dilatation generator $D = p_a \frac{\partial}{\partial p_a}$. Exponentiation reveals the finite group:

$$p_0' = \frac{\gamma\,(p_0 - v p_z)}{1 + l_P(\gamma - 1)p_0 - l_P \gamma v p_z} \tag{3}$$

⋆ Where we assume a metric signature $(+, -, -, -)$ and that all generators are antihermitian; also $a, b, c, = 0, 1, 2, 3$, and $i, j, k = 1, 2, 3$.

$$p'_z = \frac{\gamma\,(p_z - vp_0)}{1 + l_P(\gamma - 1)p_0 - l_P\gamma v p_z} \tag{4}$$

$$p'_x = \frac{p_x}{1 + l_P(\gamma - 1)p_0 - l_P\gamma v p_z} \tag{5}$$

$$p'_y = \frac{p_y}{1 + l_P(\gamma - 1)p_0 - l_P\gamma v p_z} \tag{6}$$

which reduces to the usual transformations for small $|p_\mu|$. It is not hard to see that the Planck energy is preserved by the modified action of the Lorentz group. In addition, these transformations do not preserve the usual quadratic invariant on momentum space, but preserve instead:

$$||p||^2 \equiv \frac{\eta^{ab} p_a p_b}{(1 - l_P p_0)^2} \tag{7}$$

This invariant is infinite for the new invariant energy scale of the theory $E_P = l_P^{-1}$, and is not quadratic for energies close to or above E_P. This signals the expected collapse in this regime of the usual concept of metric (i.e. a quadratic invariant).

Interestingly, despite the modifications introduced, J^i (the unmodified rotations) and K^i satisfy precisely the ordinary Lorentz algebra:

$$[J^i, K^j] = \epsilon^{ijk} K_k;\ \ [K^i, K^j] = \epsilon^{ijk} J_k \tag{8}$$

(with $[J^i, J^j] = \epsilon^{ijk} J_k$ trivially preserved). However the action on momentum space has become non-linear due to the term in p^i in (2). The new action can be considered to be a non-standard, non-linear embedding of the Lorentz group in the conformal group.

This construction can be generalized to incorporate any dispersion relations (which might be measured by experiment) relating E and p by:

$$E^2 f_1^2(E; l_P) - p^2 f_2^2(E; l_P) = m^2 \tag{9}$$

where f_1 and f_2 are phenomenological functions. Generally, for massless particles we find that $E/p = f_2/f_1$, so that the group velocity of light $c = dE/dp$ becomes energy dependent at very high energies. Although this is not necessary (and indeed (7) is a counter-example), it appears that a varying speed of light may in fact be an essential ingredient in the establishment of an invariant scale (the Planck scale), separating unambiguously the realms of classical and quantum gravity.

This unusual approach to quantum gravity, and its implications for threshold anomalies, is currently being actively investigated by a number of groups. But as shown in (Magueijo and Smolin, 2002) it is possible to design f_1 and f_2 so as to have an invariant energy scale E_P, raise the threshold for UHECRs, and realize VSL cosmology.

5. What's missing?

In spite of all this body of work there is something crucial still missing from VSL theories: a theory of structure formation. A major success of the Big Bang theory is its ability to account for the detailed structure of the Universe, such as galaxy clustering, or the temperature fluctuations in the cosmic microwave background. Paramount to this picture is the phenomenon of gravitational instability, by means of which primordial small departures from homogeneity can grow into the observed structures. Within the Big Bang theory the required primordial fluctuations are treated merely as initial conditions. However, vacuum *quantum* fluctuations in inflationary scenarios have been shown to lead to the required initial conditions, fitting current large scale structure data.

Could VSL provide the same type of initial conditions? One possible answer was provided by (Clayton and Moffat, 2002), in the context of bimetric VSL theories, and making use of quantum fluctuations in these scenarios. An alternative, currently being investigated, are thermal fluctuations in VSL scenarios where vacuum domination never occurs (Magueijo and Pogosian, 2002). One must stress that the recent observational victories in this area are a success of the Harrisson-Zeldovich spectrum plus gravitational instability, rather than 'proof' of inflation. Other scenarios providing a Harrisson-Zeldovich spectrum of initial conditions may well exist, and VSL might be one of them.

Acknowledgements

I would like to thank the organisers, in particular Carlos Martins, for an exciting and smooth-running (in a rather un-Portuguese style) workshop.

References

Adunas, G. et al.: 2000, *Phys. Lett. B* **485**, 215.
Albrecht, A. and Magueijo, J.: 1999, *Phys. Rev. D* **59**, 043516.
Alexander, S.: 2000, *JHEP* **0011**, 017, hep-th/9912037.
Alexander, S. and Magueijo, J.: 2001, hep-th/0104093.
Alexander, S., Brandenberger, R. and Magueijo, J.: 2001, hep-th/0108190.
Alfaro, J. et al.: 2000, *Phys. Rev. Lett.* **84**, 2318.
Amelino-Camelia, G. et al.: 1997, *Int. J. Mod. Phys. A* **12**, 607–624; also Amelino-Camelia, G. et al.: 1998, *Nature* **393**, 763–765.
Amelino-Camelia, G. and Majid, S.: 2000, *Int. J. Mod. Phys. A* **15**, 4301–4324.
Amelino-Camelia, G. and Piran, T.: 2001, *Phys. Rev. D* **64**, 036005.
Amelino-Camelia, G.: 2001, *Phys. Lett. B* **510**, 255–263; also gr-qc/0012051.
Barrow, J.D.: 1999, *Phys. Rev. D* **59**, 043515.
Barrow, J.D. and Magueijo, J.: 1998, *Phys. Lett. B* **443**, 104.
Barrow, J.D. and Magueijo, J.: 1999, *Phys. Lett. B* **447**, 246.

Bekenstein, J.D.: 1982, *Phys. Rev. D* **25**, 1527.
Bruno, N.R., Amelino-Camelia, G. and Kowalski-Glikman, J.: 2001, *Phys. Lett. B* **522**, 133–138.
Carlip, S.: 2001, *Rept. Prog. Phys.* **64**, 885.
Clayton, M.A. and Moffat, J.W.: 1999, *Phys. Lett. B* **460**, 263–270; gr-qc/9910112; gr-qc/0003070.
Clayton, M.A. and Moffat, J.W.: 2002, astro-ph/0203164.
Drummond, I.: 1999, gr-qc/9908058.
Ellis, J. et al.: 2000, *Astrophys. J.* **535**, 139–151.
Ellis, J., Mavromatos, N.E. and Nanopoulos, D.: 2001, *Phys. Rev. D* **63**, 124025; ibidem astro-ph/0108295.
Finkbeiner, D., Davis, M. and Schlegel, D.: 2000, *Atroph. J.* **544**, 81.
Forste, S.: 2001, hep-th/0110055.
Gambini and Pullin: 1999, *Phys. Rev. D* **59**, 124021.
Kiritsis, E.: 1999, *JHEP* **9910**, 010, hep-th/9906206.
Lukierski, J. and Nowicki, A.: 2002, hep-th/0203065 and references therein.
Magueijo, J.: 2000, *Phys. Rev. D* **62**, 103521.
Magueijo, J. and Pogosian, L.: 2002, in preparation.
Magueijo, J. and Smolin, L.: 2002, *Phys. Rev. Lett.* **88** (190403); also gr-qc/0207085.
Moffat, J.: 1993, *Int. J. Mod. Phys. D* **2**, 351.
Moffat, J.: 2001, astro-ph/0109350.
Murphy, M.T. et al.: 2001, *MNRAS* **327**, 1208.
Olive, K. and Pospelov, M.: 2001, hep-ph/0110377.
Polchinski, J.: 1996, hep-th/9611050.
Rovelli, C.: 1998, *Living Rev. Rel.* **1**, 1.
Sandvik, H.B., Barrow, J.D. and Magueijo, J.: 2002, *Phys. Rev. Lett.* **88**, 031302; also Barrow, J.D., Sandvik, H.B. and Magueijo, J.: astro-ph/0109414.
Webb, J.K., Flambaum, V.V., Churchill, C.W., Drinkwater, M.J. and Barrow, J.D.: 1999, *Phys. Rev. Lett.* **82**, 884.