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Varying alpha and the electroweak model

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Abstract

Inspired by recent claims for a varying fine structure constant, alpha, we investigate the effect of "promoting coupling constants to variables" upon various parameters of the standard model. We first consider a toy model: Proca theory of the massive photon. We then explore the electroweak theory with one and two dilaton fields. We find that a varying alpha unavoidably implies varying W and Z masses. This follows from gauge invariance, and is to be contrasted with Proca theory. For the two dilaton theory the Weinberg angle is also variable, but Fermi's constant and the tree level fermion masses remain constant unless the Higgs potential becomes dynamical. We outline some cosmological implications.
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1. Introduction

There is currently much interest in cosmological theories where the conventional "constants" of Nature may actually vary in space and time. The most observationally sensitive of these "constants" is the electromagnetic fine structure constant, α. The new observational many-multiplet technique of Webb et al., has provided the first evidence that the fine structure constant may change throughout cosmological time [1–3]. The trend of these results is that the value of α was lower in the past, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ for redshifts $z \approx 0.5$–3.5. Other investigations have claimed preferred non-zero values of $\Delta\alpha < 0$ to best fit the cosmic microwave background (CMB) and Big Bang Nucleosynthesis (BBN) data at $z \approx 10^3$ and $z \approx 10^{10}$, respectively [4,5].

A varying fine structure constant (defined to be $\alpha = e^2/4\pi\hbar c$) may be interpreted as a varying electric charge in a theory where $\hbar$ and c are held fixed. A simple varying e theory may be set up by *prescribing* that e become a dynamic field, the so-called minimal coupling prescription [6]. This electromagnetic varying e theory, reviewed here in Section 2, has been thoroughly explored [6–11], and a formal rearrangement shows that it is a particular type of dilaton theory. It is a theory in which the dilaton (a massless and gauge neutral scalar that interacts with matter at strengths comparable to that of gravity) couples to the electromagnetic F^2 term in the Lagrangian, but not to the other gauge fields [7,17].

Given that we already know that electromagnetic and weak interactions are unified, a natural question is how this electromagnetic theory extends to the standard model of particle physics, which rests on various "constants" in addition to e. The sheer multitude of "arbitrary" parameters within the standard model has

been a source of displeasure among theorists. Thus, in considering the electroweak extension of the electric model of [6], we are led to wonder whether some of these parameters become variables, and which are independent. Similar issues, in the context of the QCD, grand-unification, and the quantum vacuum energy, have been considered in [12–16].

In this Letter we extend the work of [6,7] and promote the couplings in the electroweak theory to dynamical fields. In preparation for this, in Section 3 we consider general non-abelian gauge groups with a varying coupling, and in Section 4 Proca theory. In the latter, a "gauge" boson acquires mass by explicitly breaking gauge invariance. It is possible to simultaneously have a varying e and a constant boson mass in this case. We then propose a version of the electroweak theory in which the $SU(2)$ gauge charge, $g'(x^\mu)$, in addition to the $U(1)$ gauge charge, $g(x^\mu)$, become dynamical according to a prescription similar to the one used in [6,7]. Again, a simplifying formal rearrangement converts the theory into a dilaton theory, this time with two dilaton fields that couple to the $SU(2)$ and $U(1)$ gauge fields. A single dilaton variation is also considered.

We find that the variable couplings inevitably lead to a theory in which the W and Z masses vary. This is to be contrasted with Proca theory and is directly related to gauge invariance. In the two dilaton case the Weinberg angle becomes a variable too. However, Fermi's constant and the tree level fermion masses remain constant, unless we also promote to variables the parameters in the Higgs potential. We outline some astrophysical and cosmological consequences.

2. Varying electromagnetic alpha and dilaton theories

In the varying α theories proposed in [6,7] one attributes variations in α to changes in e, or the permittivity of free space. This is done by letting e take on the value of a real scalar field which varies in space and time $e_0 \to e(x^\mu) = e_0 \epsilon(x^\mu)$, where $\epsilon(x^\mu)$ is a dimensionless scalar field and e_0 is a constant denoting the present value of $e(x^\mu)$. One then proceeds to set up a theory based on the principles of local gauge invariance, causality, and the scale invariance of the ϵ-field.

Since covariant derivatives take the form $D_\mu \phi = (\partial_\mu + ie A_\mu)\phi$, for gauge transformations of the form $\delta\phi = -i\chi\phi$ one must impose $\epsilon A_\mu \to \epsilon A_\mu + \chi_{,\mu}$. The gauge-invariant electromagnetic field tensor is then

$$F_{\mu\nu} = \frac{(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}}{\epsilon}, \tag{1}$$

which reduces to the usual form for constant ϵ. The electromagnetic Lagrangian density is still

$$\mathcal{L}_{\text{em}} = -\frac{F^{\mu\nu} F_{\mu\nu}}{4}, \tag{2}$$

and the dynamics of the ϵ field are controlled by the kinetic term

$$\mathcal{L}_\epsilon = -\frac{1}{2}\omega\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2}, \tag{3}$$

where the coupling constant ω is introduced into the Lagrangian density for dimensional reasons and is proportional to the inverse square of the characteristic length scale of the theory, $\omega \sim \ell^{-2}$, such that $\ell \geqslant L_p \approx 10^{-33}$ cm holds [6]. This length scale corresponds to an energy scale $\hbar c/\ell$, with an upper bound set by experiment. Note that the metric signature used is $(-, +, +, +)$.

A simpler formulation of this theory [7] can be constructed by defining an auxiliary gauge potential $a_\mu \equiv \epsilon A_\mu$, and field tensor $f_{\mu\nu} \equiv \epsilon F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. The covariant derivative then assumes the familiar form, $D_\mu = \partial_\mu + ie_0 a_\mu$, and the dependence of the Lagrangian on ϵ occurs only in the kinetic term for ϵ and in the $F^2 = f^2/\epsilon^2$ term, not in the rest of the matter Lagrangian $\mathcal{L}_m$ (where it could only have appeared via the covariant derivative D_μ). To simplify further, we can redefine the variable, $\epsilon \to \psi \equiv \ln\epsilon$. The total action then becomes

$$S = \int d^4x \left(\mathcal{L}_{\text{mat}} - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}e^{-2\psi}f_{\mu\nu}f^{\mu\nu}\right), \tag{4}$$

where the matter Lagrangian $\mathcal{L}_{\text{mat}}$ does not contain ψ. This is a dilaton theory coupling to the electromagnetic f^2 part of the Lagrangian only. Note the scale invariance of the action and ψ (that is, their invariance under the transformation $\epsilon \to k\epsilon$ for any constant k).

Given this mathematical trick, one may wonder which of the two sets of variables are the physics ones? The question is obviously irrelevant regarding A_μ and a_μ, because both are unphysical due to

gauge invariance. On the other hand, both $F_{\mu\nu}$ and $f_{\mu\nu}$ are "physical" and may be used as convenient (the problem is similar to the use of field **E** or displacement **D** in dielectric electrostatics). Note that the homogeneous Maxwell equations, $\epsilon^{\mu\nu\alpha\beta}\partial_\nu f_{\alpha\beta} = 0$, are not valid for $F_{\alpha\beta}$.

3. Varying couplings for non-abelian gauge groups

The tools derived for electromagnetism carry over trivially to non-abelian groups (see [12] for an example based on QCD). We take as an example $O(3)$. Let $\mathbf{\Phi}$ be a 3-vector, with covariant derivative

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} + g' \mathbf{W}_\mu \wedge \mathbf{\Phi}. \tag{5}$$

Here the gauge boson $\mathbf{W}_\mu$ is a 3-vector. Under a gauge transformation corresponding to a rotation defined by vector $\mathbf{\Xi}$, we have:

$$\delta \mathbf{\Phi} = -\mathbf{\Xi} \wedge \mathbf{\Phi}, \tag{6}$$

$$g' \delta \mathbf{W}_\mu = \partial_\mu \mathbf{\Xi} - \mathbf{\Xi} \wedge g' \mathbf{W}_\mu. \tag{7}$$

Written in this form, these equations are preserved even if g' becomes variable, $g' \to g'(x^\mu) = \eta(x^\mu) g_0$. The field tensor is now

$$\mathbf{W}_{\mu\nu} = \frac{1}{g'} \big[\partial_\mu (g' \mathbf{W}_\nu) - \partial_\nu (g' \mathbf{W}_\mu) \big] \\ + g' \mathbf{W}_\mu \wedge \mathbf{W}_\nu, \tag{8}$$

so that it is covariant,

$$\delta \mathbf{W}_{\mu\nu} = -\mathbf{\Xi} \wedge \mathbf{W}_{\mu\nu}, \tag{9}$$

and a possible Lagrangian is

$$\mathcal{L}_{\mathbf{W}} = -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu}. \tag{10}$$

As before, we can define an auxiliary gauge boson $g' \mathbf{W}_\mu \equiv g_0' \mathbf{w}_\mu$ and an auxiliary field $g' \mathbf{W}_{\mu\nu} \equiv g_0' \mathbf{w}_{\mu\nu}$, or equivalently $\eta(x^\mu) \mathbf{W}_\mu \equiv \mathbf{w}_\mu$ and $\eta(x^\mu) \mathbf{W}_{\mu\nu} \equiv \mathbf{w}_{\mu\nu}$. With these definitions, the field $\eta(x^\mu)$ does not appear in the gauge derivative

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} + g_0' \mathbf{w}_\mu \wedge \mathbf{\Phi}, \tag{11}$$

and thus not in the matter Lagrangian. The gauge Lagrangian becomes

$$\mathcal{L}_{\mathbf{w}} = -\frac{1}{4} e^{-2\chi} \mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} \tag{12}$$

with $\chi \equiv \log(g'(x^\mu)/g_0')$, or $g'(x^\mu) = g_0' e^\chi$. As we can see, with a couple of trivial modifications, the tools previously developed for the $U(1)$ gauge group may be adapted to any non-abelian gauge theory.

4. The Proca theory and explicit breaking of gauge invariance

Another interesting extension of the varying e electromagnetic model of [6,7] is Proca theory of the massive photon. It will prove useful as a contrast to the electroweak results, where gauge bosons acquire a mass via a quite different mechanism.

The Proca Lagrangian, with a dynamic electromagnetic coupling given by $e(x^\mu) = e_o e^\psi$, may be written as:

$$\mathcal{L}_P = \mathcal{L}_m - \frac{\omega}{2} \psi_{,\mu} \psi^{,\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu, \tag{13}$$

for a photon with mass m, assumed to be a constant parameter. The covariant derivative appearing in $\mathcal{L}_m$ is, say, $D_\mu \phi = (\partial_\mu + i e A_\mu) \phi$ for transformations of the form $\delta \phi = -i \chi \phi$ and $\epsilon A_\mu \to \epsilon A_\mu + \chi_{,\mu}$. Even though the mass term breaks gauge invariance it is still possible to define a gauge-invariant electromagnetic field tensor according to (1).

As before we can define $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, with $a_\mu \equiv \epsilon A_\mu$, leading to Lagrangian:

$$\mathcal{L}_P = \mathcal{L}_m - \frac{\omega}{2} \psi_{,\mu} \psi^{,\mu} - \frac{1}{4} e^{-2\psi} f_{\mu\nu} f^{\mu\nu} \\ - \frac{1}{2} e^{-2\psi} m^2 a_\mu a^\mu, \tag{14}$$

where the matter Lagrangian now does not contain ψ (since the covariant derivative becomes $D_\mu \phi = (\partial_\mu + i e_0 a_\mu) \phi$). In terms of these variables it is easy to find that the dynamical equation for ψ is

$$\Box \psi = -\frac{1}{\omega} \left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + m^2 A_\mu A^\mu \right). \tag{15}$$

With loss of gauge invariance, the question of which of A_μ or a_μ is the physical field acquires relevance. From the a_μ formulation it seems that the photon mass has to be variable in this theory; the opposite conclusion is reached from the A_μ formulation. We can quickly see, however, that a variable mass is not the correct physical interpretation. Varying the Lagrangian with

respect to, say, A^μ leads to Maxwell's equations

$$\epsilon \partial_\mu \frac{F^{\mu\nu}}{\epsilon} - m^2 A^\nu = J^\nu. \tag{16}$$

The equation of current conservation is now $\partial_\mu(J^\mu/\epsilon) = 0$, so that one must have $\partial_\mu(A^\mu/\epsilon) = 0$. The wave equation in free space is therefore

$$\left(\Box - m^2\right) A^\nu - \cdots = 0, \tag{17}$$

where the ellipse refers to terms in $\partial_\mu \psi$. One may check by direct substitution into (15) and (17) that there are plane wave solutions for all reasonable amplitudes and wavelengths (note that $m^2 \ll \omega$). Their dispersion relations (for either A^μ or a^μ waves) are $E^2 - \boldsymbol{k}^2 = m^2$, that is, the photon has a constant mass m regardless of the field variable used. This is also the mass that appears in the propagator for this theory. More generally, the physical mass should be identified from the Lagrangian written in terms of variables such that the gradient terms have no prefactor.

The conclusion is that a varying charge does not imply a varying boson mass if the latter is obtained by explicitly breaking gauge invariance. Of course, we can, if we wish, also have a varying photon mass, by promoting m to a dynamical field; but this is not necessary. The situation will be different for the vector boson masses in the electroweak extension of the Bekenstein model, where gauge invariance is preserved.

5. The electroweak model

We are now ready to consider the electroweak sector of the standard model [18–20]. Its fundamental degrees of freedom are massless spin 1/2 chiral particles Ψ_i, and the gauge symmetry group is $SU(2)_L \otimes U(1)$, where $SU(2)$ is weak isospin (acting on left handed fermions only) and $U(1)$ is the weak hypercharge. The coupling constants are g_0 and g'_0 for the $U(1)$ and $SU(2)$ interactions, respectively.

As before, we promote the gauge couplings to fields, writing $g'(x^\mu) = \eta'(x^\mu) g'_0$ and $g(x^\mu) = \eta(x^\mu) g_0$. We may then define fields ψ and χ via:

$$g'(x^\mu) = g'_0 e^\psi, \tag{18}$$

$$g(x^\mu) = g_0 e^\chi. \tag{19}$$

Again, we may avoid the presence of ψ and χ in covariant derivatives by defining auxiliary gauge boson fields,

$$g'(x^\mu) \mathbf{W}_\mu = g'_0 \mathbf{w}_\mu, \tag{20}$$

$$g(x^\mu) Y_\mu = g_0 y_\mu. \tag{21}$$

Then, considering for example the Higgs field (a complex doublet Φ), the derivative:

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} g' \mathbf{t} \cdot \mathbf{W}_\mu - \frac{i}{2} g Y_\mu\right) \Phi, \tag{22}$$

(where $\mathbf{t}$ are the $SU(2)$ generators) becomes

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} g'_0 \mathbf{t} \cdot \mathbf{w}_\mu - \frac{i}{2} g_0 y_\mu\right) \Phi. \tag{23}$$

We may also define field tensors:

$$\mathbf{w}_{\mu\nu} = \partial_\mu \mathbf{w}_\nu - \partial_\nu \mathbf{w}_\mu - g'_0 \mathbf{w}_\mu \wedge \mathbf{w}_\nu, \tag{24}$$

$$y_{\mu\nu} = \partial_\mu y_\nu - \partial_\nu y_\mu, \tag{25}$$

or similar expression for $\mathbf{W}_{\mu\nu}$ and $Y_{\mu\nu}$, written in terms of $\mathbf{W}_\mu$ and Y_μ (see (1) and (8)).

The core electroweak Lagrangian may now be written as $\mathcal{L} = \mathcal{L}_{\mathbf{w}y} + \mathcal{L}_\Phi + \mathcal{L}_{\psi\chi}$. The gauge field Lagrangian is:

$$\begin{aligned} \mathcal{L}_{\mathbf{w}y} &= -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} \\ &= -\frac{1}{4} e^{-2\psi} \mathbf{w}_{\mu\nu} \cdot \mathbf{w}^{\mu\nu} - \frac{1}{4} e^{-2\chi} y_{\mu\nu} y^{\mu\nu}, \end{aligned} \tag{26}$$

and (as in Proca theory) using variables $\mathbf{W}^\mu$ and Y^μ (in terms of which the gauge Lagrangian has no prefactor) facilitates identifying the physical masses that appear in the dispersion relations and in the propagators. The Higgs Lagrangian is

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \tag{27}$$

with potential

$$V(\Phi) = \frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} \left(\Phi^\dagger \Phi\right)^2. \tag{28}$$

As in Proca theory with varying electric charge, the potential parameters m and λ may be assumed to be constant. Finally, the fields χ and ψ acquire dynamics via

$$\mathcal{L}_{\psi\chi} = -\frac{\omega'}{2} \psi_{,\mu} \psi^{,\mu} - \frac{\omega}{2} \chi_{,\mu} \chi^{,\mu}. \tag{29}$$

We could also consider a simpler, one-dilaton variation of this theory by identifying χ and ψ, and keeping just one of the terms in $\mathcal{L}_{\psi\chi}$.

In order to induce spontaneous symmetry breaking of the $SU(2)$ gauge group we should choose $m^2 < 0$. Then the potential has a minimum at $|\Phi|^2 = v_0^2 \equiv -m^2/\lambda \neq 0$, so the vacuum state may be at $(\Phi)_0 = \binom{0}{v_0}$. Given that the symmetry is local, a perturbative expansion around the vacuum can always be written as

$$\Phi(x^\mu) = \begin{pmatrix} 0 \\ v_0 + \frac{\sigma(x^\mu)}{\sqrt{2}} \end{pmatrix}. \tag{30}$$

One may now expand the Lagrangian. The crucial term for identifying the boson masses is the Higgs gradient term:

$$(D_\mu \phi)^\dagger (D^\mu \phi)$$
$$= \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{v_0^2}{4}(g')^2 \left[\left(W_\mu^1 \right)^2 + \left(W_\mu^2 \right)^2 \right]$$
$$+ \frac{v_0^2}{4} \left(g' W_\mu^3 - g Y_\mu \right)^2. \tag{31}$$

From this expression we define a massless gauge field, A_μ, and its orthogonal field, Z_μ, with respect to the fields W_μ^3 and Y_μ

$$Z_\mu \equiv \frac{g' W_\mu^3 - g Y_\mu}{\sqrt{g'^2 + g^2}} = \cos\theta_W\, W_\mu^3 - \sin\theta_W\, Y_\mu,$$

$$A_\mu \equiv \frac{g W_\mu^3 + g' Y_\mu}{\sqrt{g'^2 + g^2}} = \sin\theta_W\, W_\mu^3 + \cos\theta_W\, Y_\mu, \tag{32}$$

where θ_W is the weak mixing angle, or Weinberg angle, given by

$$\tan\theta_W = \frac{g}{g'} = \frac{g_0}{g'_0} e^{\chi - \psi}. \tag{33}$$

In the two-dilaton theory this is a variable. We could have defined a similar (but constant) rotation in terms of the fields w_μ^3 and y_μ, and this would still diagonalize the mass matrix for a Lagrangian (31) rewritten in terms of these variables. However, such a rotation would induce photon–Z couplings in the kinetic terms (24) and therefore would not be the correct Weinberg rotation. Thus the Weinberg angle must be variable in the two-dilaton theory. Obviously, in the single dilaton theory (where χ and ψ are identified) θ_W remains constant.

Once this rotation is performed we find the following tree-level masses for the gauge bosons:

$$m_W = \frac{v_0}{\sqrt{2}} g' \propto e^\psi, \tag{34}$$

$$m_Z = \frac{v_0}{\sqrt{2}} \sqrt{g'^2 + g^2} = \frac{v_0}{\sqrt{2}} \sqrt{g_0'^2 e^{2\psi} + g_0^2 e^{2\chi}}, \tag{35}$$

$$m_A = 0, \tag{36}$$

where we used the charged $W_\mu^\pm = (W_\mu^1 \pm i W_\mu^2)/\sqrt{2}$. We shall not discuss in this Letter radiative corrections to these formulae.

The variability of these masses is to be contrasted with Proca theory. There, mass and charge are essentially independent and so it is possible to have a constant photon mass and a varying electric charge. In the standard model, on the other hand, gauge invariance precludes an explicit mass term. Gauge bosons acquire a mass because they couple to the "charged" Higgs field via the covariant derivative and the Higgs field undergoes spontaneous symmetry breaking. Thus, the gauge bosons have mass only because the Higgs field has charge, and a varying charge necessarily implies a concomitant varying gauge boson mass.

One may wonder whether the actions presented in this section may be derived from a more fundamental theory. It is well known that string theory produces a dilaton field in what is usually taken to be its first stage of compactification. But it is also true that in further stages of compactification it produces a multitude of dilaton fields, generally coupling differently to the different terms in the Lagrangian, even at tree level. This is also true of brane theories. A possible toy model for our construction may be found in [27], where action terms associated with different 2-forms become multiplied by exponentials of different linear combinations of dilaton fields. It is difficult to find a fundamental theory leading exactly to our model because it is awkward to derive (or even incorporate) the standard model into string/M-theory.

6. Lepton charges and masses

We now consider the leptonic sector of the theory. For the sake of brevity we will consider only the electron and the electron neutrino, but our considerations can be easily extended to include muon and tau leptons, as well as quarks. The left handed fermions are

placed in weak isospin doublets $L = \binom{\nu_e}{e_L}$ whereas the right handed fermions are singlets e_R. Bearing in mind that the covariant derivatives are:

$$D_\mu L = \left(\partial_\mu - \frac{\iota}{2}g'\tau \cdot \mathbf{W}_\mu + \frac{\iota}{2}gY_\mu\right)L, \tag{37}$$

$$D_\mu R = \partial_\mu R + \iota gY_\mu R, \tag{38}$$

we arrive at the free fermion Lagrangian

$$\mathcal{L}_f = \iota\bar{R}\gamma^\mu(\partial_\mu + \iota gY_\mu)R$$
$$+ \iota\bar{L}\gamma^\mu\left(\partial_\mu + \iota gY_\mu - \frac{\iota}{2}g'\tau \cdot \mathbf{W}_\mu\right)L. \tag{39}$$

After rotation (32) this becomes:

$$\mathcal{L}_f = \iota\bar{e}\gamma^\mu\partial_\mu e + \iota\bar{\nu}\gamma^\mu\partial_\mu\nu - g'\sin\theta_W\bar{e}\gamma^\mu eA_\mu$$
$$+ \frac{g'}{\cos\theta_W}\left(\sin^2\theta_W\bar{e}_R\gamma^\mu e_R\right.$$
$$\left. - \frac{1}{2}\cos(2\theta_W)\bar{e}_L\gamma^\mu e_L + \frac{1}{2}\bar{\nu}\gamma^\mu\nu\right)Z_\mu$$
$$+ \frac{g'}{2}\left[\left(\bar{\nu}\gamma^\mu e_L W_\mu^{+\dagger}\right) + \text{h.c.}\right], \tag{40}$$

where h.c. denotes hermitian conjugate. This expression allows us to identify the electromagnetic and weak currents. We find that the field A^μ is indeed the electromagnetic field, and that the electric charge is given by

$$e = g\cos\theta_W = g'\sin\theta_W = \frac{gg'}{\sqrt{g^2 + g'^2}}. \tag{41}$$

The fine structure "constant" α is therefore fixed by a nontrivial combination of the fields ψ and χ, should there be two dilatons. In the single dilaton case this reduces to $e = e_0 e^\chi$.

One may also identify the weak currents to find the expression for the Fermi constant. One finds

$$G_F = \frac{\sqrt{2}}{4}\frac{g'^2}{M_W^2} = \frac{1}{2\sqrt{2}v_0^2}. \tag{42}$$

Interestingly, this does not vary. Fermi's constant is determined by the Higgs potential only, and so, for as long as its parameters are held fixed, varying couplings in the standard model do not lead to a varying Fermi constant.

Finally, we consider the Higgs-fermion interaction Lagrangian, through which fermions acquire their masses once the Higgs acquires a vacuum expectation value. This may be written as

$$\mathcal{L}_{\Phi_{int}} = -G_e\left(\bar{L}\Phi R + \bar{R}\Phi^\dagger L\right)$$
$$= -G_e\left(v_0 + \frac{\sigma(x^\mu)}{\sqrt{2}}\right)(\bar{e}_L e_R + \bar{e}_R e_L), \tag{43}$$

where we have used the vacuum expectation value for Φ chosen earlier, and where G_e is the Higgs–lepton coupling strength for the electron. The electron mass is therefore given by $m_e = v_0 G_e$. Again, if the Higgs potential parameters are kept fixed and the parameters G_i are not promoted to dynamical variables, the tree level fermion masses remain constant even if the couplings g and g' are promoted to fields.

7. Equations of motion and applications

We reserve to a future publication a complete study of the cosmological and astrophysical implications of this theory, but here we outline some areas of interest.

The Einstein's equations for this theory are:

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{\text{EW}'}e^{-2\psi} + T_{\mu\nu}^{\text{EW}}e^{-2\chi}\right.$$
$$\left. + T_{\mu\nu}^\psi + T_{\mu\nu}^\chi + T_{\mu\nu}^{\text{mat}}\right), \tag{44}$$

that is, one must add the stress energy tensor of fields χ and ψ to the right-hand side. In addition, we have

$$\Box\psi = -\frac{1}{2\omega'}e^{-2\psi}\mathbf{w}_{\mu\nu}\cdot\mathbf{w}^{\mu\nu}, \tag{45}$$

$$\Box\chi = -\frac{1}{2\omega}e^{-2\chi}y_{\mu\nu}y^{\mu\nu}. \tag{46}$$

For the single dilaton theory one identifies ψ and χ and the last two equations are replaced by

$$\Box\chi = -\frac{1}{2\omega}e^{-2\chi}\left(\mathbf{w}_{\mu\nu}\cdot\mathbf{w}^{\mu\nu} + y_{\mu\nu}y^{\mu\nu}\right). \tag{47}$$

We can analyze these equations for two general cases. The first is for spatially-varying, time-independent coupling fields ψ and χ, for which we can find a spherically symmetric solution to the equations of motion (an extension of the considerations in [11,21–23]). These can then be applied to scenarios in which weak interactions are non-negligible, such as around massive objects like neutron stars and black holes. The second case is for time-varying, spatially-independent fields ψ and χ, which is applicable to cosmological scenarios (an extension of the work in [7]).

From this exercise we may expect that the Webb results imply significant variations in the W and Z masses and in the Weinberg angle in the very early universe, in neutron stars, or near black holes and their accretion disks. This has obvious implications for the physics of neutron stars, BBN, and the electroweak phase transition. But perhaps the most dramatic implication may be the stability of solitonic solutions in the standard model. Semi-local strings are defects that owe their stability to non-topological considerations [24]. They are present in the electroweak theory, and their region of stability has been studied [25,26]. It appears that this region does not include the parameter values observed in the "actual" standard model. However, according to the theory presented in this Letter, these parameter values are not constants of Nature. It is conceivable that the region of stability for electroweak strings may be realized in the very early universe or near neutron stars.

We include a final word on the applicability of this theory. As in [7], we find that the Webb et al. results [1–3] imply that the couplings ω and ω' are of the order of the Planck scale. Thus ψ and χ should be regarded as gravitational fields (fifth force type) and the applicability of this theory (should it be confirmed by further astrophysical observations) is similar to that of general relativity. At the quantum level the interactions proposed—just like general relativity—are likely to be non-renormalizable. Hence, we should always treat ψ and χ—and the metric—as classical fields, and we should never consider phenomena at energy scales near the Planck scale. We should regard our theory as an effective phenomenological theory (a view that may also be applied to general relativity), useful for studying particle cosmology but not quantum cosmology.

However, unlike general relativity, the couplings proposed violate the equivalence principle. But as shown in [11] current experiments are not sensitive enough to rule out this type of theories. Interestingly, upcoming satellite experiments would be sensitive to this important effect.

8. Conclusions

In this Letter we examined the implications of a varying alpha in the light of the electroweak theory.

We already know that electromagnetism and weak interactions are unified. Hence, a varying alpha *implies* variability for the two coupling "constants" of the electroweak theory. These variations may be controlled by one or two independent "dilaton" fields.

We found that with coupling variability, the gauge boson masses must also vary. This conclusion is hardly surprising and can be qualitatively understood. In Proca theory an explicit mass term is added to the "photon" Lagrangian, thereby breaking gauge invariance. This mass term is independent of the charge couplings and so it is possible to accommodate both a varying electromagnetic coupling and a constant "photon" mass.

The origin of the $W^{\pm}$ and Z masses is quite different. In the standard model, gauge invariance is fully preserved, and gauge bosons have mass because the Higgs field undergoes spontaneous symmetry breaking. But more important, *the gauge bosons only have mass because the Higgs field carries charge*, that is, it couples to the gauge bosons that *will* acquire mass. Thus, it is impossible to have a standard model with varying charges without passing this variability on to the $W^{\pm}$ and Z masses, and ultimately also the Weinberg angle (in the two-dilaton case).

The situation is again different for the tree level lepton masses. These are not due to charge, but to the interaction with the Higgs field via "Yukawa" couplings. Unless the Higgs potential becomes dynamical, fermion masses do not change even if the couplings do. It would be interesting to explore a variation of the theory proposed in this Letter where the Higgs potential becomes dynamical and so the fermion masses can vary too. Perhaps such a theory could explain the mystery of the fermion masses, but this is merely a speculation.

In summary, we have explored the implications of a varying alpha for other parameters of the standard model. If in a future experiment we were to find that the observed variations in alpha are not accompanied by specific variations in the W and Z masses we should be very worried indeed. Such a finding would imply a violation of gauge invariance and contradict the standard model. If we found that the Weinberg angle did not change that would be less apocalyptic. It would simply imply that the observed variation in alpha is due to a single dilaton field within the framework of the standard model. If, on the other

hand, we were to find that the Fermi constant varied, or that the fermion tree-level masses varied, then we would know that the theory presented in this Letter is too tight a framework. We would need to "promote to variables" the parameters in the Higgs potential. The Mexican hat would have to become dynamical.

Acknowledgements

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