Variation of the fine structure constant caused by the expansion of the Univerce.

Anton A. Lipovka, Ivan A. Cardenas
Department of Investigation for Physics,
Sonora University, Sonora, Mexico.
e-mail: aal@cifus.uson.mx

May 15, 2018

Abstract

In present paper we evaluate the fine structure constant variation, that should take place as the Universe expands and its curvature is changed adiabatically. Such variation of the fine structure constant is attributed to an energy losses by an extended physical system (consist of baryonic component and electromagnetic field) due to expansion of our Universe. Obtained ratio $\dot{\alpha}/\alpha = -1 \cdot 10^{-18}$ (per second) is only five times smaller than actually reported experimental limit on this value. For this reason obtained variation can probably be measured within a couple of years. To argue the correctness of our approach we calculate the Planck constant as adiabatic invariant of the electromagnetic field propagated on a manifold characterized by slowly varied geometry, in the framework of the pseudo-Riemannian geometry. Finally we discuss the double clock experiment based on Al^+ and Hg^+ clocks carried out by T. Rosenband et al. (Science 2008). We show that in this case (when the fine structure constant is changed adiabatically) the method based on double clock experiment can not be applied to measure the fine structure constant variation.

PACS numbers: 06.30.Ft,06.20.Jr,32.60.+i,37.10.Ty

1 Introduction

The extremely important problem of the fundamental constants variation attracts great attention of the scientific community for the last decades. Every year a lot of papers on this subject are published both in theory as well in measurement methods (see [1,2] and references therein). Such an interest in the subject is due to the huge importance of the problem of the fundamental constants variation for understanding foundation of physics. Particular attention is paid for the variation of the fine structure constant, because it is basic parameter for QED and because the experimental measurements have reached

1

unprecedented accuracy. It is well known, that the search for variations under discussion is carried out both in laboratories [3-7] and by using the cosmological data obtained from observed spectra of distant quasars [1,2,8,9,10]. Unfortunately up to now, such variations have not been detected yet, but it is important to note that in the last decade, the accuracy of laboratory measurements has approached closely to the limit of variation of fundamental constants, that must take place through the adiabatic change in the geometry of our Universe. For this reason, the need for a correct theoretical estimate of the fine structure constant variation due to adiabatic change in geometry is clearly visible.

In this paper we fill this gap and suggest the calculation of the fine structure constant variation on time, which must take place due to the adiabatic changes of scalar curvature provoked by expansion of our universe. In order to confirm the correctness of the obtained result, we calculate by the same method the adiabatic invariant for free electromagnetic field (propagating on the Riemannian manifold characterized by adiabatically changed curvature) which actually is the Planck constant. As it was mentioned, all calculations are carried out in the framework of pseudo - Riemannian geometry and for this reason obtained value is differ slightly (by factor 3/2) of their real value calculated for the Finslerian manifold [11]. Finally we explain why this variations are not detected in the experiments based on comparison of two different frequencies like those discussed in recent papers [6,7].

2 Changing of the fine structure constant due to expansion of the Universe

Let us consider a system that consists of a classical field located on the Riemannian manifold characterized by the adiabatically changed curvature. In this case as it was previously shown [11, 12] (see also the next part of this paper) such a system is characterized by an adiabatic invariant, which for the electromagnetic field is actually the Planck constant. Moreover, this adiabatic invariant depends on the scalar curvature of the Universe measured in the point of observation and for this reason is varied over time [11, 12]. The fine structure constant in turn depends on h ($\alpha = e^2/hc$) and for this reason its value also must changes over time. It should be stressed here, this consideration is applied not only to the classical fields (particularly to the electromagnetic field), but also to any adiabatically isolated system consisting of fields and baryonic matter interacting by means of this field. In this case, parameters of the system as a whole depend adiabatically on the geometry of manifold. So as the Universe expands, any physical system (for example an atom) will lose its energy adiabatically.

How large this variation of energy is? To begin with let us make very preliminary and simple estimation of the effect we are interested in. Consider a system which consists of the classical field and characterized by energy E distributed over volume V (we can put $V = 1 cm^3$). In this case the changing of the energy due to expansion of the Universe is

$$\frac{\delta E}{E} = -\frac{\delta V}{V} = -3\frac{\delta l}{l} \quad . \tag{1}$$

But in consistence with Hubble relation

$$\delta l = Hl\delta t \quad . \tag{2}$$

For this reason we can evaluate

$$\frac{\delta E}{E} \approx \frac{\delta \alpha}{\alpha} \approx -3H\delta t = -7 \cdot 10^{-18}\delta t \quad . \tag{3}$$

This very simple estimation gives us an idea about the value of variation we should expect to obtain in general case.

Now let M be an 3-dimensional C^∞ manifold characterized by scalar curvature $\mathcal{R} = 2/R^2$, where R is the curvature radius, x be a local coordinate on an open subset $U \subset M$. $T_p(M)$ and $T_p^*(M)$ are respectively tangent and cotangent bundles on M, where $P_\alpha \in T_p(M)$ and $P^\alpha \in T_p^*(M)$ are covariant and contravariant components of corresponding 4-momentum.

We are interested in variation of the 4-momentum components P as functions of the Universe radius R and, consequently, of time t. By taking into account relation $x = R\varphi$, (here x actually is the size of resonator for the electromagnetic field, R is the radius of the universe and φ is corresponding small angle in radians), we can write projection of x on tangent and cotangent bundles of M as

$$P^\alpha = \xi R \sin\varphi \tag{4}$$

$$P_\alpha = \xi R \tan\varphi \quad , \tag{5}$$

where coefficients $\xi = \frac{2c}{\kappa}$ (here $\kappa = 8\pi G/c^2$ is the coupling constant for the Einstein field equations) are written to comply $\mathcal{R} = \frac{\kappa}{c^2}T$ in classical limit, and factor 2 appears from relation $\mathcal{R} = 2/R^2$. In this case the absolute value of the momentum can be written as

$$P = \sqrt{P_\alpha P^\alpha} = \frac{2c}{\kappa}R\frac{\sin\varphi}{\sqrt{\cos\varphi}} \quad , \tag{6}$$

where $R = x/\varphi$ is for the local (effective) radius of curvature of the universe in the point in which our system is localized.

By taking into account that $\varphi \ll 1$ for any reasonable laboratory system, we can restrict our consideration by first and second terms of the expansion of $\sin\varphi$ and $\sqrt{\cos\varphi}$, then we get

$$P = \xi R\left(\varphi + \frac{\varphi^3}{12}\right) \quad . \tag{7}$$

3

As our manifold M expands, the value of P also changes and taking into account that $x = R\varphi$, we immediately obtain from (7):

$$\delta P = -\frac{c^3}{24\pi G R^3}\delta R \quad . \tag{8}$$

It should be stressed here - we write this expression for propagating electromagnetic field localized within a unit volume. Actually this relation describes the momentum losses by system due to adiabatic changing of the manifold's curvature.

To evaluate this expression, we need to re-express R through the observable parameters. Actually we have such a parameter, named as Hubble constant H. But H give us relation for passing trajectory l: $\delta l = H l \delta t$.

To establish relation between R and l let us imagine a fly walking over globe with velocity c, whereas we inflate the globe such that $\dot{R} = c$ too. It is easy to show that in this simple case the integrated length l is $l = 2R$. Actually this is the length which pass a photon when it propagates on manifold while its curvature is changing due to expansion.

So in this case our expression can be rewritten as:

$$\delta P = -\frac{cH^3}{6\pi G}\delta t \quad . \tag{9}$$

To evaluate variation of the fine structure constant α, it should be noted that historically it was introduced by Sommerfeld as $\alpha = v/c$, where v is the electron velocity at the first Bohr orbit for the hydrogen atom. This definition is correct for classical limit $v << c$ up to 3-rd digit, and by taking into account the fact that we are interested in the first digit (actually we calculate the order of magnitude of the variation), we may accept this definition for our calculation. For this reason the momentum of electron is

$$P = \frac{m\alpha c}{\sqrt{1 - \alpha^2}} \quad , \tag{10}$$

and varying it we obtain a losses of momentum by electron on the first Bohr orbit due to adiabatically changing curvature governed by expansion of our universe (see also [11, 12, 13])

$$\delta P = \frac{mc}{\left(1 - \alpha^2\right)^{3/2}}\delta\alpha \quad . \tag{11}$$

By substituting this expression into (9), we find

$$\delta\alpha = -\frac{\left(1 - \alpha^2\right)^{3/2} H^3}{6\pi G m}\delta t \tag{12}$$

This is the variation of the fine structure constant on time due to adiabatically changed curvature of the Riemannian manifold.

It should be stressed here, this expression for $\delta\alpha$ coincide well with that obtained in [12] (see also [11]), within the framework of the Einstein-Cartan geometry , if we write it for the Riemannian manifold (i.e. when $\Lambda = 0$).

Namely we have in [12] ($\Lambda = 0$):

$$\alpha = \frac{c^2}{32\pi^2 Gm}\mathcal{R} \tag{13}$$

By varying this expression we immediately obtain

$$\delta\alpha = -\frac{H^3}{2\pi^2 Gm}\delta t \tag{14}$$

that perfectly agree with above obtained expression (12).

Direct calculation for $H = 73 \ kms^{-1}Mpc^{-1} = 2.4 \cdot 10^{-18}s^{-1}$ give us value $\dot\alpha/\alpha = -1.7 \cdot 10^{-18}$ (in 1 second).

This value is about 5 times smaller if compared with reported sensitivity $\dot\alpha/\alpha < 5 \cdot 10^{-18}$ [3], but the difference is not so large and we hope the required sensitivity will be achieved within a couple of years.

3 Planck constant from the first principles

Einstein [14] and later Debye [15] at the beginning of XX century have shown from thermodynamics that electromagnetic field is quantized and this fact do not depends of the oscillators properties (properties of baryonic matter). Unfortunately there was not paid duly attention to this result and historically it was the baryonic component that was quantized first whereas the electromagnetic field was quantized much later in 1950 by Gupta [16] and Bleuler [17].

In this part of paper we show how the electromagnetic field is quantized on the pseudo - Riemannian manifold with adiabatically changed scalar curvature. Namely we obtain from the geometry of our Universe the adiabatic invariant for Electromagnetic field (which should be identified with the Planck constant). As it was mentioned in the introduction, the calculation of the Planck constant value is made with the same method and for this reason the obtained result serves as an independent verification of the validity of the applied method.

As we have seen from the first part of this paper, the momentum P and energy of electromagnetic field propagating on the manifold with adiabatically changed curvature are changed on time. This variation proceeds adiabatically and can be considered as linear function, that is, we retain only the first term of the expansion and neglect the corrections of subsequent orders of smallness.

$$\frac{\delta E}{E} = -\frac{\delta t}{t} \tag{15}$$

From this expression we can immediately write the adiabatic invariant we are interested in

$$Et = -\frac{\delta E}{\delta t}t^2 \tag{16}$$

But for free electromagnetic field we have

$$\delta E = c\delta P \tag{17}$$

By substituting δP obtained before into this expression we can write finally for energy in 1 $cm.^{-3}$

$$Et = \frac{c^2 H^3}{6\pi G} t^2 = 9.93 \cdot 10^{-27} \, (erg \cdot s.) \tag{18}$$

for one second in unit volume. It is a very good coincidence with real value $h = 6.6 \cdot 10^{-27} \, (erg \cdot s.)$ for such a simple model we have considered here within the framework of the Riemannian geometry which is differ of the Finsler geometry by the absence of the cosmological constant. It should be stressed, we do not include the cosmological constant Λ into consideration because on the one hand it naturally appears only in the complete Finsler geometry, on the other hand, this paper is dedicated mainly to the problem of the fine structure constant variation in the Riemann geometry, and it is difficult discus here all details of real geometry of our Universe and nature of cosmological constant. We just note here that if the actually measured value $\Lambda = 1.7 \cdot 10^{-56}$ is taken into account, the obtained here value of the Planck constant will decrease slightly and reach actually measured value $h = 6 \cdot 10^{-27} \, (erg \cdot s.)$. The reader can see these details in our previous works [11, 12].

To conclude this part we stress again that we prove geometrically the fact that the electromagnetic field is quantized alone even on the expanded Riemannian manifold. To do this we need not oscillators and baryonic matter. The only we need for free electromagnetic field to be quantized is adiabatically changed curvature of manifold.

4   The Hg^+ and Al^+ optical clocks experiment

In first part of the paper we have shown that the fine – structure constant variation due to adiabatically changed curvature of manifold is $\dot{\alpha}/\alpha = 1.7 \cdot 10^{-18} \, (s^{-1})$. As it was mentioned above, at present time the experimental constrain on the $\dot{\alpha}/\alpha$ is very close to calculated value and consist $\dot{\alpha}/\alpha < 5 \cdot 10^{-18} \, (s^{-1})$ [3], so, probably within a couple of years experimental facilities will be able to measure the variation of fine structure constant caused by expansion of our Universe, discussed above.

However there is another type of experiments based on comparison of frequencies variation of two optical clocks. Most precise measurements of this kind were reported by Rosenband et al in 2008 [6] (see also paper [7] for the same problem) for Al^+ and Hg^+ single-ion optical clocks. In this paper the preliminary constraint on the temporal variation of the fine-structure constant $\dot{\alpha}/\alpha < 5 \cdot 10^{-17} \, (yr^{-1})$ were suggested, that actually corresponds to variation $\dot{\alpha}/\alpha < 3 \cdot 10^{-25} \, (s^{-1})$. In this case a reasonable question arises: why variation we calculate $\dot{\alpha}/\alpha = 10^{-18} \, (s^{-1})$ was not measured, whereas (as we have seen

before) it inescapably should appears due to expansion of the Universe? The answer on this question is simple: because the variation proceeds adiabatically. Let us consider this issue in details by taking as an example the paper [6] (the same way one can explain the negative result reported in [7]). The authors of paper [6] reported that they were measuring variation of ratio of frequencies, i.e. $\delta\left(\nu_{Al^+}/\nu_{Hg^+}\right)$.

To make our expressions more clear, let us write 1 for Al^+ and 2 for Hg^+. In this case the measured variation can be written as:

$$\delta\left(\frac{\nu_1}{\nu_2}\right) = \frac{E_1}{E_2}\left(\frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}\right) \tag{19}$$

where E_1 and E_2 are the energies of transitions $i \to f$ for Al^+ and Hg^+ respectively. So

$$\left(\frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}\right) = \frac{\delta\left(E_{1i} - E_{1f}\right)}{E_{1i} - E_{1f}} - \frac{\delta\left(E_{2i} - E_{2f}\right)}{E_{2i} - E_{2f}} = \tag{20}$$

$$= -\frac{\delta E_{1i}}{E_{1i}}\frac{E_{1i}}{E_{1f}}\frac{1}{1 - \frac{E_{1i}}{E_{1f}}} + \frac{\delta E_{1f}}{E_{1f}}\frac{1}{1 - \frac{E_{1i}}{E_{1f}}} + \frac{\delta E_{2i}}{E_{2i}}\frac{E_{2i}}{E_{2f}}\frac{1}{1 - \frac{E_{2i}}{E_{2f}}} - \frac{\delta E_{2f}}{E_{2f}}\frac{1}{1 - \frac{E_{2i}}{E_{2f}}} \quad .$$

But for adiabatic variation we have $\frac{\delta E_{1i}}{E_{1i}} = \frac{\delta E_{1f}}{E_{1f}} = \frac{\delta E_{2i}}{E_{2i}} = \frac{\delta E_{2f}}{E_{2f}}$, thus

$$\frac{\delta E_{1i}}{E_{1i}} - \frac{\delta E_{1i}}{E_{1i}} = 0 \tag{21}$$

and therefore

$$\delta\left(\frac{\nu_{Al^+}}{\nu_{Hg^+}}\right) = 0 \quad . \tag{22}$$

So one can conclude that the geometrical adiabatic variation cannot be observed in such experiments, when the frequencies of two single-ion optical clocks are compared.

5   Conclusions

In present paper we calculate variation of the fine structure constant which must take place due to expansion of the Universe. For the pseudo – Riemannian manifold it consist $\dot{\alpha}/\alpha = 1.7 \cdot 10^{-18}\left(s^{-1}\right)$ that only 5 time smaller than currently established constrains on this value $\dot{\alpha}/\alpha < 5 \cdot 10^{-18}\left(s^{-1}\right)$ [3].

We also show that on the pseudo – Riemannian manifold there exist adiabatic invariant for electromagnetic field which depends on the curvature and has a value very close (it differ by factor 3/2) to the laboratory measured Planck constant. Exact value for the Planck constant, as function of curvature and

cosmological constant, can be calculated only within the framework of the complete Finslerian geometry and can be found in [11] and [12] . This suggests that we live not in the (pseudo-) Riemannian world, but in Finsler one.

It is shown that double clock experiment is not appropriate for measurement of adiabatically changed values (particularly it can not be applied to measure the fine structure constant variation).

References

[1] S. A. Levshakov. (2016) arXiv:1603.01262

[2] P. Bonifacio, H. Rahmani, J.B. Whitmore, M. Wendt, M. Centurion, P. Molaro, R. Srianand, M.T. Murphy, P. Petitjean, I.I. Agafonova, S. D'Odorico, T.M. Evans, S.A. Levshakov, S. Lopez, C.J.A.P. Martins, D. Reimers and G. Vladilo. (2014) Astronomische Nachrichten Volume 335, Issue 1, pages 83–91.

[3] D. R. Leibrandt, M. J. Thorpe, C. W. Chou et al. (2013) Phys. Rev. Lett. 111, 237402.

[4] Shon Cook, Till Rosenband, and David R. Leibrandt. (2015) Phys. Rev. Lett. 114, 253902.

[5] Y. V. Stadnik and V. V. Flambaum. (2016) Phys. Rev. A 93, 063630

[6] T. Rosenband, D. B. Hume, P. O. Schmidt, C. W. Chou, A. Brusch, L. Lorini, W. H. Oskay, R. E. Drullinger, T. M. Fortier, J. E. Stalnaker, S. A. Diddams, W. C. Swann, N. R. Newbury, W. M. Itano, D. J. Wineland, J. C. Bergquist. (2008) Science. 28;319(5871)

[7] R.M. Godun, P.B.R. Nisbet-Jones, J.M. Jones, S.A. King, L.A.M. Johnson, H.S. Margolis, K. Szymaniec, S.N. Lea, K. Bongs, and P. Gill. (2014) Phys. Rev. Lett. 113, 210801.

[8] P. A. R. Ade, N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. J. Banday, R. B. Barreiro, E. Battaner et al. (160 more) (2015) Astronomy & Astrophysics, 580, A22

[9] Rahmani, H.; Srianand, R.; Gupta, N.; Petitjean, P.; Noterdaeme, P.; Vasquez, D. Albornoz. (2012) Monthly Notices of the Royal Astronomical Society, Volume 425, Issue 1, pp. 556-576. DOI: 10.1111/j.1365-2966.2012.21503.x

[10] Levshakov, S. A.; Combes, F.; Boone, F.; Agafonova, I. I.; Reimers, D.; Kozlov, M. G. (2012) Astronomy & Astrophysics , Volume 540, id.L9, 4 pp. DOI: 10.1051/0004-6361/201219042

[11] Lipovka, A. (2017) Physics on the Adiabatically Changed Finslerian Manifold and Cosmology. Journal of Applied Mathematics and Physics, 5, 582-595. doi: 10.4236/jamp.2017.53050. arXiv:1608.04596

[12] Lipovka, A. (2014) Journal of Applied Mathematics and Physics, 2, 61-71. doi: 10.4236/jamp.2014.25009.

[13] Lipovka, A. (2016) Journal of Applied Mathematics and Physics, 4, 897-902. doi: 10.4236/jamp.2016.45098.

[14] Einstein, A. (1905) Annalen der Physik, 17, 132-148.

[15] Debye, P. (1910) Annalen der Physik, 33, 1427-1434.

[16] Gupta, S. (1950), Proc. Phys. Soc. 63A (7): 681–691, Bibcode:1950PPSA...63..681G doi:10.1088/0370-1298/63/7/301

[17] Bleuler, K. (1950), Helv. Phys. Acta (in German) 23 (5): 567–586, doi:10.5169/seals-112124