Einstein centennial review article / Article de synthèse commémoratif du centenaire de l'année miraculeuse d'Einstein

Fundamental physical constants: looking from different angles[1]

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Abstract: We consider fundamental physical constants that are among a few of the most important pieces of information we have learned about Nature after intensive centuries-long study. We discuss their multifunctional role in modern physics including problems related to the art of measurement, natural and practical units, the origin of the constants, their possible calculability and variability, etc.

PACS Nos.: 06.02.Jr, 06.02.Fn

Résumé : Nous étudions les constantes physiques qui sont les porteurs des plus importants renseignements que nous ayons glanés sur la Nature et ce, après des siècles d'étude. Nous analysons leur rôle multifonctionnel en physique moderne et leur relation avec l'art de la mesure, les unités naturelles et conventionnelles, l'origine de ces constantes, la possibilité de les calculer, leur variance, etc

[Traduit par la Rédaction]

1. Introduction

> **'You needn't say "please" to *me* about 'em,' the Sheep said, ...'I didn't put 'em there, and I'm not going to take 'em away.'   Lewis Carroll**

There are a number of ways to understand Nature. One can approach it with logic, with guesses, and with imagination. One way scientists and, especially, physicists address the problem is based on a comparison of ideas and reality via measurements. Experiment inspires and verifies theory. The soul of the physical approach is neither logic, nor even a quantitative approach, but common sense. The latter

Received 25 January 2005. Accepted 19 July 2005. Published on the NRC Research Press Web site at http://cjp.nrc.ca/ on 16 August 2005.

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[1]This article is one of a series of invited papers that will be published during the year in celebration of the World Year of Physics 2005 — WYP2005.

 doi: 10.1139/P05-047

is based on centuries-long experience in the investigation of Nature. It tells us to be sceptical. It tells us that even complicated phenomena are often based on simple pictures, and most of them allow for the estimation of effects in terms of certain fundamental quantities. It tells us that those simple pictures are good to start with but should involve more and more detail once we desire a more accurate agreement between any of our theories and the measured reality.

Fundamental constants play a crucial role in physics in a few different ways and we consider their significance in this paper. However, to start the subject we need to agree on what the fundamental constants are. We discover a great variety of approaches to the problem based on a particular role of a particular constant in a specific field of physics.

Two polar points of view are related to “practical” and “fundamental” physics.

- The practical view addresses the art of measurement, which makes physics physics. There are a number of beautiful laws such as Maxwell equations or the Dirac equation that pretend to describe Nature. However, as a quantitative method of exploring the World, physics needs some quantitative values to be measured. That requires that certain parameters enter basic equations. We also need certain quantities to be used as units to make proper comparisons of different results. Some of these parameters enter a number of equations from different branches of physics and are universal to some extent. That is a “practical” way to define what the fundamental constants are. A very important property of such constants is that they should be measurable. The fundamental constants understood in such a way are a kind of an interface to access Nature quantitatively and apply basic laws to its quantitative description.

- However, not every such constant is truly fundamental. If we, for example, need a unit, we can consider a property of such a nonfundamental object as the caesium atom. The approach of fundamental physics is based on the idea that we can explain the World with a few very basic laws and a few very basic constants. The rest of the constants should be calculable or expressed in terms of other constants. Such constants are our interface to really fundamental physics but most of them have a very reduced value in real measurements, because they are often not measurable. To deduce their values from experiment, one has to apply sophisticated theories and, sometimes, certain models.

A good illustration of a difference between these two approaches is the situation with the Rydberg constant

$$R_\infty = \frac{\alpha^2 m_e c}{2h} = \frac{e^4 m_e}{8\epsilon_0^2 h^3 c} \tag{1}$$

which is expressed in a simple way in terms of certainly more fundamental quantities. However, this exactness is rather an illusion, because the constant is not measurable in a direct way. The most accurately measured transition in hydrogen is the triplet $1s$–$2s$ transition (see, for example, ref. 1)

$$\nu_H(1s-2s,\ F=1) = 2466\,061\,102\,474\,851(34)\ \mathrm{Hz} \qquad \left[1.4\times10^{-14}\right] \tag{2}$$

and it may be only approximately related to the Rydberg constant (see Fig. 1). To obtain its value, one has to apply quantum electrodynamics (QED) theory and perform some additional measurements. At the present time, the fractional accuracy (the number in square brackets) in the determination of this constant [2]

$$R_\infty = 10\,973\,731.568\,525(73)\ \mathrm{m}^{-1} \qquad \left[6.6\times10^{-12}\right] \tag{3}$$

Fig. 1. It is a relation between the $1s$–$2s$ transition frequency $\nu_{\rm H}(1s–2s)$ and the Rydberg constant R_∞. A correction for the difference between the center of gravity of the $1s$ and $2s$ hyperfine multiplets and their triplet component is not included. This figure is an example of a complicated relationship, and is not intended to be read.

$$
\begin{aligned}
\nu_H(1s-2s) = \frac{3}{4}\,c\,\mathrm{R}_\infty \Bigg\{ 1
&+ \left[\frac{11}{48}(Z\alpha)^2 + \frac{43}{384}(Z\alpha)^4 + \frac{851}{12288}(Z\alpha)^6 + \dots\right] \\
&+ \frac{m_e}{m_p}\left[-1 - \frac{13}{24}(Z\alpha)^2 - \frac{17}{64}(Z\alpha)^4 + \dots\right] \\
&+ \left(\frac{m_e}{m_p}\right)^2\left[1 + \frac{41}{48}(Z\alpha)^2 + \dots\right] \\
&+ \left(\frac{m_e}{m_p}\right)^3\left[-1 + \dots\right] \\
&+ \frac{(Z\alpha)^3}{\pi}\frac{m_e}{m_p}\left[-\frac{7}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{9}\ln k_0(2s) + \frac{64}{9}\ln k_0(1s) - \frac{112}{3}\ln 2 - \frac{805}{54}\right] \\
&+ \frac{(Z\alpha)^3}{\pi}\left(\frac{m_e}{m_p}\right)^2\left[\frac{7}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{8}{3}\ln k_0(2s) - \frac{64}{3}\ln k_0(1s) + 112\ln 2 + \frac{889}{18}\right] + \dots \\
&+ \frac{\alpha}{\pi}(Z\alpha)^2\left[-\frac{28}{9}\ln\frac{1}{(Z\alpha)^2} - \frac{4}{9}\log k_0(2s) + \frac{32}{9}\log k_0(1s) - \frac{266}{135}\right] \\
&+ \frac{\alpha}{\pi}(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{28}{3}\ln\frac{1}{(Z\alpha)^2} + \frac{4}{3}\log k_0(2s) - \frac{32}{3}\log k_0(1s) + \frac{14}{5}\right] \\
&+ \frac{\alpha}{\pi}(Z\alpha)^2\left(\frac{m_e}{m_p}\right)^2\left[-\frac{56}{3}\ln\frac{1}{(Z\alpha)^2} - \frac{8}{3}\log k_0(2s) + \frac{64}{3}\log k_0(1s) - \frac{14}{15}\right] \\
&+ \alpha(Z\alpha)^3\left[\frac{14}{3}\log 2 - \frac{2989}{288}\right] \\
&+ \alpha(Z\alpha)^3\frac{m_e}{m_p}\left[-14\log 2 + \frac{2989}{96}\right] \\
&+ \frac{\alpha}{\pi}(Z\alpha)^4\left[\frac{7}{8}\ln^2\frac{1}{(Z\alpha)^2} + \left(-\frac{208}{9}\ln 2 + \frac{347}{90}\right)\ln\frac{1}{(Z\alpha)^2} + 71.626974\right] + \dots \\
&+ \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^2\left[-\frac{7}{2}\pi^2\ln 2 + \frac{70\pi^2}{81} + \frac{15253}{1944} + \frac{21}{4}\zeta(3)\right] \\
&+ \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^2\frac{m_e}{m_p}\left[\frac{21}{2}\pi^2\ln 2 - \frac{70\pi^2}{27} - \frac{15253}{648} - \frac{63}{4}\zeta(3)\right] \\
&+ 50.2976\left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^3 \\
&+ \left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^4\Bigg[\frac{56}{81}\ln^3\frac{1}{(Z\alpha)^2} + \frac{1}{27}\ln^2\frac{1}{(Z\alpha)^2} \\
&\qquad + \left(-\frac{246337}{32400} - \frac{385\pi^2}{81} + \frac{1126}{135}\ln 2 - \frac{7\pi^2}{4}\ln 2 - \frac{248}{27}\ln^2 2 - 34.845333\right)\ln\frac{1}{(Z\alpha)^2} \\
&\qquad + 147(25)\Bigg] + \dots \\
&+ \left(\frac{\alpha}{\pi}\right)^3(Z\alpha)^2\Bigg[-\frac{248659831}{279936} + \frac{1765757\pi^2}{29160} - \frac{11137\pi^4}{9720} + \frac{7952}{27}\ln 2 - \frac{33509\pi^2}{324}\ln 2 \\
&\qquad + \frac{1673\pi^2}{405}\log^2 2 + \frac{497}{81}\log^4 2 + \frac{588497}{6912}\zeta(3) + \frac{847\pi^2}{216}\zeta(3) - \frac{595}{72}\zeta(5)\Bigg] + \dots \\
&+ \frac{\alpha(Z\alpha)^3}{\pi^2}\frac{m_e}{m_p}\left[\frac{3136}{81} - \frac{245\pi^2}{108} + \frac{14\pi^2}{3}\ln 2 - 14\zeta(3) - \frac{14}{9}\pi(Z\alpha)\ln^2\frac{1}{(Z\alpha)^2}\right] \\
&- \frac{14}{9}(Z\alpha)^2\left(\frac{m_e c R_p}{\hbar}\right)^2 \\
&+ \dots \Bigg\}
\end{aligned}
$$

is much lower than that of the measurement of $\nu_{\rm H}(1s–2s)$ (cf. (2)). One might indeed redefine the Rydberg constant in some other way, for example,

$$\widetilde{R} = \frac{4}{3\,c}\,\nu_{\rm H}(1s-2s) \tag{4}$$

making the accuracy of its determination higher. However, a relationship with more fundamental constants would be more complicated and not exactly known. The practical and fundamental approaches cannot easily meet each other because we can very seldom both calculate exactly and directly measure some quantity, which has a certain nontrivial meaning. A choice between the practical and fundamental options is a kind of trade-off between measurability and applicability, on one side, and calculability and theoretical transparency, on the other.

There are a number of approaches that lie between the two approaches mentioned above. For example, quantum electrodynamics (QED) at the very beginning of its development met the problem of divergencies in simple calculations. A response to the problem was the idea of renormalization, which states that QED theory should express observable values (such as, for example, the Lamb shift in the hydrogen atom or the anomalous magnetic moment of an electron) in terms of certain observable properties (such as the charge and mass of an electron). The measurable charge and mass are definitely more fundamental than most practical constants such as the caesium hyperfine constant. However, they are certainly less fundamental than similar quantities defined at the Planck scale. Such an approach is in a formal sense not an ab initio calculation of a quantity under question (for example, the Lamb shift), but rather an ab initio constraint on observable quantities (the energy shifts, the charge, and the masses).

The point that definitely unifies all approaches is that the fundamental constants are dimensional or dimensionless quantities that are fundamentally important to understand, investigate, and describe our world. However, the *importance* is often understood differently. In a finite-size paper, it is not easy to consider the whole range of problems related to the fundamental constants and part of the discussion missing here can be found in another recent review of the author [3] (see also ref. 4).

2. Physical constants, units, and art of measurement

Speak in French when you can't think of the English for a thing. Lewis Carroll

When one does a measurement, certain units should be applied to arrive at a quantitative result. A measurement is always comparison and in a sense we deal with dimensionless quantities only. "Mathematically", this point of view is true, however, it is counterproductive. To compare two similar quantities measured separately, we have to go through a number of comparisons. Instead of that, it has been arranged to separate a certain part of the comparisons and use them to introduce *units*, certain specific quantities applied worldwide for a comparison with similar quantities under question. The units (or a system of units) find an endower as a coherent system of certain universally understood and legally adopted measures and weights that can be used to measure any physical quantity.

One should not underestimate problems of measurements. Access to the quantitative properties of Nature is a crucial part of physics and it is a problem of fundamental importance to improve and extend our accessibility.

Since the very discovery of the world of measurable quantities, we used natural units. But their degree of naturality was different. We started with values related to our essential life:

- parameters of human beings;
- parameters of water, the most universal substance around us;
- parameters of Earth itself;
- parameters of Earth as a part of the Solar system.

This approach had been followed until the introduction of the *metric system* two centuries ago, and, in fact, the very metric system was originally based on properties of the Earth: the metre[2] was defined in such a way that a length of a quadrant (a quarter of a meridian) of a certain meridian was equal to 10 000 km (exactly); the second was obviously defined by a day and a year; the gram was then understood as a mass of one cubic centimetre of water and so on.

The metric treaty was signed in 1875 in Paris. Since then, we have changed the contents of our units but tried to keep their size. Few changes took place after the SI was adopted in 1960 (SI means *Système International d'Unités* – International System of Units). The latest version is presented in an CIPM[3] brochure [5]. For example, the SI unit of length, the *metre*, was originally defined via the size of the

[2] There are two different spellings for this term: the *meter* is used in USA, while the *metre* is used in UK and most of other English speaking countries and in international literature (see, for example, ref. 5). The latter is also traditionally used in metrological literature.
[3] CIPM is the International Committee for Weights and Measures.

Earth, later was related to an artificial ruler, then to a hot optical emission line, and now to the hyperfine structure interval in cold caesium atoms

$$\nu_{\rm HFS}(^{133}{\rm Cs}) = 9\,192\,631\,770\ {\rm Hz} \quad ({\rm exactly}) \tag{5}$$

and a fixed value of the speed of light

$$c = 299\,792\,458\ {\rm m/s} \quad ({\rm exactly}) \tag{6}$$

One also has to remember that this unit was introduced as a substitute for numerous units based on details of the shape of a human body, such as the *foot* (ft) and the *yard* (yd) and takes their magnitude (in a general scale) from them

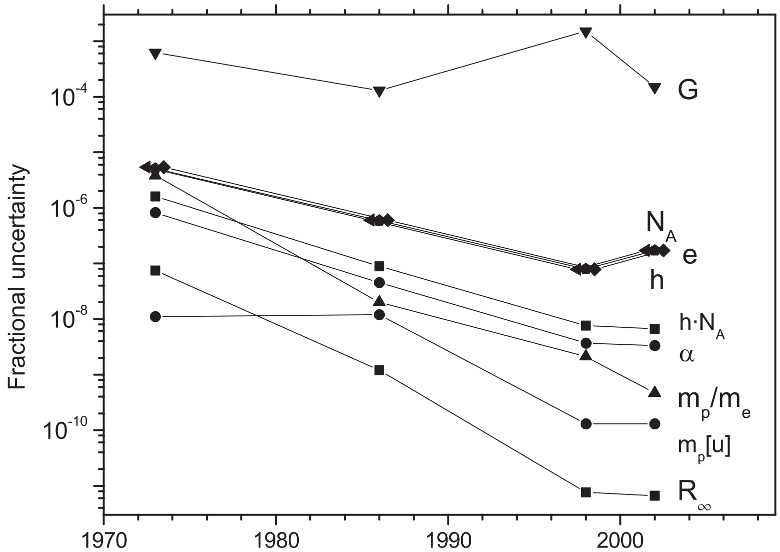
$$1\ {\rm m} \simeq 1.1\ {\rm yd} \simeq 3.3\ {\rm ft} \tag{7}$$

This evolution in the definition of the metre clearly demonstrates two great controversies of the SI: *changing stability* and *advanced simplicity*. First, there has been no single SI at all. We have seen with time since the very appearance of the metric convention, a number of various, but similar, systems of units. While the hierarchy and basic relations between the units were roughly the same all the time, the units themselves and related standards changed drastically. However, for practical reasons, the size of the units during those revolutionary redefinitions was kept the same as much as possible. Secondly, the system of units is indeed a product created first of all by nonphysicists for nonphysicists. I dare say, we, physicists, even do not care what the SI units actually are. For example, most of us have learned about the *mole* at a time when we could not recognize that excitations, binding effects in the solid phase, kinetic energy, etc. would change the mass of the sample (but indeed not a number of particles). Later, after we learned about all these effects, we assumed that the SI definition is properly adjusted to them. But it is unlikely that most of us checked the SI definitions for that. Actually, all SI definitions come historically from nonrelativistic classical physics and similar to their appearance, we also learned them for the first time as classical nonrelativistic stuff. We do not care about actual SI definitions partly because we do not consider seriously the legal side of SI and because of that we believe that we may ourselves interpret and correct SI definitions if necessary.

Physicists serve as experts only while decisions are made by authorities. The SI has been created for legal use and trade rather than for scientific applications. Because of that crucial features of the SI convention should be expressed as simply as possible. Meanwhile, these “simply defined” units should be allowed to apply to the most advanced physical technologies. That makes the SI a kind of iceberg with a stable and simple visible part, while the underwater part is sophisticated, advanced and changes its basic properties from period to period. The changes in the definition of the SI metre have demonstrated a general trend in physical metrology: to use more stable and more fundamental quantities and closely follow progress in physics. Eventually, we want units to be related to the quantized properties of natural phenomena and most of all, if possible, to values of fundamental constants. We already have natural definitions of the metre and the second, and are approaching a natural definition of basic electric units and, maybe, the kilogram.

Note, however, that a choice of units is not restricted to the International system SI. There are a number of options. Certain units, such as the *unified atomic mass unit*, are accepted to be used together with SI units [5]. There are a number of units such as the *Bohr magneton* $\mu_{\rm B} = e\hbar/2m_{\rm e}$ and the *nuclear magneton* $\mu_N = e\hbar/2m_{\rm p}$, which do not need any approval since they are well-defined simple combinations of basic fundamental constants. A number of quantities are measured in terms of fundamental constants. For example, the electric charge of nuclei and particles is customarily expressed in that of the positron. Sometimes, instead of introducing units, new values with special normalization are introduced, such as angular momentum in quantum physics, which is equal to the actual angular momentum divided by the reduced Planck constant $\hbar$.

Fig. 2. Progress in the determination of fundamental constants: the time dependence of the fractional uncertainty (see the recent paper, ref. 2, and also ref. 6 for earlier results by the CODATA task group). Here, G stands for the Newtonian gravity constant, N_A for Avogadro's constant, and m_p [u] for the proton mass in the unified atomic mass units.



Fundamental constants (as units) play a very important role in precision measurements and in special cases. The latter corresponds to a situation when conventional methods cannot be applied. For example, sometimes to determine a temperature we can not use a thermometer properly calibrated using primary thermodynamical standards. In some cases, when, for example, the temperature is too high or too low, or if we have to deal with a remote object, we may rely on the Boltzmann distribution and measure frequency and the spectral intensity of emitted photons. To interpret the frequency in terms of temperature, we have to use the values of the Planck constant h and the Boltzmann constant k.

3. Physical constants and precision measurements

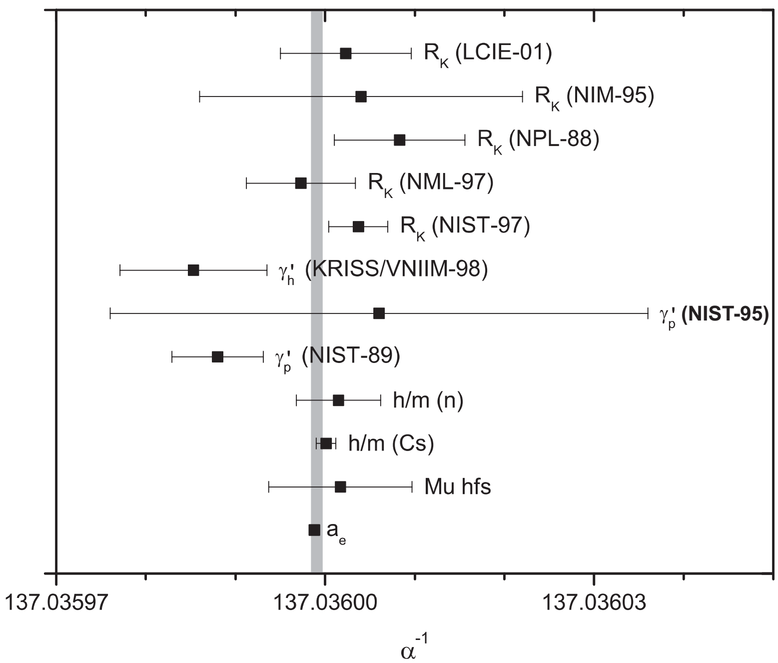
'You needn't say "exactly," ' the Queen remarked: 'I can believe it without that. Now I'll give *you* something to believe.' Lewis Carroll

A precision measurement is another case very closely related to fundamental constants. As we mentioned, those constants are universal and some may appear in measurements in different branches of physics. That offers us a unique opportunity to verify our understanding of Nature in a very general sense. We know that any particular theory is an approximation. Our basic approach always involves certain laws and certain ideas on what the uncertainty of our consideration is. The most crucial test of the whole approach is to check if values from different areas of physics agree with each other.

This test has been regularly performed by CODATA[4] task group on fundamental constants, which publishes its *Recommended Values of the Fundamental Constants* [2] (see also previous CODATA papers [6]). The progress in the determination of the most important fundamental constants for about 30 years (since the establishment of the CODATA task group) is shown in Fig. 2. The responsibility of the group is to compare results from different fields and to deliver the most accurate values of constants important for "precision" measurements of "essential" quantities. Indeed, the precision threshold is different for different quantities. Note that constants related to cosmology, astronomy, and some from

[4]CODATA is the Committee for Data for Science and Technology of the International Council for Science.

Fig. 3. Determination of the fine structure constant α by different methods as discussed in ref. 2. Among the results: a free QED value from the anomalous magnetic moment of electron (a_e), a bound QED value from the muonium hyperfine structure (Mu), an atomic interferometer value (Cs), a value involving a lattice parameter (n), and values dealing with calculable capacitor (R_K) and gyromagnetic ratio of proton and helion (γ), measured in SI units with the help of macroscopic electric standards. The grey vertical strip is related to the CODATA-2002 value [2].



particle physics such as the Hubble constant, astronomical unit, and Cabibbo angle are traditionally excluded from the consideration as not being related to precision physics. Meanwhile, certain properties of light nuclei (deuterium and both stable helium isotopes ^{3}He and ^{4}He) are included.

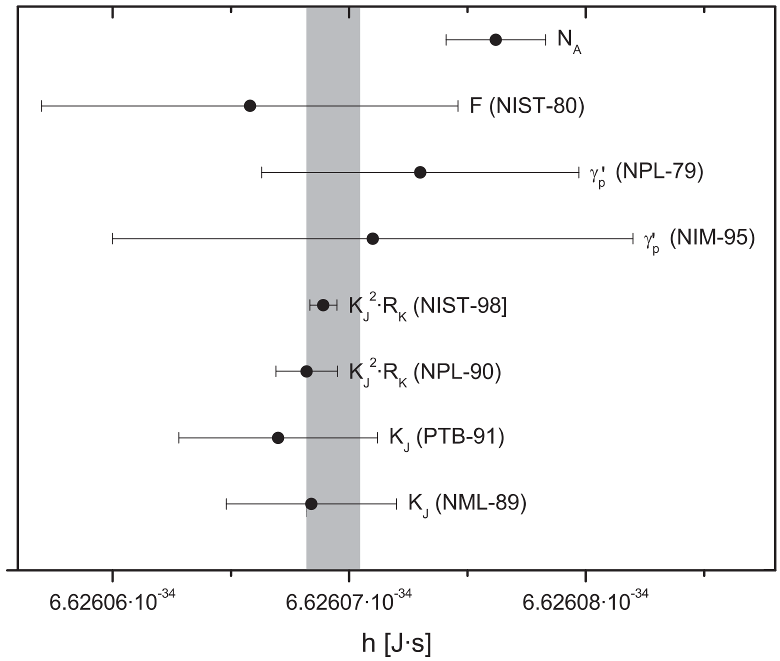
The most important lesson we have learned from CODATA's work is not just their recommended values, but evidence of the overall consistency of our approach to a quantitative description of Nature. That is illustrated in Figs. 3 and 4 showing different approaches to the determination of the fine structure constant α and the Planck constant h [2], which play a central role in the adjustment of the fundamental constants [2].

Why are the values of these two constants so significant for the CODATA adjustment? The answer is that when we consider physics from the fundamental point of view, the electron and proton are just certain particles among many others. However, when we do our measurements, we deal not with matter in general but, mainly, with atomic substances where electrons and protons are the fundamental "bricks". In such a case, electron and proton properties are as fundamental as h and c or even more so. In particular, these properties determine results of the spectroscopy of simple atoms and macroscopic quantum electromagnetic effects (see Sect. 5). The experiments, the results of which are presented in Figs. 3 and 4, involve, directly or indirectly, such constants as the Rydberg constant, the electron and proton masses, the electric charge, and the magnetic moments of an electron and a proton, the Planck constant, and the speed of light.

Let us consider the Planck constant in more detail. The accuracy of the determination of the most important fundamental constants is summarized in Fig. 2. We note, that the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \tag{8}$$

Fig. 4. Determination of the Planck constant h by different methods as discussed in ref. 2. The results come from the measurement of the Faraday constants $F = eN_A$, the Avogadro constant N_A, the gyromagnetic ratio of proton (γ_p), the volt (K_J), and the watt ($K_J \cdot R_K$) balances. The grey vertical strip is related to the CODATA-2002 value [2].



and the molar Avogadro constant $h \cdot N_A$ are better known than some of their constituents, such as h, e, and N_A. That means that an experimental determination of h is equivalent to a determination of e and N_A and thus, from the experimental point of view, the Planck constant h also becomes related to classical electrodynamics and atomic and molecular physics of substance. In a practical sense, h is even more universal than in a fundamental sense. More discussions on this subject can be found in ref. 3.

4. The international system of units SI: Vacuum constant ϵ_0, candela, kelvin, mole, and other questions

From the point of view of fundamental physics, the SI system [5] is unnecessarily complicated. It has seven basic units:

- *metre*, *second*, and *kilogram* (which beyond any doubt are crucial units for any system of units for physical quantities);
- *ampere* (which is already questionable and, in fact, a number of physicists (see, for example, ref. 7) strongly believe that it is much better to set $\epsilon_0 = 1$ and to measure electrical quantities in terms of three basic mechanical units);
- *kelvin* and *mole* (which are, in a sense, unnecessary units since the thermodynamic energy and the number of particles can be measured without introducing any special units);
- *candela* (which looks like a worse case — an unnecessary unit for a quantity related to the sensitivity of the human eye, an object, which is rather outside of physics).

4.1. "Unnecessary" units

'I didn't say there was nothing *better*,' the King replied. 'I said there was nothing *like* it.' Lewis Carroll

Let us start with the "unnecessary" units. From the philosophical point of view, any measurement is a comparison of two quantities of the same dimension and thus is a *relative* measurement. However, as mentioned above, if we do not like to create a chain of comparisons every time, we should introduce certain units. A measurement in terms of these units, although still a comparison, is a very special comparison and we qualify it as an *absolute* measurement. To be more precise, we like to have a coherent system of units and thus it is not enough to define any units, we have to define a certain system of units with

- one unit for each kind of quantity (each dimension);
- most units derived from a few basic units (for example, the newton, a unit of force, is defined through the metre, the second, and the kilogram: $1\ \mathrm{N} = 1\ \mathrm{kg} \times 1\ \mathrm{m} \times 1\ \mathrm{s}^{-2}$).

Meanwhile, in certain areas the relative measurements are so much more accurate (or much easier, or have other big advantages) than the absolute measurements that we face a hard choice: either to support a minimized coherent system of units, or to introduce some "extra" units (inside or outside the system). There are several options for a solution. A choice made in the case of temperature and amount of substance was to extend the system and to introduce new base units. For the mass of atoms and molecules, the unified atomic mass unit has been introduced as a unit outside of SI, but officially recognized and recommended for use. Nuclear magnetic moments are customarily measured in units of the nuclear magneton, which has never been included in any official recommendation of units.

One may think that since the kelvin appeared a long time ago[5] (before we realized that temperature is a kind of energy), it is kept now for historic reasons only. That is not correct. An example of the use of the *foot* in the USA shows how the problem is treated. There is no independent *foot* — this traditional unit[6] is defined as an exactly fixed part of the *metre*

$$1\ \mathrm{ft} = 0.3048\ \mathrm{m} \qquad \text{(exactly)} \tag{9}$$

As a result, for everyday life the use of feet and metres is not quite the same. Meanwhile, for scientific applications and industrial precision mechanics and electronics, their simultaneous use may be quite confusing, but it is completely equivalent: the same information, the same accuracy, the same actual basic definitions. On the contrary, the use of the kelvin and the joule is not the same – interpreting data from one unit to the other changes the accuracy of the results.

4.2. "Human-related" units

It was labelled 'Orange marmalade', but to her great disappointment it was empty. Lewis Carroll

The case of *candela* presents an additional problem, which is not a question of units, but a question of quantities. Why did the *original* foot and similar units fail? One of the reasons is that they were ill-defined. But that is half-truth only. The truth is that they were related to nonphysical quantities. The

[5] To be more precise, the Celsius temperature scale is meant since the value of degrees Celsius and Kelvin is the same.

[6] Actually there are a number of different versions of the foot. Equation 9 corresponds to the *international foot*. There is also the *U.S. survey foot* which is equal to 1200/3937 m. The number is chosen in such a way that 1 m = 39.37 in (see Sect. B.6 in ref. 8 for legal details; historical details can be found in ref. 9).

original foot was first related to the size of the foot of a particular person (a king/queen), later some approaches were related to an "average" person. And only eventually, the "human" foot was substituted by an artificial ruler.

When we rely on properties of a particular or average human being, we are dealing with a biological object. If we now reverse the problem and try to check a value related to the foot in its original sense, we meet a biological problem. We need to make a decision on the selection of people, to address the problem that the result may depend on geography etc. Eventually, the ultimate decision will choose either a "conventional foot" (as it is), which is not related anymore to any person, or a "conventional person", which should be a subject of a real measurement. In other words, even measuring some property in well-defined units, we may need in certain cases an *arbitrary* agreement on what this property is. We qualify this kind of agreement as *arbitrary* because within certain margins we are free to adopt any parameters.

In fact, the problem of the average size of a human body is not so important now, however, there are a number of questions due to ecology, safety, and medicine that involve interactions of certain physical effects and a human being. We can easily characterize these phenomena by a complete description of their physical properties. However, for obvious practical reasons, we often need an *integral* estimation of the influence that involves a number of parameters, which values vary in a broad range (such as frequency), and we certainly know that the human sensitivity depends on frequency and other various parameters. Such integral characteristics are not of a physical nature. To determine them we need to perform two kinds of measurements on

- physical details of the effects (a kind of the spectral distribution);
- spectral sensitivity of a human being.

If we accept the sensitivity as a real quantity, which is determined by effects beyond physics, the whole integral characteristic is not purely physical but becomes a combined nature: physics + biology. If we accept a model for the sensitivity, we can do simple calculations within this model and obtain a pure physical result, which will be related to the model rather than to reality. In other words, the result will be in well-defined units but for a conventional quantity. In some cases (for example, in radiology [10]) real and conventional quantities are clearly distinguished. In others, the separation is less clear. But in any case, quantities, related to the *sensitivity* of an [*average*] human eye, cannot be accepted as physical quantities and it is does not matter how their units are defined.

What is also important is the status of the SI as an international treaty. Everything related to the SI is a part of this agreement. Otherwise, it is not a part of the SI. A unit has to be a unit for a certain quantity. If a quantity is not well-defined, the unit is also ill-defined. If we deal with a quantity for which an additional agreement is needed, we have to put it into the SI, because it has to be a part of a definition of the related unit.

The question of the candela is very doubtful. The candela itself is defined as a part of the SI in "rigid physical terms".[7] As we mentioned above, there may be a need to have a convention on properties, but never on all physical quantities of one kind. We define the metre in the SI and we can use it. Length, in general, is well defined and does not need any additional agreement. However, if we like to measure particular properties of certain objects, which are related to length, we may need an additional agreement on these properties. For example, when we deal with an average parameter of a human body.

This problem of a "conventional" characteristic or a "conventional" object is not only for human-related (or life-related) cases. It is due to the peculiarity of classical objects. A number of well-known not-life-related examples of conventional properties are related to those of Earth such as the "standard

[7] *The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency* 540×10^{12} *hertz and that has a radian intensity in that direction of 1/683 watts per steradian* [5].

acceleration due to gravity” ($g_n = 9.806\,65$ m/s^2), adopted by the General Conference on Weights and Measures, the “standard pressure” (of 1 atm $= 101\,325 \times 10^5$ Pa), various conventional days and years. From a formal point of view, these values have nothing to do with the real acceleration of free fall, which varies from place to place and during the day. As we mentioned in Sect. 2, the metric treaty first relied to properties of Earth (the metre, the second, and, indirectly, the kilogram), which were believed to be well defined. Later, it was realized that they are not and the units were redefined via artificial objects (which are now partly substituted by natural quantum objects). Earth as a whole also presents an example of a conventional object, when its shape is simplified and a number of properties are “projected” to sea level. A convention is not necessary related to a peculiar object, a reason for a convention may be the specific conditions of an experiment. A recommendation for the practical realization of the metre [11] gives a list of accurately measured optical atomic and molecular transitions. However, some transitions are to be measured under specific conditions, i.e., their given frequencies are *related* to real transition frequencies *not* necessarily identical to them.

The case of the candela and photometry is very specific and quite different from any other basic unit. Without accepting, as a part of the SI, a convention either on the spectral sensitivity of a human eye[8]or on what are “the same sensation” and “an [average] human eye” we see a very reduced field for measurements. Actually, there are two kinds of photometrical quantities: visual and physical. To introduce the candela as a unit for both, we need both kinds of conventions.

The candela and photometrical quantities were designed to deal with all visible frequencies. However, at the present time the SI does not include any convention that allows us to go beyond the frequency of 5.4×10^{14} Hz at which the candela is defined. That means that the candela definition as an SI unit is incomplete and completely compromises it as such, because the SI denies any quantity to be measured in candelas. Within the SI alone, we cannot measure photometrical quantities related to, for example, red light.

For really physical quantities (such as electrical current or amount of substance) those definitions are rigid, however, for the human-related quantities, the definitions are quite flexible and need additional assumptions to be adopted. There may be different opinions on what the best way to treat the candela is and how to modify the SI for that. However, we have to acknowledge that in the current version of the SI [5] the candela, as an SI unit, is much compromised and cannot be used as an SI unit for any application. The physical quantities are defined through physical laws and that means that they are “defined by Nature”, not by us.

4.3. Vacuum constant ϵ_0 and Gaussian units

‘You can call it “nonsense” if you like’, she said, ‘but *I’ve* heard nonsense, compared with which that would be as sensible as dictionary!’ Lewis Carroll

Although the candela is the most questionable among the basic SI units, it has never been the subject of a world-wide discussion as the ampere and the Gaussian units have been. Obviously, that is because of the significance of electromagnetic phenomena in modern physics. There is no doubt that the Gaussian units, in which $\epsilon_0 = 1$, are better for the understanding of electrodynamics. However, there are a number

[8]Appendix 2 of the official SI booklet [5] contains some details of practical realizations of all basic SI units. In the case of the candela, it reads: *The definition of the candela given on page 98* [of ref. 5] *is expressed in strictly physical terms. The objective of photometry, however, is to measure light in such a way that the result of the measurement correlates closely with the visual sensation experienced by a human observer of the same radiation. For this purpose, the International Commission on Illumination (CIE) introduced two spectral functions $V(\lambda)$ and $V'(\lambda)$*... One of them, $V(\lambda)$, is applied in photometry. However, the recommendations of CIPM on practical realizations do never (except of the case of the candela) contain any information which is *needed* for the realization. They are supposed to deliver certain information which follows from the main body of the SI brochure and *may* be used to simplify the realization (see, for example, Sects. 5.4 and 9).

of units very well suited for some classes of phenomena (see, for example, Sect. 8) and that does not mean that these units are proper units for general purposes. In this short chapter, I will try to explain why the units with $\epsilon_0 = 1$ have never been good for general use.

First, we have to be reminded that the units are needed mainly to express results of measurements (done or predicted). If these practical units are not good for theory, we may do calculations in more appropriate units, but in the end, we have to present the final results in some practical units.

Secondly, we remark that there are some areas and, in particular, a field of electrotechnical measurements (of electric potential, current, resistance, inductance, and capacity), where relative measurements can be taken much more easily and accurately (in respect to absolute measurements of the same values). Why is it so? The answer is simple: both the SI and Gaussian definitions of the basic electromagnetic units involve calculations of the magnetic or electric fields and the building of a macroscopic bulk setup with well-controlled values of these fields. In other words, the absolute measurements deal with completely different kind of experiments. The absolute measurements correspond to electrodynamics, while the relative measurements of quantities listed above are related to electrotechnics.

For this reason, nearly all electric measurements are realized as relative measurements done in special "electrotechnical" units. Separate experiments are performed in a limited number of metrological laboratories to cross check these units and to calibrate them properly in terms of the SI. At earlier times, the standards were built on classical objects. They were artificial and in this sense similar to the present standard of mass. However, in contrast to a prototype of the kilogram, they were much more vulnerable. There are a number of effects that may affect the properties of classical objects and shift them. However, it is much easier to "break" an electric device than a weight. Thus, the electrical units evolved and their calibration was not a simple procedure. They were quasi-independent. That produced a strong need to have an independent unit for electrical effects and to provide it one has to have $\epsilon_0 \neq 1$. A value of ϵ_0 has been fixed within the SI, but it was unknown in practical units and had to be measured.

Now, we have taken an advantage of the application of macroscopic quantum effects (see Sects. 5.4 and 9 for details) and may be sure that the practical electrical units do not evolve, but still we need to calibrate them. While a value of ϵ_0 is calculable in terms of SI units

$$\begin{aligned}\epsilon_0 &= \frac{1}{c^2\mu_0}\\ &= \frac{10^7}{4\pi\,(299\,792\,458)^2}\ \mathrm{F/m}\\ &= 8.854\ 187\ 817... \times 10^{-12}\ \mathrm{F/m}\end{aligned} \tag{10}$$

it is still unknown in practical units (such as, for example, ohm-1990, Ω_{90}, [12, 13]) and is a subject of measurement. Likely in future, we will decide to reverse a situation accepting quantum definitions of the ohm and the volt. That will upgrade today's practical units Ω_{90} [12, 13] and V_{90} [14] up to the status of SI units, but will make ϵ_0 a measurable quantity and will substitute the prototype of the kilogram by an electrical balance. In such a scenario, the values of the Planck constant h and the elementary charge e would be fixed and, with a value of the speed of light already fixed, one can see that

$$\epsilon_0 = \frac{e^2}{4\pi\alpha\hbar c} \tag{11}$$

where the fine structure constant α as a dimensionless constant has an unknown value, which is a subject of measurement. Thus, ϵ_0 becomes a measurable quantity certainly not equal to unity in any sense. That is why we should not like to set a simple identity $\epsilon_0 = 1$ now.

There is one more issue about the constants of vacuum ϵ_0 and the fine structure constant α. We may wonder whether the fine structure constant is calculable or not. We cannot answer this question now, however, there is a certain constraint on a scheme, how α might be predicted. The most expected scenario is that we would be able to predict α_0 (a value of the fine structure constant related to the Planck scale) as a kind of a geometric factor (see Sect. 7, 10 and 13). In such a case, the electric charge would not be a new independent property, but a kind of a derivative from mechanical properties and should be, in principle, measured in mechanical units with $\epsilon_0 = 1$. However, if α has a value chosen because of the spontaneous breakdown of symmetry (see Sects. 10 and 11) it may be any. Perhaps, we should treat the electric charge as a new independent quantity and measure it in separate units with a dimensional value of ϵ_0 as a consequence. A situation with the Coulomb (or Ampere) law is different from, for example, that with the Newtonian gravity. For the latter, we already know that the gravitational charge (i.e., the gravitational mass), due to a deep physical reason (the equivalence principle), is not a new property, but a derived property completely determined by the inertial mass. Indeed, the question of the calculability of α and thus of the origin of the electric charge will not be answered soon and, indeed, such a general view would never affect any decision on units, because of the practical importance of the question. The practice of electrical measurements has obviously pointed to the proper choice.

4.4. "Unnecessary" units, II

While discussing necessary and unnecessary units, we would like to mention a point that is important for practical use. When we speak about most phenomena, we often apply a "jargon" dropping important words. In such cases to understand anything properly, redundant information would be helpful. For example, we often speak about a magnetic field not clearly discerning magnetic induction B and magnetic field strength H (which are not simply related in media), or even about a field not specifying whether we mean magnetic or electric field. In such a case, the use of different units is very helpful to understand the practical situation. The same story is true for units and their biological equivalents, which from a theoretical point of view should be the same. However, naming the unit we immediately explain which property of, for example, radiation we have in mind: their energy or their effect on a human body. A choice of a unit plays the role of a flag allowing us to drop a number of words. Use of four different units for the electromagnetic field (for E, D, B, and H) makes theory less transparent and unnecessary complicated, however, these four units may be helpful in describing an experiment in a much shorter way.

5. Physical phenomena governed by fundamental constants

It's as large as life and twice as natural! Lewis Carroll

We mentioned above a quantum approach to standards of electrical units. They superseded classical standards providing universal values that do not depend upon the time or location of the measurement.[9] It is also very useful that we can determine them from certain experiments not related to electricity. That is because they are based on fundamental constants. However, fundamental constants, if they are really fundamental in some sense, should show themselves only at a fundamental level, while any particular measurements deal with objects and phenomena far from fundamental. How can we access any fundamental quantity? The obvious answer is that we have to try to find a property of a certain nonfundamental object, which we can calculate. There are two general kinds of such objects.

- First, we can study relatively simple objects, whose properties can be calculated by us. The simplest are particles, and only recently have we learned how to study a single particle in a trap. In earlier times, we dealt with beams and clouds of interacting particles trying to eliminate their interactions. The next in the row of simple objects are simple atoms and simple molecules.
- Another option are macroscopic quantum effects, such as, for example, the Josephson effect. Once we realized proper conditions, we can see the same result for various samples and the result is

[9] We discuss possible time and space variations of fundamental constants in Sects. 11 and 12.

simply expressed in terms of fundamental constants. An important feature of this kind of effects is that when conditions are not perfect, they often make the effect harder or even impossible to observe, but seldom affect the basic parameters of the effect. So, certain quantities coming from this kind of effect are quite immune to the conditions of the experiment.

Before we discuss any application of properties of elementary and compound particles, let us underline that an important detail for the interpretation of such measurements is that particle properties are the same for each species. A measurement may be even of classical nature aiming to determine the Avogadro or Faraday constant, however, the output is very different in the classical and quantum framework. For example, the Faraday constant

$$F = e\,N_{\mathrm{A}} \tag{12}$$

in the classical consideration is defined for an *average* charge carried by the Avogadro number of electrons (correcting for the sign of the charge) or single-charged ions, while N_{A} is in turn an average number of carbon atoms needed to form 12 g of a carbon material. In the quantum case, we know that carbon atoms or electrons are the same and we can drop a word average. In the quantum case the electron charge e is certainly a fundamental constant, while from a classical point of view it is about the same as an average mass of dust particles or rain drops.

Classical physics is unable to deal with identical objects. What does "identical" mean? If there is no interference, we can always distinguish between two electrons. From the point of view of classical statistics "identical" means "different recognizable objects for which in a particular consideration we do not care which is which".[10] But if we did care, we could always recognize them. If two electrons have approximately the same mass and charge, classical mechanics cannot check if they are the same exactly or approximately, because there is always an experimental uncertainty. It may be in principle reduced to any level but never removed completely.

Quantum physics introduces interference between particles. The physics of two slightly different electrons and two identical electrons is by far not the same. And what is very important, we do not need to perform any interferometric experiments. They have already been performed for us by Nature. The Pauli principle governs the atomic levels and nuclear shells. All electrons are identical as well as all protons and neutrons. The very existence of lasers showed the identity of all photons as particles. The identity of objects of the same class makes their properties to be natural constants. However, if we want them to be really fundamental from a theoretical point of view, we have to study simple objects.

5.1. Free particles

Studying free particles offers relatively limited access to the fundamental constants. We can measure their masses and magnetic moments. Their electric dipole moments have been searched for (because of different supersymmetrical theories) but have not yet been detected. Their charge is known in relative units and seems to be trivial. Sometimes, but very seldom, they have calculable properties. Two of the most important of them are related to the anomalous magnetic moment of the electron and the muon.

Briefly speaking, if we like to learn something beyond basic properties (such as mass) of an object, we have to study interactions. An interaction with a classical field is not a good case because we can hardly provide configurations of the classical electromagnetic field controlled with a high accuracy. Only one kind of classical fields is suited for precision measurements, namely, a homogeneous magnetic field with a fixed, but unknown (in the units of SI) strength. That allows us to compare masses and magnetic moments. A quantum interaction is under much better control, because its strength is controlled by Nature and not by us. Measuring the mass or the magnetic moment, one determines certain fundamental

[10] Historically, statistical analysis appeared in an attempt to describe social phenomena dealing with people. Definitely, that is just a case when the objects are clearly distinguishable.

Table 1. Fundamental constants determined through properties of elementary and compound particles.

Particle	Constant	Comment
Electron	m_{e}	via comparison to proton
	α	via QED
Muon	a_μ	via comparison to proton
	μ_μ/μ_{p}	via comparison to proton
Neutron	α	via comparison to lattice spacing and R_∞
Deuteron	m_{n}	via a measurement of binding energy and comparison to proton
Caesium	α	via atomic interferometry and Raman scattering

parameters directly, while dealing with a calculable interaction and calculable properties, we access fundamental constants indirectly. A quantum electrodynamical self-interaction allows us to present the anomalous magnetic moment of an electron a_{e} in terms of the fine structure constant with a high accuracy [15]

$$a_{\mathrm{e}} = \frac{1}{2}\frac{\alpha}{\pi} - 0.328\,478\,696\left(\frac{\alpha}{\pi}\right)^2 + 1.181\,241\left(\frac{\alpha}{\pi}\right)^3 - 1.737(39)\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$
$$+ \text{ near negligible effects of weak and strong interactions} \quad (13)$$

A measurement of a_{e} [16]

$$a_{\mathrm{e}} = 1.159\,652\,188\,3(42) \times 10^{-3} \qquad \left[3.2 \times 10^{-9}\right] \quad (14)$$

delivered the most accurate value of α

$$\alpha^{-1}_{\text{g–2}} = 137.035\,998\,80(52) \qquad \left[3.4 \times 10^{-9}\right] \quad (15)$$

In Table 1, we list the fundamental constants that may be obtained studying particles. An example of a nonelementary particle is the deuteron. Measuring its binding energy and masses of proton and deuteron, one can obtain the neutron mass (see Sect. 5.3).

5.2. Simple atoms and molecules

From the very beginning of studies of classical effects, we distinguished kinematics (i.e., the theory of particle motion due to a given force or a given potential field) and dynamics (i.e., the theory of forces: fields, their sources and their interactions with particles). The classical theory of Maxwell equations is the dynamics of charged particles and the kinematics of photons. Quantum mechanics introduced quantized properties and identical objects. That, in addition to dynamics and kinematics, opens up an opportunity for a prediction of the structure of certain objects. Indeed, classical physics could successfully consider compound objects, which consist of “simple” constituents such as the Solar system formed by the Sun and the planets. However, there has been no chance for any ab initio calculation, because physics deals only with particular objects and in classical physics any particular object is a peculiar object: the parameters are peculiar and the initial conditions are peculiar. Quantum physics introduced fundamental particles: a few kinds of particles that form everything in our World. We still need to know some of their parameters, but with these few parameters determined, we can try to calculate everything. We can also reverse the problem: predicting the structure properties in terms of the fundamental parameters of the constituent particles and determining the properties experimentally, we deduce actual values for the parameters.

If we study a few-particle system, we have a better chance of studying the interactions of its constituents. However, different simple atoms are treated within QED in very different ways. The most obvious case is lepton quantum electrodynamics (electron and muon are the most important leptons in QED). It needs very few input data: the elementary charge and the lepton masses. Any other property (for example, the magnetic moment) can be, in principle, derived from these few.

Indeed, the proton cannot be treated this way. Generally speaking, QED is a theory of electromagnetic interactions of leptons, photons and "external" sources. A proton is such a source and we need a number of parameters to describe it: its charge $q_{\rm p}$, magnetic moment $\mu_{\rm p}$, details of its charge and magnetic moment distribution as well as more sophisticated parameters. What we only know is that this approach is consistent — we can measure these parameters and they are the same for very different kinds of experiments.

The theory of hydrogen-like atoms involves a few dimensionless parameters (which actually are small parameters for a number of important applications and allow various perturbative expansions):

- the fine structure constant α, the exponential factor of which indicates how many QED loops are taken into account;
- the Coulomb strength $Z\alpha$, with the nuclear charge Z changing in a very broad range from $Z = 1$ to $Z \simeq 90$;
- the mass ratio of the orbiting particle to the nucleus, which is $m_{\rm e}/Am_{\rm p} \simeq 10^{-3}/2A$ (A is the nuclear mass number) for a conventional ("electronic") atom; $m_{\rm e}/m_\mu \simeq 1/207$ for muonium (the nucleus is a positive muon); unity for positronium (the nucleus is a positron); and $m_\mu/Am_{\rm p} \simeq 1/9A$ for a muonic atom;
- various parameters related to the nuclear structure.

It is clear that an exact calculation with all these parameters is not possible. Some calculations for conventional atoms are exact in $Z\alpha$, while positronium calculations apparently must be done exactly in the electron-to-nucleus mass ratio. Still, an expansion in other small parameters has to be applied and an accurate theory is possible not for any simple atoms. A proper estimation of uncalculated terms is sometimes a difficult problem [17]. Various details of theoretical calculations can be found in the review, ref. 18.

In light atoms, the perturbative approach is dominant and to demonstrate how far we can go with theoretical predictions, we summarize in Table 2 crucial (for a comparison with experiment) orders of QED theory for energy levels in various two-body atoms. We note that the leading nonrelativistic binding energy is of order of $(Z\alpha)^2 m_{\rm e}c^2$.

Various simple atoms and certain simple molecules can deliver much more information on fundamental constants than free particles, because we are able to express their properties in terms of such fundamental constants such as the Rydberg constant R_∞, the fine structure constant α, various masses ($m_{\rm e}$, $m_{\rm p}$, m_μ, m_π, etc.), magnetic moments (μ_p, $\mu_{\rm d}$, μ_μ, etc.) and some other constants. Working with atoms and molecules we can apply various spectroscopic methods, which are the most accurate at the moment.

Simple molecules are much more complicated than simple atoms and their use is rather limited. For example, studies of hydrogen deuteride (HD) provide us with the most accurate value of $\mu_{\rm p}/\mu_{\rm d}$ [19]. A summary on the use of simple atoms and molecules to determine precision values of various fundamental constants is given in Table 3. More details on simple atoms can be found in refs. 20 and 21, while a popular history of applications of hydrogen to fundamental problems is presented in ref. 22.

5.3. Free compound particles

Free particles, which we can study, are not necessarily elementary particles. We can treat nuclei, atoms, and molecules as compound particles and study their simplest properties (such as the mass or the

Table 2. Crucial (for a comparison of QED theory and experiment) orders of magnitude for corrections to the energy levels in units of $(Z\alpha)^2 m_e c^2$ (see ref. 17 for details). Here: M stands for the nuclear mass.

Value	Order [in units of $(Z\alpha)^2 m_e c^2$]
Hydrogen, deuterium (gross structure)	$\alpha(Z\alpha)^5$, $\alpha^2(Z\alpha)^4$
Hydrogen, deuterium (fine structure)	$\alpha(Z\alpha)^5$, $\alpha^2(Z\alpha)^4$
Hydrogen, deuterium (Lamb shift)	$\alpha(Z\alpha)^5$, $\alpha^2(Z\alpha)^4$
$^3He^+$ ion (2s HFS)	$\alpha(Z\alpha)^5 m_e/M$, $\alpha(Z\alpha)^4 m_e^2/M^2$, $\alpha^2(Z\alpha)^4 m_e/M$, $(Z\alpha)^5 m_e^2/M^2$
$^4He^+$ ion (Lamb shift)	$\alpha(Z\alpha)^5$, $\alpha^2(Z\alpha)^4$
N^{6+} ion (fine structure)	$\alpha(Z\alpha)^5$, $\alpha^2(Z\alpha)^4$
Muonium (1s HFS)	$(Z\alpha)^5 m_e^2/M^2$, $\alpha(Z\alpha)^4 m_e^2/M^2$, $\alpha(Z\alpha)^5 m_e/M$
Positronium (1s HFS)	α^5
Positronium (gross structure)	α^5
Positronium (fine structure)	α^5

Table 3. Fundamental constants determined through simple atomic and molecular systems

System	Constant	Comment
Muonium	α	via bound state QED
	m_μ/m_e	via bound state QED
	μ_μ/μ_p	via bound state QED and comparison to proton
Hydrogen	R_∞	via bound state QED
	μ_p/μ_e	via bound state QED
Deuterium	R_∞	via bound state QED
	μ_d/μ_e	via bound state QED
Helium	α	via bound state QED
Hydrogen-like carbon	m_e/m_p	via bound state QED
Hydrogen-like oxygen	m_e/m_p	via bound state QED
Muonic atoms	m_μ/m_e	via bound state QED
Pionic atoms	m_π/m_e	via bound state QED
HD molecule	μ_d/μ_p	via bound state QED
HT molecule	μ_t/μ_p	via bound state QED

magnetic moment). We can also rely on conservation laws. For example, the best value for the neutron mass comes from deuteron studies. The deuteron mass [23]

$$m_d = 2.013\,553\,212\,70(35)\ \mathrm{u} \qquad \left[1.7\times10^{-10}\right] \tag{16}$$

combined with an accurate value of its binding energy E_d of approximately 2.2 MeV [24] and the proton mass [25]

$$m_p = 1.007\,276\,466\,89(14)\ \mathrm{u} \qquad \left[1.4\times10^{-10}\right] \tag{17}$$

provides us with such a possibility [2, 23][11]

$$m_\mathrm{n} = m_\mathrm{d} - m_\mathrm{p} - \frac{E_\mathrm{d}}{c^2} = 1.008\,664\,915\,60(55)\ \mathrm{u} \qquad \left[5.5 \times 10^{-10}\right] \tag{18}$$

Another example of an application of compound particles is the determination of the fine structure constant α via the scattering of photons on a caesium atom and measuring their recoil shift [26]. In this experiment, one should deal with absorption and stimulated emission, however, dynamical details of the interaction of the photons and the atom are unimportant. Once we know the direction and frequency of the photons, we need to know only about the atom as a whole: its energy and momentum. If we treat the atom in such a way as a compound particle, it is “simple” in a sense.

We are reminded that the properties of certain atoms and molecules play a crucial role in the definition of SI units: the hertz is defined via the hyperfine interval in the caesium-133 atom and the kelvin via the triple point of water. The unified atomic mass unit (a non-SI unit acceptable along with the SI) and the mole are defined via the mass of the carbon-12 atom. However, considering atomic properties as units, we should indeed not care if they may be calculated.

5.4. Macroscopic quantum phenomena

Macroscopic quantum phenomena offer us certain properties that may be presented in terms of fundamental quantities. For example, the Meissner effect provides us with a quantized magnetic field. The magnetic flux through a superconducting loop can take only very specific values such as

$$\Phi_n = n\Phi_0 \tag{19}$$

where

$$\Phi_0 = \frac{h}{2e} \tag{20}$$

is the magnetic flux quantum and n is an integer number. As we have mentioned, for applications it is most important to consider macroscopic quantum phenomena that are related to the quantized values of the electrotechnical properties. Two such effects play a crucial role in practical use. These are the quantum Hall effect and the Josephson effect. The former offers us a quantized value of the resistance

$$R_n = \frac{R_\mathrm{K}}{n} \tag{21}$$

proportional to the von Klitzing constant

$$R_\mathrm{K} = \frac{h}{e^2} \approx 25.8\ \mathrm{k}\Omega \tag{22}$$

while the latter allows to quantize the voltage related to some frequency ν

$$U_{n'} = \frac{n'}{K_\mathrm{J}} \cdot \nu \tag{23}$$

where

$$K_\mathrm{J} = \frac{2e}{h} \approx 483.6\ \mathrm{THz/V} \tag{24}$$

is the Josephson constant and n and n' are certain integer numbers.

We note that a direct measurement of these constants in SI units is very complicated, and consider different approaches to the application of these two quantized phenomena in Sect. 9.

[11] This value of the neutron mass also involves data similarly related to other, more complicated, nuclei [23]. Note that the data on the binding energy may differ from the originally published results because of the recalibration of the lattice parameter.

Table 4. CODATA 2002 recommended values of some fundamental constants [2] and methods applied to achieve their values. Here MQE stands for macroscopic quantum effects and ADN for effects due to the atomic discrete nature of the substance

Constant	CODATA 2002 values	Method
R_∞	10 973 731.568 525(73) m^{-1} [6.6×10^{-12}]	bound state QED
$m_{\rm p}$	1.007 276 466 88(13) u [1.3×10^{-10}]	free particles
$m_{\rm p}/m_{\rm e}$	1 836.152 672 61(85) [4.6×10^{-10}]	bound state QED, free particles
$a_{\rm e}$	1.159 652 185 9(38) $\times10^{-3}$ [3.2×10^{-9}]	QED, free particles
α^{-1}	137.035 999 11(46) [3.3×10^{-9}]	QED, free particles, bound state QED, MQE
$\mu_{\rm p}/\mu_{\rm B}$	1.521 032 206(15) $\times10^{-3}$ [1.0×10^{-8}]	bound state QED
$m_\mu/m_{\rm e}$	206.768 283 8(54) [2.6×10^{-8}]	bound state QED
$\mu_\mu/\mu_{\rm p}$	−3.183 345 118(89) [2.6×10^{-8}]	bound state QED
e	1.602 176 53(14) $\times10^{-19}$ C [8.5×10^{-8}]	MQE, ADN
F	96.485 383 3(83) $\times10^{23}$ C/mol [8.6×10^{-8}]	MQE, ADN
$\mu_{\rm p}$	1.410 606 71(12) $\times10^{-26}$ J/T [8.7×10^{-8}]	free particles, MQE, ADN
h	6.626 069 3(11) $\times10^{-34}$ J s [1.7×10^{-7}]	MQE, ADN
$N_{\rm A}$	6.022 141 5(10) $\times10^{23}$ mol^{-1} [1.7×10^{-7}]	MQE, ADN
a_μ	1.165 919 81(62) $\times10^{-3}$ [5.3×10^{-7}]	free particles, QED

5.5. Atomistics and discrete classical phenomena

As already mentioned, even certain classical constants such as the Avogadro and Faraday constants have quantum origins. First, they originate from the atomic nature of a substance, which is a consequence of quantum mechanics, and, secondly, they receive their meaning because of the identity of species of the same kind.

We collect in Table 4 the most accurately known fundamental constants [2] that have been obtained through studies of calculable objects.

6. Fundamental constants and renormalization: operational philosophy of physics

And she tried to fancy what the flame of a candle is like after the candle is blown out, for she could not remember ever having seen such a thing. Lewis Carroll

Quantum mechanics appeared after the brilliant success of relativity and indeed it was understood that a nonrelativistic quantum theory should be extended to the relativistic case. However, the development met a problem that perturbative calculations have involved certain divergencies. The problem was solved by the introduction of the procedure of renormalization.

The solution to this problem has a philosophical side and before addressing the problem, let us discuss some philosophical aspects of physics and, first of all, answer the question what are the objectives of physics? Let us do that pragmatically. We will not discuss what various sciences pretend to aim at, we will check what they really do. Both philosophy and physics pretend to understand Nature. Philosophy picks out the most significant questions about the very existence of Nature, however, it does not care if we have enough data to answer them. And actually, similar to the truly fundamental constants, the fundamentality never shows itself for measurement. Physics also pretends to understand Nature, however, in reality it does not care what Nature, matter, or any particular object such as a photon and an electron are. Physics questions not what various objects are, but how they interact to each other. It studies not what Nature is, but how it operates. We, physicists, certainly believe that something really exists in an "absolute" sense since the same experiments produce the same results. However, we cannot say that anything particular exists until we measure it. It is close to the positivistic philosophy. However, that is not the philosophy of physicists, but a kind of modus operandi in physics. This kind of double standard is often met in everyday personal and professional life: there is a philosophy, which provides us with a general view on events, and there is an operational scheme, which determines our reaction to the events. The philosophical views of physicists on Nature may be very different from each other, while their professional operational scheme is nearly the same for everybody. This scheme is based on a kind of a "short-range" philosophy. I call the philosophy beyond the operational scheme "operational philosophy".

Indeed, we may say that matter exists. Or that an electron exists. But that gives us no real piece of information at all. If we could say that a certain particle with specific properties existed that would contain certain information, and could be correct or not. But to learn that we have to perform an experiment.

The philosophical breakthrough of special relativity was the idea that simultaneity of the events are unmeasurable. Quantum mechanics said that the trajectory is unmeasurable. That we cannot distinguish between two identical particles. That we cannot measure certain properties simultaneously. That we cannot do "exact" measurements without certain consequences. After we had learned that, we changed our view on what exists and what does not.

Nonrelativistic quantum mechanics succeeded with a perturbative approach. We start with an unperturbed equation with unperturbed parameters and introduce various small perturbations which shift the properties of the result. In quantum mechanics we are able "to switch off" most of perturbations for real quantum mechanical problems or at least vary their parameters. On the contrary, in quantum electrodynamics (QED), we cannot turn off the self-interaction. It is proportional to a small parameter $\alpha \sim 1/137$, but because of the divergencies the perturbative correction is not small. It cannot be even calculated properly because it involves the physics of high momenta. QED says that since the unperturbed "bare" parameters (such as the electron mass m_0 and charge e_0) are not measurable, we should not care if they are finite or divergent, well-determined or model-dependent. In a sense they do not exist since they are certain abstract results of our imagination. What we have to care about are only measurable quantities, i.e., "dressed" (perturbed) parameters. We are able to express measurable energy shifts in terms of the measurable electron mass m and charge e without any divergencies and any need for knowledge of physics at the high-momentum scale. This kind of expression of one measurable quantity in terms of others means a QED calculation, a successful QED calculation.

All these examples follow the idea of some equality between the very existence of a quantity and the possibility for a measurement of its value. This approach is the backbone of physics, its operational philosophy.

It finds its realization in the approach of effective potentials, which are used for various problems in particle physics. It may be an effective phenomenological potential for pions, or an effective quantum field theory produced on the way of going down to our energy from the Planck scale or from the supersymmetry scale. The story is that we believe that, for various reasons, the fundamental physics is determined at certain much shorter distances and higher energies and momenta than the ones we deal with in our experiments. Dealing with low energies, we can see only a certain effective theory. That is not a true fundamental theory but that is all that we have in an experimental sense. We have to be successful, otherwise physics would have no sense until we reached the fundamental scale of distances and energies. We trust that it is enough to determine some parameters at our low energies and any further calculations can be performed in their terms. In other words, we expect that low-energy physics is complete (in a sense that parameters determined at low energy are enough for the low-energy calculations) and consistent. If that is not correct, we should interpret that as the existence of something unmeasurable that affects our world in an unpredictable way.

7. On calculable physical constants

There was nothing so *very* remarkable in that; …it occurred to her that she ought to have wondered in this, but at the time it all seemed quite natural. Lewis Carroll

If we look at the list of the recommended values of the fundamental constants [2], it is unlikely that we will find there any constant that can be calculated exactly ab initio. We can then assume that there is no calculable constant that has a practical sense. We have already mentioned in Sect. 1, that considering the hydrogen atom we should deal with either a calculable quantity (R_∞), or with a measurable one ($\nu_{\rm H}(1s-2s)$). If a spectral property can be measured directly, it cannot be calculated ab initio exactly. We may then conclude that there is no constant that is both exactly calculable and directly measurable.

We note, however, that a reason for this conclusion comes in part from psychology. Let us give an example of a similar situation. It is well known that the theory of a point-like particle with a nonzero anomalous magnetic moment is inconsistent. Meanwhile, we believe, that the electron, being point-like, still possesses an anomalous magnetic moment and its theory is consistent. The only inconsistency here is in the terminology. We say "the electron has an anomalous magnetic moment", because it may be directly measured and because it was first measured and next understood theoretically. We say "the electron is a point-like particle", because its structure cannot be actually measured in any straightforward way and because it was first calculated and next certain previously measured effects (such as the Lamb shift in the hydrogen atom and the helium ion) were understood as consequences of the internal structure of the electron. From the theoretical point of view, the same effects are responsible for the anomalous magnetic moment of the electron and its internal structure and in a sense we can speak either about a point-like electron with $g=2$ or about an electron that has both the anomalous moment and the structure. But for historical and psychological reasons we have chosen another way to express the situation.

A similar problem in terminology is for the calculability of the constants. We know that, for example, an internal angular momentum of Earth and Moon could take arbitrary values and their ratio is a kind of constant to characterize our Earth–Moon system. If the internal angular moment (spin) were measured for quantum objects (such as electrons or atoms) before the appearance of quantum mechanics (still it is hard to imagine how), we could be surprised that $S_{\rm e}/S_{\rm p}=1$. Quantum mechanics would explain this constant. However, in reality, first, a quantum theory of the angular momentum was created and next we measured the spin (or rather interpreted some results as a determination of the spin of an electron and a proton). In the time of quantum mechanics the identity $S_{\rm e}/S_{\rm p}=1$ is trivial, and now we do not consider the ratio of the spins as a fundamental constant.

Another example is the famous Einstein identity $E_0=mc^2$. This equation appeared as a result of special relativity and was first seen experimentally through a relativistic correction to the kinetic energy. There was no way to measure it directly. Now, we can measure the binding energy $E_{\rm B}$ (of nuclei, such as the deuteron, or even of atoms — see, for example, ref. 27) and check whether the mass of a bound system is the same as a sum of the masses of its composites. We indeed know and can now verify experimentally that the mass is reduced by a value of $E_{\rm B}/c^2$. We study the mass and the binding energy as static properties and do not need to perform any relativistic experiment to check $E_0=mc^2$. Another possibility to reach $E_0=mc^2$ without any relativistic experiments is to measure the annihilation energy of positronium. The energy is determined as the energy of two gamma-quanta and the positronium mass is twice the electron mass (with a correction due to the atomic binding energy). If that was measured before Einstein's theory of relativity, we would write it as $E_0=k_1\cdot mc^2$ and interpret the theory as a calculation of $k_1=1$.

One more example is a comparison of the properties of a particle and its antiparticle (like, for example, their charges, masses, etc.). That is a result of the CPT invariance, which is a consequence of the Lorentz invariance. In early times, even the very existence of antiparticles was first proved theoretically and next discovered experimentally. We may say that we are able to calculate the electron-to-positron mass ratio and it should be unity.

As we see from the examples above, very often a question of calculability of a constant is related to history and psychology: we should first recognize a certain property as a constant of Nature and next calculate it. Generally speaking, the most fundamental constants such as the speed of light c or the Planck constant h enter a great number of very different equations. If any of these equations were discovered before Einstein's relativity and quantum mechanics, we should introduce a number of constants $c_1, c_2 \dots$ and $h_1, h_2 \dots$ instead of two basic constants c and h. Reducing the numerous coefficients in different equations to these two, we, in a sense, calculate these constants stating $c_1 = c_2 = \dots = c$ and $h_1 = h_2 = \dots = h$ (as it is discussed above for $k_1 = 1$). And actually, that is one of the most likely situations in future for exactly calculable constants.

Perhaps, the most important example of a similar situation is related to the "elementary electric charge". We accept for practical applications that the absolute value of the electron and proton charges are the same. For example, the CODATA adjustment [2] does not distinguish between the proton charge and the positron charge. However, no theory, confirmed by the experiment, implies that. Conservation of the electric charge only urges that

$$q_e + q_p = q_n + q_\nu \tag{25}$$

In other words, a small disbalance of the electron and proton charges is permitted if the neutron and (or) the neutrino possesses a small electric charge. In earlier times, we believed that the neutrino was massless and thus should be neutral since a massless charged particle would cause certain problems in conventional QED. Now we have learned that the neutrino has a nonzero mass, but it is suspected that this is the so-called Majorana mass, which also implies neutrality of the neutrino. However, we can say nothing about the neutron (from a theoretical point of view). Meantime, from experiment we know that [28]

$$\frac{|q_e + q_p|}{q_p} \leq 1.0 \times 10^{-21} \tag{26}$$

The various limitations on $|q_e + q_p|$ involve certain assumptions and we have to be very careful with the results. However, the orders of magnitudes are quite clear. Briefly speaking, when we consider an interaction of two hydrogen atoms at a long (in a macroscopic sense) distance, the gravitational interaction is 37 orders of magnitude weaker than the electromagnetic Coulomb interaction of two protons. That means that, if $|q_e + q_p|/q_p \geq 10^{-18}$, the electromagnetic H–H interaction would dominate over gravity. We know that the interaction of bulk "neutral" substances is apparently Newtonian's gravitation. If we suggest for simplicity that $q_\nu = 0$ (what is most probably true), then a small value of $q_e + q_p = q_n$ would effectively produce an $1/r$ force coupled to the baryon charge of the bulk matter. We know (from various tests of the equivalence principle) that this force (if any) is substantially weaker than Newtonian gravity. That sets a limit on the residual electric charge of the "neutral" hydrogen atom (and of the neutron) at the level of a few orders of magnitude below $10^{-18} q_p$. We note that the limit in (26) is only approximately three orders of magnitude stronger than the limit of $10^{-18} q_p$ related to the dominance of gravity in the interaction of the neutral particles. That is because of two reasons: first, the measurements are related to the coupling constant, which is proportional to $(q_e + q_p)^2$, and secondly, the mass itself is approximately proportional to the baryon charge. Only small corrections, due to a difference $(m_p + m_e) - m_n$ and a nuclear binding energy, violate the equation $M_{atom} = A m_n$. That considerably weakens the use of the equivalence principle.

If we believe in a certain unification theory (such as, for example, SO(10)), we can *derive*

$$q_e + q_p = 0 \tag{27}$$

So considering different unification theories, we are approaching a calculation of $q_e + q_p$, but it is likely that, once we succeed, we will (for psychological reasons) say again "that is not a calculation since it is a trivial consequence of the unification theory."

Let us return to the Rydberg constant. Have we calculated anything real? Or did we just give a special name to a certain experimentally meaningless combination of e, c, h, and m_e? To answer this question, we need to consider one more approach for the ab initio calculation of properties in terms of the fundamental constants: an approximate calculation. Such a calculation is quite important for applications since we apply a perturbative approach to numerous problems. Historically, the Rydberg constant was introduced to describe certain hydrogen energy levels (the Balmer series) and this constant was later calculated. However, with a substantial increase in the accuracy of theory and experiment we arrived at a point when a choice had to be made: to deal with a measured quantity or to introduce a special value that would be used in perturbative calculations. So, at the present time, the constant itself is not a real property of any atom, and we can say that we gave a special name to a specific combination of the more fundamental values. However, we can say that there is a constant that describes (without any corrections made) the hydrogen and deuterium energy levels with an uncertainty below one part in 10^3 and this constant has been calculated. We may introduce certain corrections and bridge the Rydberg constant and the hydrogen energy levels with much higher accuracy. But the significance of the application of Rydberg constant is first of all that this constant approximately describes a number of transitions and that is an important success of ab initio calculations.

The significance of this constant is also that it describes the order of magnitude for any gross-structure transition in any neutral atom and molecule (see Sect. 12.2). A constant characterizing an effect in general is also an important and nontrivial result. That is one more facet of the calculability of natural constants. A famous example of such a calculation of order of magnitude was the consideration by Schrödinger of the size of atoms and living cells [29]. He tried to answer the question, why are the atoms so small? Indeed, that is rather the question why we are so big? Schödinger considered some reasons for that. This consideration is also important to understand the numerical values of the atomic constants, since the practical units and, in particular, the SI units are defined in such a way that anything related to a human being should be in a sense of order of unity.

Other examples: a prediction of the order of magnitude of the electrical voltage that arose from molecular and atomic phenomena: it is a few volts — that takes its origin from early studies of similar phenomena and from the fact mentioned above that all molecular and atomic energy levels in the neutral atoms have the same order of magnitude (related to the Rydberg energy which is approximately 13 eV). Actually, because of that, the volt is only a natural unit if we mean its order of magnitude.

Ab initio calculations in the leading order, or even a rough approximation, may be also useful when one looks for a possible time variation of the fundamental constants. In such a case it is necessary to be able to perform a calculation of the dependence of energy levels on the fundamental constants rather than the energy levels themselves. We discuss this issue in Sect. 12.

8. Natural units

And it certainly *did* seem a little provoking ('almost as if it happened on purpose,' she thought). Lewis Carroll

How many units and standards do we need? Theoretically, we need only the base units of SI, and even not all of them. In particular, we can reproduce the metre through the second and the ampere through the kilogram, the metre, and the second. However, practically, we need a lot. A measurement is a comparison, and the most fortunate case is a comparison of a quantity under question with a "probe quantity" of the same kind. However, when we do different measurements of "the same" kind of quantities such as, for example, the distances, we notice a big range in their values. Astronomical and atomic distances are related to not quite "the same" kind. And indeed, for obvious practical reasons, we measure them quite differently and like to apply different "probe quantities", i.e., different units.

Indeed, these units cannot be independent and we need to properly calibrate them. However, the calibration is not always important, because quite seldom we are really interested in a comparison of, for example, the mass of a hydrogen atom and the mass of the Sun. Because of that, we can leave their units, which must be related in a formal sense, to be practically unrelated.

Still, for a number of measurements a proper calibration is needed. In the case of classical phenomena, we have to perform this calibration regularly, to take care that the unit is unchanged during the experiment etc. We should also take care that the units in different laboratories are properly compared. However, quantum physics opens another option. It offers quantum natural units that are stable and universal.

Actually every dimensional fundamental constant is a kind of a natural unit [7], and a substantial number of the dimensionless constants (and certain dimensional constants such as the Boltzmann constant k) can be considered as conversion factors[12] (see [3]). So, there is a large variety of various natural units and even natural systems of units[13]. Two kinds of natural systems have been used.

- First, a system can have natural units for any dimension, such as the atomic units and the Planck units. Indeed, some systems are incomplete because they do not care about all phenomena. But every unit, which is really needed for a description of a certain class of phenomena, may be or may not be related to the fundamental constants. In the first kind of natural units all necessary units are related.

- Secondly, a system can apply natural constants together with other units. In such a case, the natural parameters or the fundamental constants set a certain constraint on the units, such as in the case of systems in which $\hbar = c = 1$ (relativistic units) or $\epsilon_0 = 1$ (for example, Gaussian units).

Are natural units (or a natural system of units) a good choice? In their "complete form" they are as good as any other units. However, in physics, we widely use various "jargons" [7]. We can indeed measure similar (but not the same) quantities in the same units, such as a measurement both of the time intervals and the distances in seconds (or both in metres). However, we know these are very different quantities that are measured differently. That means that in saying $c = 1$, we use a jargon, and in reality, we mean something like c is equal to one light year per year or so. Jargon, as a special kind of language, it differs from normal language being designed for special use only. In this special use (for a special kind of phenomena), it offers a more short and clear description. Meantime, often the very use of the "words" differs from the normal use and the jargon sentences are "wrong" or meaningless literary. The same in physics. We like to measure frequency, energy, momentum, and mass in different units in a general case. They are closely related in the case of relativistic quantum physics. However, in the general case they correspond to very different properties and assume different experimental techniques to deal with them. They also suggest different modifications for applications to continuous media (which is rather unimportant for fundamental physics, but significant for experiment). The practice of jargon often deals with numerous hidden substitutions for quantities (confusingly keeping their names) such as

$$\begin{aligned} t &\rightarrow x_0 = ct \\ m &\rightarrow E_0 = mc^2 \end{aligned} \tag{28}$$

Such hidden substitutions, which are equivalent to use of the same units for different quantities (as the energy units for the mass), would be misleading and not very helpful in a general case because of destroying the advantages of the dimensional analysis method. We like to distinguish the distinguishable

[12] We do not mean that these constants are just the conversion factors and nothing else.

[13] We note that what is customarily referred to as a "systems of units" is in fact a system of units and quantities. For instance, the rationalized and irrationalized CGSG systems deal with the same electrical units, but with differently defined quantities. Natural systems of units sometimes assume normalization of certain quantities different from the SI.

quantities. However, in quantum relativistic physics, where constants c and h may appear in any equation, we can hardly use dimensional analysis and it is worthwhile to present all these quantities in, for example, energy units without losing even a bit of information.

Atomic units,

$$\begin{aligned}
m_{\text{a.u.}} &= m_{\text{e}} = 9.109\ldots\times10^{-31}\ \text{kg}\\
e_{\text{a.u.}} &= e = 1.602\ldots\times10^{-19}\ \text{C}\\
l_{\text{a.u.}} &= a_0 = \frac{\hbar}{\alpha m_e c} = 5.291\ldots\times10^{-11}\ \text{m}\\
E_{\text{a.u.}} &= E_{\text{h}} = \alpha^2 m_e c^2 = 4.359\ldots\times10^{-18}\ \text{J}\\
t_{\text{a.u.}} &= \frac{\hbar}{E_{\text{h}}} = 2.418\ldots\times10^{-17}\ \text{s}
\end{aligned}\tag{29}$$

which present a very natural, physical, and logical coherent system of units, are very well adjusted to atomic and molecular phenomena and most of the quantities there are of the order of unity. However, those units indeed are not convenient for other phenomena. These units present a case of the "theoretically" natural units. We choose them to simplify theory (see, for example, Sect. 12.2). The other kind of natural units are "practical" natural units. Choosing them, we do not care too much about their fundamentality. Our concern is our ability to apply them. Examples of the "practically" fundamental units are the caesium HFS interval, the carbon atomic mass, the Bohr and nuclear magnetons, the masses and the magnetic moments of an electron and a proton, and the von Klitzing and Josephson constants. We partly consider a question of the "practically" fundamental units in the next section (Sect. 9).

The choice of units, we use in physics, is quite simple. We use various "theoretically" natural units when we do calculations. Some of them are very helpful also for education. The more complicated the calculations are, the more useful are the related natural units. However, once we refer to a quantity to be measured, we switch to "general" (SI) or natural "practical" units.

9. Definitions and *Mise en Pratique* for SI units: A back door for natural units

> **'When *I* use a word,' Humpty Dumpty said in rather a scornful tone, 'it means just what I choose it to mean — neither more nor less.' Lewis Carroll**

Currently, practical recommendations issued by CIPM for the most important units [5] such as the metre [11], the ohm [12, 13], and the volt [14] are based on certain natural units.

Why do we need such recommendations? The problem is that the original SI definitions [5] cannot be used in an easy way. As we mentioned, the idea of the units comes from the fact that a measurement is a comparison and to compare the results of different measurements we need to go through a chain of comparisons. The introduction of the units means that an essential (and the "universal") part of the comparisons is separated from the rest and recognized in a very specific way. It is a responsibility of metrological institutions around the World to take care of the standards and the units. The output of this work should be a certain set of quantities, convenient for further use. Unfortunately, the SI definition of certain units is not suited for that. The practical recommendations are designed to cover the gap between the rigorous SI definitions and practical accessability by a relatively broad range of users. However, the recommendations are not a part of the SI in a sense: they aim to arrange additional conventional units and to simplify a measurement in the SI units as long as the users agree with a reduced accuracy.

Let us give an example of such a recommendation. As we mention in Sect. 5.4, certain macroscopic quantum effects (the quantum Hall effect and the Josephson effect) may be very helpful in establishing natural units of the resistance and the electric potential. For that one has to know the values of the fundamental constants R_{K} (the von Klitzing constant) and K_{K} (the Josephson constant) in the units of the SI. That is not a simple issue and three basic strategies may be applied to take advantage of these effects.

Scenario # 1 suggests, that we fix values of these two constants. Since that is not possible for units of the SI, that means an introduction of certain conventional units, in which

$$R_\mathrm{K} = 25\,812.807\ \Omega_{90} \qquad \text{(exactly)} \qquad \text{ref. 12}$$
$$K_\mathrm{J} = 483\,597.9\ \mathrm{GHz/V_{90}} \qquad \text{(exactly)} \qquad \text{ref. 14} \tag{30}$$

They are not SI units and that means that if we check the Ampere law with magnetic constant

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} \tag{31}$$

we should fail. We would also arrive at a discrepancy in the energy measurements since

$$\frac{V_{90}^2}{\Omega_{90}} \neq \mathrm{kg\,m^2/s} \tag{32}$$

But for most of the applications these mismatches with the SI are not important and the units volt-1990 and ohm-1990 are sufficient for most of the measurements, but not for all.

If we really need to deal with the SI units, we should do something else.

Scenario #2 suggests, that we use the same units based on these two quantum effects, however, we determine two necessary parameters R_K and K_J from additional experiments. For trade and legal applications one may use the recommendations [13, 14], which suggest an uncertainty of the numerical values of (30) as related to the SI units

$$R_\mathrm{K} = 25\,812.807\,0(26)\ \Omega \qquad \left[1 \times 10^{-7}\right] \qquad \text{ref. 13}$$
$$K_\mathrm{J} = 483\,597.9(2)\ \mathrm{GHz/V} \qquad \left[4 \times 10^{-7}\right] \qquad \text{ref. 14} \tag{33}$$

The two scenarios above are based on the CIPM recommendations. The recommendations, however, as well as all the legal metrology, are not designed for scientific use. What is important for physics is not the subject of any legal agreement. For nonprecision absolute measurements, or for relative measurements, one can use the CIPM [12–14] or CODATA [2] recommendations just for convenience. Meanwhile, for precision scientific applications, we should avoid any particular values of R_K and K_J in the SI units at all. Instead, the results should be presented as related to more complicated values, which contain factors $(R_\mathrm{K})^n(K_\mathrm{J})^m$, taking into account that we have measured a certain quantity in the units related to the quantum natural units, determined by R_K and K_J. One can see such an approach in the CODATA adjustment of the fundamental constants [2], which dealt with the most accurate measurements of the fundamental constants.

10. Fundamental constants and geometry

…The Multiplication Table doesn't signify: let's try Geography. Lewis Carroll

Speaking about the constants of Nature, we cannot avoid the question if the number π is one of them. Our answer is, in a sense, it is. To present our point of view, we address geometry. We remember from high school, that geometry is based on twelve axioms, the statements that are above any proofs. However, in physics, we should prove (experimentally) everything. We know that general relativity states that space-time is flat if there are no gravity sources around. However, a correct statement is "locally flat". Globally, the Universe may have a geometry that does not allow the flat geometry "universewide" (like, for example, a surface of sphere). We have to check if the actual geometry is flat and the present point of view is that it is close to being flat within a certain uncertainty. That does not help much, because according to the inflatory model [30, 31] the related parameters should be very close to the values for the flat case. Topologically, the Universe may be either open or closed (or even of a more complicated topological structure than just a closed four-dimensional space). The closure or openness is closely related to the density of the energy, which is known at the level of a few-percent (see Sect. 14). It is consistent with the flat value. Now, we may, in principle, check whether the sum of angles of a triangle studied in our space (after doing corrections to remove gravity effects) is equal to π. This statement may be verified and, thus, π is a fundamental constant of our space. However, it is a fundamental constant not in the same way as, for example, α. The fine structure constant is (at least now) a constant without any relation to theory (of its origin). We measure α and use its value to develop a QED phenomenology. A value of α is in a sense not critical. If it were discovered that $\alpha = 1/136$ or $1/138$, that would certainly change the theoretical predictions numerically, but would not change our general view of fundamental physics. For the number π or the number of the space dimensions ($d = 3$) we already have a theory sensitive to their values. We can indeed say that π or 3 are just mathematical numbers, imbedded into a flat three-dimensional geometry. However, a physical statement is that this geometry is ours. Actually, to measure the angles geometrically or trigonometrically means to accept a part of geometrical ideas. In such a sense, any test of the sum of angles to be π is rather a test of the validity of the three-dimensional Euclidean flat geometry in application to our world.

The relation between the geometry and the predictability of the fundamental constants has a broader context. In a sense, any symmetry is related to a kind of geometry.

Trying to build relativistic gravity theory, we strengthen the significance of the equivalence principle, which says that a ratio of the inertial and gravitation masses is a universal parameter (which we set to unity). A wish to build a linear equation for a relativistic particle led to a prediction of positrons with "calculable" properties

$$
\begin{aligned}
&q_{\bar{\mathrm{e}}} = -q_{\mathrm{e}} \\
&m_{\bar{\mathrm{e}}} = m_{\mathrm{e}} \\
&S_{\bar{\mathrm{e}}} = S_{\mathrm{e}} \\
&\mu_{\bar{\mathrm{e}}} = -\mu_{\mathrm{e}}
\end{aligned}
\tag{34}
$$

The positron actually was the first particle ever predicted. The Dirac equation (as well as the other linear equations) suggests that in the leading approximation the g factor of a point-like (i.e., structureless) particle is equal to two. However, this value is perturbed. For the free leptons (electrons and muons) the nonzero anomalous contribution is small and can be perturbatively calculated up to certain accuracy and also accurately measured. The present combined results for the anomalous magnetic moments are [2]

$$
\begin{aligned}
&\left(\frac{g-2}{2}\right)_{\mathrm{e}} = a_{\mathrm{e}} = 1.159\,652\,185\,9(38) \times 10^{-3} \qquad \left[3.2 \times 10^{-9}\right] \\
&\left(\frac{g-2}{2}\right)_{\mu} = a_{\mu} = 1.165\,919\,81(62) \times 10^{-3} \qquad \left[5.3 \times 10^{-7}\right]
\end{aligned}
\tag{35}
$$

When the unification theory SU(5) was suggested, one of its successes was an explanation of the Weinberg angle Θ_{W}. The value was predicted for a certain high-energy scale (related to $E_{\mathrm{un}} \sim 10^{14}$ GeV). A measurable value is related to a much lower energy scale and thus should be renormalized. An accurate theory of its perturbation by the radiative corrections together with accurate experimental data should provide us with a constraint on the unification theory. The SU(5) theory happens to be incorrect since it disagrees with a number of observed effects (such as a "too-long" proton lifetime τ_{p}, presence of the neutrino oscillations etc.). It is believed, nevertheless, that a similar unification scheme will eventually explain the neutrality of the hydrogen atom and the value Θ_{W}.

It is quite likely that most, if not all, calculable constants may be predicted only via symmetrical and thus geometrical ideas.

The symmetries and conservation laws are very strongly related to the fundamental constants and their constancy. Because of that, we would like to note that, in addition to direct violations of symmetries, quantum physics offers additional ways to violate the symmetries in "smooth" matter: via the quantum anomaly or spontaneous breaking.

- **Quantum anomaly:** Quantum field theory suggests that we should substitute the wave function for a field operator $\Psi(x)$, which is quite singular and consequently a quantity $J[\Gamma] = \overline{\Psi}(x)\Gamma\Psi(x)$ (where for fermions Γ is an arbitrary combination of the Dirac gamma-matrices) is ill-defined. The quantity J is significant because any electron or quark current is of such a form. It was discovered that there may be a special kind of quantum violation of symmetry — the anomaly. It is realized in such a way that

  (*i*) there are two currents J^1_μ and J^2_μ, which are conserved within classical physics ($\partial J^a_\mu/\partial x_\mu = 0$), and thus two symmetries are presented at the classical level;

  (*ii*) the currents are singular and thus ill-defined at the quantum-field level;

  (*iii*) there is no regularization that supports the conservation of both currents and both symmetries.

  As a result, one of the symmetries (for example, related to J^2_μ) is to be violated and the nonconservation term $\partial J_\mu/\partial x_\mu \neq 0$ is proportional to the Planck constant h (see ref. 32 for more details). A well-known example is the Adler anomaly for the axial current, which plays an important role in the properties of the π^0-meson.

- **Spontaneous breakdown:** The spontaneous breakdown of symmetry is another example of how a classical symmetry can be broken in quantum field theory. Let us suggest that the interactions (potentials) are invariant with respect to a certain symmetry. There is no symmetrical state with minimal energy, but instead there is a family of nonsymmetrical minimum-energy states. It is similar to, for example, the magnetization of bulk iron. The theory is isotropic, however, a minimum energy is related to the case with a certain nonzero value of the macroscopic magnetic moment. Any direction of the moment is related to a minimum in the energy ("vacuum"), however, only one direction can take place at any particular case. Indeed, there are domains with different directions, but if our observable universe is inside such a domain, we would not see the other domains. We note that it is different from the simplest problems in quantum mechanics. Quantum mechanics can also deal with such a potential, however, because of the overlap of the vacuum states there is a "fine structure" and the actual minimum of the energy is related to a certain superposition of these states (for example, their symmetric sum). The particular asymmetric vacuum state is to be presented as a sum of the superposition of compound vacuum states and its evolution via certain oscillations will lead, most probably, to the lowest state of this fine structure. However, with an increase of the phase volume (with increase of the number of degrees of freedom) the probability of the tunnel transitions between the vacuum states goes down very fast. The evolution time becomes so long that we can see no evolution et all; for example, we cannot detect any oscillation between left-hand and right-hand organic molecules. In the case of the quantum field the characteristic evolution time is infinite because of the huge volume of the Universe (see refs. 30–33 for more detail).

There is also a specific kind of phenomena, which may lead to an "observational" violation of such symmetries as, for example, the Lorentz invariance. The phenomena are related to the fact that it is unlikely that any symmetry will be observed directly, but we study certain consequences of the symmetry, and, if we do not know the complete theory, we can be misled.

Let us explain that with an example of nonrelativistic quantum and classical mechanics. They have mainly the same symmetries and conservation laws (conservation of momentum, angular momentum, and energy). However, from a classical point of view, conservation means that we can measure, for example, all three components of the angular momentum in two separate moments of time and the result must the same. Classically, we also expect that we can do two “fast” separate measurements of the energy, as precisely as we like, and that allows us to check whether the energy is conserved exactly and in any particular phenomena or the conservation takes place approximately and (or) on average. And “fast” means that the measurement time may be as short as we like. That is by far not the same in the quantum case. In both cases, doing “classical” experiments for the conservation of the “whole” angular momentum and the “exact” conservation of energy, one will fail to confirm the conservation laws once we arrive at the level of accuracy where quantum effects enter into the game. The symmetries and conservations still take place, but their observed consequences differ from the naive classical expectations.

We know, that at the Planck scale, the geometry of space-time quite likely differs from what we see around us.[14] We do not know what it really is. Indeed, certain symmetries can be broken there. However, some most “sacred” symmetries might be realized in such a way that their consequences alternate from our expectations and, thence, the experimental results might “observe” certain violations of these symmetries and conservations.

11. Constancy of fundamental constants

…They began running when they liked, and left off when they liked. Lewis Carroll

The fundamental constants, most of them, appear in physics with quantum mechanics. Newton’s constant G came earlier, but only considering the Planck-scale effects, we can imagine how fundamental it is. They were called “constants” and it was believed that they should be such by default. To vary them, one should rather expect an exceptional reason. That was the situation, when Dirac and later Gamov suggested that the “constants” may not be constant.

However, the truth is that there is no strong reason why the “constants” of Nature are constant. We know that the ratio of the electron and proton spins is unity and cannot vary. If it were possible to switch off the QED corrections, we should expect that the g factor of an electron is a trivial constant equal to two. Thus, there may be only one theoretical reason for their constancy — that would be an explanation of their origin. For the most important constants we have none. The constancy of the constants is merely an experimental fact and an a priori trust in the domination of symmetry in the nature of Nature. The former, indeed, can never be final and we need to check that again and again with a more broad range of phenomena and with a greater accuracy. The latter is in a formal sense rather wrong: we recognize the inflation as a basic element of modern cosmology. Inflation [30,31] had urged the electron mass and charge to vary in the very remote past. If we accept that the constants were varying once, we should rather consider them as changing quantities at a default situation, and need a reason for them not to vary again; or not to vary quickly. A once nonconstant is forever not a “trusted” constant.

[14] I have heard that numerously and in particular a statement about a “noncommutative space” and this is one more example of physical jargon. We, for example, clearly understand that quantum mechanics does not change the phase space. In the one-dimensional case, the plain $\{x - p\}$ is just the same as in the classical case. However, classical states are rather point-like. We sometimes assign them a finite volume because of the experimental uncertainty in our data or because of their statistical treatment, which is in classical physics also a result of an uncertainty in our description. Meanwhile, the volume presents only a spot where the point may be, but any state still is point-like and the uncertainty is, in principle, avoidable in classical physics. The Heizenberg inequality implies that a quantum state has a minimal finite volume determined by the Planck constant $\hbar$. The same for the three-dimensional angular momentum. The only point-like quantum state in the three-dimensional angular-momentum space corresponds to the case of zero angular momentum. But nevertheless — the space is the same, the allowed states are different.

We recognize the existence of dark matter, which may interact "very" weakly with our matter. We do not know what the dark matter is and how weak this "very" weak interaction may be. Due to a number of such unclear phenomena, we need to distinguish between:

- effects such as a violation of the local position invariance (and in particular a violation of the local time invariance)
- and a variation of the constants.

One may expect that a violation of the local invariance means that the results of measurements would depend on time, and location upon the measurement and that is the same as a time and space dependence of the fundamental constants. However, these two situations are not quite the same.

The results of an experiment may be affected by an environment. In earlier times, an "environment" for a laboratory-scale experiment was also laboratory-scaled; exceptions were the gravitational and magnetic fields of the Earth. However, they were not significant: since the former was nearly a constant (which did not depend on the location at the level of then achievable accuracy) and for the latter there may have been a shield. Now, doing high-precision balance experiments, one can clearly see the effects of the motion of the Sun and the Moon on this scale of experiments. Indeed, the existence of the surf has been known for centuries. But the surf is a result of an accumulation of these effects over a "big" detector, which is of the Earth scale. Until the very recent time it was not possible to see such effects with "small" detectors.

Now, we are sensitive to the environment on a very large scale. We know that we live in a changing universe (the environment item number one), going through a bath of 2.7 K cosmic microwave background and a similar background radiation of known (neutrino) and, maybe, unknown massless particles (the environment item number two), and dark matter and dark energy presented around (the environment item number three), etc. We would never qualify any effect of interactions with them as a real violation of Lorentz symmetry, but we may want to qualify a variation of certain natural parameters induced by them as a variation of the constants. In principle, we can say that there was no variation of truly fundamental constants during the inflation, but only "environmental effects", caused by cooling of the Universe. However, we prefer to say that the electron mass has changed.

As we mentioned, the Earth's gravity field is nearly a constant and the free fall acceleration g was considered as a universal and fundamental constant for a while. Now we know it is neither constant nor universal and fundamental.

Kepler found that any planetary orbit satisfies a condition

$$\frac{R^3}{T^2} = \text{[Kepler's] constant} \tag{36}$$

with the same universal constant for any planet. We now know that Kepler's "universal" constant, which governs the motion of all planets, is a specific constant related to our solar system only and nothing more.

These two examples show how important it is to understand the nature of the constants. We now have a great number of fundamental parameters, the origin of which is unclear: the Yukawa Higgs coupling constants, the Cabibbo–Kobayashi–Maskawa matrix (CKM) parameters, parameters of a lepton analogue of CKM, cosmological constants, etc. These constants have been observed and studied. There are also a number of important constants that have not yet been detected, but strongly expected as, for example, the mass of the Higgs particle.

Albert Einstein believed that all the constants are, in principle, calculable. That should be expected in a world, where the equations determine everything. But that apparently is not our world. We know, that some symmetries of our world have been spontaneously broken. That happens, when the symmetric state is unstable, while a family of nonsymmetric states has the same minimal energy. The vacuum falls to one of these minimum energy states. We know a number of examples in classical physics. For example, we already mentioned a bulk piece of iron with a nonzero value of the residual magnetic moments. A massive piece used to consist of domains — numerous smaller pieces, in which there is a nonvanishing macroscopic magnetic moment. The Hamiltonian and all equations that describe any domain are isotropic. However, the state with zero macroscopic magnetic moment is unstable. Stable states are those with a magnetic moment directed to somewhere. Where? We cannot predict. It may be any direction and in fact the directions in different domains are different. One may think that a direction is not as important for observable quantities as the magnitude (indeed until we do not look for a violation of the isotropy — which may not be important if we have in mind not our space but a certain functional space like, for example, the space of isotopic spin). However, there is a simple example of how to transform the direction into a magnitude. It is enough to imagine a situation when there are two independent values similar to the iron's magnetic moment, completely independent for the vacuum states, but coupled together to the same matter field (i.e., to a certain particle). In such a case, the angle between them is related to a scalar that can affect the value of the energy of the particle.

This example shows that certain properties cannot be predicted through the equations. We acknowledge a spontaneous breakdown of symmetry in the Standard Model of the electroweak interactions [31–33]. We expect that our world has a larger symmetry group than we actually observe. And a nonobserved part of the symmetry has been destroyed by one or few spontaneous breakdowns. It may happen that certain "fundamental" parameters of our world are a direct result of such breakdowns and they could take, in principle, different values in another place or another version of the evolution of the Universe.

If that is the case, certain parameters are not predictable and by discussing them we approach a framework of the so-called *anthropic principle*. There have been a number of various modifications of it, including not only physical, but also philosophical ideas. We are not very enthusiastic about these ideas. However, a "minimal" physical part of the principle is the *selection principle*: we observe only what we can observe and the very presence of our species, as the observers, sets a certain constraint on the observable properties. That is like Kepler's second law: if we would learn neither the Newtonian gravity theory, nor data about planets outside of our solar system, we should consider the Kepler's constant in (36) as a universal constant — the universal constant for all observable planets.

Once we allow variations of the constants of Nature, we remark that the units are also vulnerable. From first glance, we should prefer to speak about dimensionless constants such as

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \tag{37}$$

They are clearer to discuss phenomenologically and easier to detect. Sometimes, it is even stated that we can only look for a variation of dimensionless quantities.

Indeed, variations of the dimensional constants may also be detected, but the experiments are much more complicated, because they should directly address time and space gradients of such constants [4, 34]. A well-known example is the famous Michelson–Morley experiment, which checked whether the dimensional constant (the speed of light) in the same in every directions.

Let us leave the general discussion on the search for the variability of the constants at this point and first look at how we can describe their variations. If one tries to seriously consider the varying constants, we have to introduce changes from the very beginning. It is not enough just to accept the equations derived under a conventional assumption of the constancy of the natural constants and then allow them to vary slowly. We can easily arrive at a contradiction. Let us consider a simple example: a situation when, in a specific inertial frame, the Planck constant is slowly changing globally with time, so $h = h(t)$. Meantime, the laws of physics are still isotropic and in particular $\partial h/\partial \boldsymbol{x} = 0$. However, the angular momentum[15] is quantized

$$L_z = \hbar \cdot l_z \tag{38}$$

We arrive at an obvious contradiction: the angular momentum L_z should be conserved (because of the isotropy), while l_z is also not changing (being an integer or semi-integer number) and, meantime, their ratio, the Planck constant $\hbar$, is changing. This inconsistency comes directly from the assumption that we can accept the known equations and allow their constants to vary slowly.

There is no straightforward way to deal with the variable constants. First of all, when the constants are *constant*, we can redefine operators via their effective renormalization as, for example,

$$F_{\mu\nu} \to \frac{1}{e} F'_{\mu\nu} \tag{39}$$

etc. If we like to introduce slowly varying constants into the basic equations, we do not know even into which equation. Because of the "renormalizations", such as in (39), we do not have a single set of the basic equations but quite a broad family of equations that are equivalent as long as the "constants" are constant.

We used to describe most of the quantities that are calculable with help from such equations. However, in our opinion, the starting point to adjust our basic phenomenology to the case of the variable constants is the path integral (the functional integral over field configurations). The conventional approach reads that the path integral

$$Z = \int \mathrm{e}^{-iS'}$$

presents a matrix element of the evolution operator [32,35]. To study a particular evolution, we should integrate over all available configurations ("trajectories") with proper initial and final conditions. The action S' is normalized to be dimensionless. This operator has a transparent physical sense: we have to sum over all possible trajectories and we also know that in most of cases the dominant trajectory (trajectories) is related to the least action. The least-action trajectory for quantum mechanics is the classical trajectory. When we study quantum-field "trajectories" in the functional space the least-action trajectory is related to the field equation such as the Maxwell equations for the photon's field and the Dirac equation for the electron's field. Far from the minimizing trajectory, the phase (which is the action S') is changing fast and the contributions cancel each other. Close to the minimizing trajectory the phase is nearly unchanged and the contributions are enhanced.

Now, we can generalize the action (for example, for quantum electrodynamics, which is in a narrowed sense, substantially, a theory of electrons and photons — see Sect. 5.2) to the form

$$S'_{\mathrm{QED}} = \int \mathrm{d}^4x \left\{ \xi_3(x)\overline{\psi}\left[g_{\mu\nu}\gamma^{\mu}\left(i\frac{\partial}{\partial x_{\nu}} + \xi_4(x)A^{\nu}\right) - \xi_5(x)\right]\psi - \frac{1}{4}\xi_6(x) g_{\mu\nu}g_{\rho\lambda}F^{\mu\rho}F^{\nu\lambda}\right\} \tag{40}$$

where the metric tensor at a particular preferred frame[16] is defined as

$$g_{\mu\nu} = \begin{pmatrix} \xi_1(x) & 0 & 0 & 0 \\ 0 & -\xi_2(x) & 0 & 0 \\ 0 & 0 & -\xi_2(x) & 0 \\ 0 & 0 & 0 & -\xi_2(x) \end{pmatrix}$$

[15] We consider here the classical angular momentum $\boldsymbol{L}$, which is dimensional, and the quantum angular momentum $\boldsymbol{l}$ which is dimensionless.

[16] re are two natural options for such a frame. The first is related to the one in which the cosmic microwave background (CMB) radiation is isotropic. The other corresponds to the frame that is determined by dark matter. This latter is well defined locally, but not globally. Once we fix the frame, we can consider an analogy with electrodynamics in media, which in the simplest case can be described by two dimensionless functions $\epsilon_{\mathrm{rel}}(x)$ and $\mu_{\mathrm{rel}}(x)$.

and the electromagnetic field tensor as

$$F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

The functions $\xi_n(x)$ obviously violate several important symmetries (including the gauge invariance, the local position invariance, and the local Lorentz invariance) and allow time and space variations of the dimensional fundamental "constants". Here, the functions $\xi_n(x)$ are dimensional, but we can indeed introduce factors $h^{(0)}$, $c^{(0)}$, $e^{(0)}$, and $m^{(0)}$ in a proper way and make the ξ functions dimensionless. These four factors are related to the Planck constant, the speed of light, the elementary charge, and the electron mass as measured at a particular point $x^{(0)}$. More complicated models are also possible with, for example, a less trivial metric tensor or the electromagnetic field tensor, or with the appearance of small terms that directly violate various symmetries.

When should we use (40) and when just put time and space dependence inside conventional equations? It depends on the problem. Basically, there are three kinds of related measurements:

- One may perform a series of "fast" measurements separated in time by a "large" interval T (for simplicity we speak here about the time variations only). That is, in particular, a case of atomic, molecular, and nuclear spectroscopy. Additional terms with derivatives of the $\xi(x)$ functions should be integrated over the "short" time of the measurements τ and the value of $\tau\partial\xi/\partial t$ may be neglected in comparison with a difference $\xi(t+T) - \xi(t)$ over the "large" separation ($T \gg \tau$).
- "Long" measurements can be performed, for example, as a result of continuous monitoring of the motion of planets etc. In this case the effect of the integration of $\partial\xi/\partial x$ is comparable to the effect of the adiabatic change of $\xi(x)$ in the conventional equations.
- And indeed, one can try to deal directly with derivatives performing $\partial\xi/\partial x$-sensitive experiments. It is quite probable that it is easier to do that for space gradients rather than for time gradients. For example, we can try to perform precision measurements in space similar to the GPS measurements in the atmosphere. In the case of space-gradient terms in the law of the propagation of light, we should "observe" a nonflat geometry after interpreting the light propagation time intervals as the effective distances.

12. Search for possible time variation of fundamental constants

– …'one *can't* believe impossible things.'
– 'I dare say you haven't had much practice.' Lewis Carroll

The easiest and most transparent kind of experiments to search for a variation of the constants is indeed to measure the same quantity twice. If these measurements are "fast" and have a long separation, we can use the description with nonvarying constants and not care about possible additional terms and gradients. The validity of this approach is obvious for atomic physics: we do a series of short measurements, for which the gradients of the constants are negligible for atomic time and space scale and cannot affect the result of measurements. A different situation is for a search of a variation of the Newtonian constant G. Most of measurements are related to continuous monitoring over a long period. Instead of a large series of short atomic measurements, the gravity searches deal with a number of long measurements. In such a case, the contributions of the gradients should be important.[17]

Because of that we concentrate our attention on atomic measurements. Still there are three kinds discussed in the literature.

[17] It should be understood, that varying G and its gradients probably is not enough. For instance, the equation for a photon in a medium, presented in terms of vacuum fields, would involve not $c(x)$ and its gradients, but instead $\epsilon_{\rm rel}(x)$ and $\mu_{\rm rel}(x)$ and their gradients. Those functions have no separate sense in vacuum.

- Astrophysical comparisons deal with a relatively low fractional accuracy, but take advantage of a very big time separation (up to 10^{10} yr). This kind of observations is by far not transparent and suffers from necessary statistical evaluations in a situation when certain correlations may be present.
- Clocks based on different transitions have been developed and by comparing them one may hope to learn about the relative variations of their transition frequencies. However, the clock is just a device, built for a purpose, and its properties are not necessarily related to the atomic or molecular transition frequency in an exact sense. There may be a number of drifts and certain parameters of the clock, which determines drifts, and which are determined, in turn, by an artificial environment in a noncontrolled way. An example of such a clock is the hydrogen maser. Its frequency drift is due purely to an environmental problem (the so-called wall shift).
- Still, the frequencies of certain clocks follow atomic transition frequencies. Such clocks are similar, in this sense, to the primary caesium standard. The very basic principle of the primary caesium clock is that it should reproduce the caesium HFS transition frequency, which is related to the SI second. To deal with this kind of clock is the same as to measure an atomic or molecular frequency with high accuracy.

Below, we consider in detail these kinds of "near primary" frequency standards (and we note that maybe, in future, one of them will give us a new SI second.[18])

The significance of these clock-based experiments is in the great accuracy of the precision frequency measurements. Presently and for a while (maybe even forever), the frequency measurements are the most accurate.

Here, we consider certain details of laboratory searches. Different aspects of the possible variations of the fundamental constants and their searches are discussed in ref. 36.

12.1. Atomic clocks

In this chapter we consider frequency standards rather than clocks. In principle, a clock is a time standard. Indeed, the frequency and time intervals are closely related, however, the time measurement may be "absolute", i.e., related to the conventional "beginning of time". That involves two metrological problems for keeping the time scale: the realization of the time-interval unit and of the "zero point". The specifics of time keeping requires that "true" clocks operate continuously, otherwise the information on the initial moment would be lost. A real time standard is actually not a single device but a set of various related standards. Still, with a peripheral part of the clock operating around it, the very heart of any clock is a certain frequency standard. Presently, that is either a caesium standard or a standard calibrated against the caesium.

The best modern clocks pretend to deliver certain reference frequencies with an uncertainty at the level of one part in 10^{14} and even less. If we check the value of the linear Doppler shift related to this level, the speed of the atom is to be 3 μm/s. For a hydrogen atom a temperature of 1 K is related to a speed of approximately 100 m/s, i.e., eight orders of magnitude higher. Heavier atoms at this temperature are slower by a factor of $\sqrt{A}$, where A is the atomic mass number. That means that for an accurate clock, we have to solve the problem of the linear Doppler effect.

[18] Because of a certain conservativeness of CIPM, which should necessarily take place, and because of a variety of competitive optical candidates, we expect in the near future not a change in the definition of the SI second, but, first of all, certain CIPM recommendations. At the first stage, it could recommend values for certain optical and microwave transitions that would be advised for a practical realization of the second (compare, for example, the CIPM recommendation on the metre [11]). At the second stage, after the accuracy of a comparison of certain optical transitions to each other will supersede the accuracy of the caesium standards, a conventional second (for example, the second-2015) could be introduced.

In different clocks the problem of the linear Doppler effects is solved differently (see ref. 37 for more detail). In clocks with neutral atoms, the atoms are cooled down to a level much much lower than 1 K. Ions are trapped and that eliminates the Doppler effect — a localized trapped particle cannot have a nonzero momentum. One more approach is to study two-photon transitions, which are not sensitive to the linear Doppler effect.

If the constants are changing, not only theory should be reconsidered, but also experiment. First of all, we need to acknowledge that if certain natural constants are changing, our units and, in particular, the SI second are changing as well. For this reason, the most simple and hopeful way is to look for variation of dimensionless quantities. However, since any measurement is a comparison, we can also deal with dimensional quantities, properly specifying the units. The interpretation of the variation of the numerical values of the constants differs drastically from the interpretation of a search for their direct variation. In particular, we will speak about a variation of the numerical value of the Rydberg constant, which is closely related to the properties of the caesium atom. In fact, it is equal to

$$\begin{aligned}\{R_\infty\} &= \frac{2c\cdot R_\infty}{\nu_{\rm HFS}\left({}^{133}{\rm Cs}\right)} \times \frac{\nu_{\rm HFS}({}^{133}{\rm Cs})}{2c}\\ &= \frac{1}{\left(\nu_{\rm HFS}\left({}^{133}{\rm Cs}\right)\right)_{\rm a.u.}} \times \frac{9\,192\,631\,770}{2\times 299\,792\,458} \quad \text{(exactly)}\end{aligned} \tag{41}$$

where $\left(\nu_{\rm HFS}\left({}^{133}{\rm Cs}\right)\right)_{\rm a.u.}$ is the caesium HFS interval in atomic units and an exactly known number

$$\frac{9\,192\,631\,770}{2\times 299\,792\,458} = 15.331\,659\,494\,249\,183\,8\ldots$$

is an artifact of the SI system.

12.2. Scaling of different transitions in terms of the fundamental constants

With the help of accurate clocks, we can compare the frequencies of different transitions. What can we learn from them?

First of all, let us look at expressions for different atomic transitions in the simplest case, i.e., for the hydrogen atom

$$\begin{aligned}&f(2p-1s) \simeq \frac{3}{4}\cdot cR_\infty\\ &f(2p_{3/2}-2p_{1/2}) \simeq \frac{1}{16}\cdot\alpha^2\cdot cR_\infty\\ &f_{\rm HFS}(1s) \simeq \frac{4}{3}\cdot\alpha^2\cdot\frac{\mu_{\rm p}}{\mu_{\rm B}}\cdot cR_\infty\end{aligned} \tag{42}$$

Indeed, there are various corrections and, in particular, the relativistic and the finite-nuclear-mass corrections but they are small.

The first interval is related to the gross structure, the second is for the fine structure, and the last is for the hyperfine splitting. So, we note that if we would measure them, we can learn about variations of cR_∞, α, and $\mu_{\rm p}/\mu_{\rm B}$. To be more precise, when one measures a frequency, the result may be either absolute or relative. The latter case, when two ratios are measured for three transitions, will tell us nothing about the Rydberg constant. Measuring the intervals absolutely, i.e., in certain units, we can consider a variation of the value of the Rydberg frequency cR_∞ in these units. In the previous subsection, we explained about the physical meaning of a value of the Rydberg constant in the SI units.

Table 5. Scaling behavior of the atomic energy intervals as functions of the fundamental constants. μ stands for the nuclear magnetic moment.

Transition	Energy scaling
Gross structure	cR_∞
Fine structure	$\alpha^2 cR_\infty$
Hyperfine structure	$\alpha^2(\mu/\mu_\mathrm{B})cR_\infty$

What happens if we consider a more complicated atom? First, let us rewrite the equations above in atomic units

$$\begin{aligned} f(2p-1s)\Big|_{\mathrm{a.u.}} &\simeq \frac{3}{8} \\ f(2p_{3/2}-2p_{1/2})\Big|_{\mathrm{a.u.}} &\simeq \frac{1}{32}\cdot\alpha^2 \\ f_{\mathrm{HFS}}(1s)\Big|_{\mathrm{a.u.}} &\simeq \frac{2}{3}\cdot\alpha^2\cdot\frac{\mu_\mathrm{p}}{\mu_\mathrm{B}} \end{aligned} \tag{43}$$

The gross structure is of the order of unity. The fine structure is a relativistic effect, proportional to the factor $(v/c)^2$ and thus to α^2. The hyperfine structure is also a relativistic effect, but it is suppressed by a small value of the nuclear magnetic moment in atomic units.

If we have a more complicated atom, nothing will change except for the numerical coefficients. There is no additional small or big parameter when calculating the gross structure and it still should be of the order of unity. The electron speed is always proportional to αc. There may also be a value of the nuclear charge Z involved, but it does not change with time. So, we conclude that the scaling behavior of the atomic transitions with changes of the constants is the same as in hydrogen (see Table 5). The importance of these scalings for a search of the variation of the constants was first pointed out in ref. 38 and discussed there for astrophysical searches.

Molecular spectra are more complicated than atomic. The biggest energy intervals are related to the electron transitions and they are completely similar to the atomic gross structure. Two other kinds of the intervals (vibrational and rotational) are due to the nuclear motion.

Let us consider a diatomic molecule. In the so-called Born–Oppenheimer approximation (see, for example, ref. 39), we can consider the energy of the electronic states as a solution of the problem of the electrons in the field of two Coulomb centers with the infinite masses separated by a distance R. The result depends on this distance ($E(R)$) and the next step is to find a value of the distance R_0 that minimizes the energy. Let us now take into account nuclear motion. In the leading approximation, the Hamiltonian is of the form

$$\begin{aligned} H &= \frac{\boldsymbol{P}^2}{2M} + E(R) \\ &\simeq \frac{\boldsymbol{P}^2}{2M} - \frac{k}{2}(R_0-R)^2 + E(R_0) \end{aligned} \tag{44}$$

where we note that R_0 is about unity in atomic units, the binding energy $E(R)$ is also about unity and thus $k \sim E(R)/R_0^2$ is about unity as well. All of them do not depend on the fundamental constants (in the atomic units). M is the nuclear reduced mass. The equation is for a harmonic oscillator (in the leading approximation) and we know all the parameters (at least their dependence on the constants in the atomic units). We find that the vibrational quantum of energy scales as $\sqrt{1/M}$ in atomic units or $(m_\mathrm{e}/M)^{1/2}cR_\infty$ in SI units. Here M is a characteristic nuclear mass, but for the most of the applications,

Table 6. Scaling behavior of the molecular energy intervals as functions of the fundamental constants. M stands for an effective nuclear mass (equal to reduce nuclear mass for diatomic molecules), but for most of applications may be substituted for the proton mass m_p.

Transition	Energy scaling
Electronic structure	cR_∞
Vibrational structure	$(m_e/M)^{1/2}cR_\infty$
Rotational structure	$(m_e/M)cR_\infty$

we can neglect the nuclear binding energy and the difference between the proton and neutron masses and set $M = Am_p$. For diatomic molecules the effective atomic number is related to the reduced mass

$$A_R = \frac{A_1 A_2}{A_1 + A_2}$$

being a number, it cannot vary with time and can be dropped from any scaling equations applied for interpretation of search-for-variation experiments.

Estimation of the rotational energy is also simple. The energy is of order L^2/I where L is the (dimensional) orbital momentum and I is the moment of inertia. In atomic units, L is of the order of unity and $I \sim MR_0^2 \sim M$. Finally, we find that the rotational energy scales as $(1/M)$ in atomic units or $(m_e/M)cR_\infty$ in SI units.

The importance of different scaling of the molecular transitions is pointed out in ref. 40 due to astrophysical applications. We summarize the scaling behavior of various molecular transitions in Table 6.

This evaluation shows the great convenience of atomic units for atomic and molecular calculations and demonstrates that a calculation of the order of magnitude of an effect and its rough detail is an important issue and a part of "calculable" properties.

At the present time, all these scalings are not very hopeful for laboratory searches since only two kinds of atomic transitions, optical (gross structure) and HFS, are studied with a high accuracy. It should have a very reduced application, if the nonrelativistic scaling in Tables 5 and 6 were the only method we have.

A successful deduction of constraints on a possible time variation of $\{cR_\infty\}$ and α is possible because of the relativistic corrections, which are responsible for a different sensitivity to the α variation for various transitions of the same kind (for example, for various gross-structure optical transitions). That was first pointed out in ref. 41 and later successfully developed and applied to various atomic systems in ref. 42.

Most standards deal with neutral atoms and single-charged ions. The valent electron(s) spends most of its time outside the core created by the nucleus and the closed shells. The core charge for atoms used in the actual clocks is from one to three. The relativistic correction is of order of $(Z'\alpha)^2$, where Z' is an effective charge that a valent electron sees. If Z' is the core charge, it is still very small. However, the relativistic corrections are singular and a contribution of the short distances, where an electron interacts with the whole nuclear charge Z, is enhanced. The dominant part of the relativistic corrections comes from the short distances where the electron sees the whole charge of a bare nucleus and thus in some atoms under question the correction can be really big.

Table 7. Current laboratory constraints on the possible time variations of natural constants [37]. The results above the horizonal line are model-independent, while the validity of the results below the line depend on the applicability of the Schmidt model. The uncertainty of this application is not shown.

Constants (X)	Variation rate ($\partial \ln X/\partial t$) (yr^{-1})
α	$(-0.3 \pm 2.0) \times 10^{-15}$
$\{cR_\infty\}$	$(-2.1 \pm 3.1) \times 10^{-15}$
$\mu_{\rm Cs}/\mu_{\rm B}$	$(3.0 \pm 6.8) \times 10^{-15}$
$\mu_{\rm Rb}/\mu_{\rm Cs}$	$(-0.2 \pm 1.2) \times 10^{-15}$
$\mu_{\rm Yb}/\mu_{\rm Cs}$	$(3 \pm 3) \times 10^{-14}$
$m_{\rm e}/m_{\rm p}$	$(2.9 \pm 6.2) \times 10^{-15}$
$\mu_{\rm p}/\mu_{\rm e}$	$(2.9 \pm 5.8) \times 10^{-15}$
$g_{\rm p}$	$(-0.1 \pm 0.5) \times 10^{-15}$
$g_{\rm n}$	$(3 \pm 3) \times 10^{-14}$

12.3. Current laboratory limits

Optical measurements delivered to us data for a few elements (mercury ion [43], hydrogen [44], ytterbium ion [45], and calcium [46]) and there are also promising results for the strontium ion [47], neutral strontium [48], and more data on strontium and other transitions are expected. The data already available [43–46] are related to transitions with very different relativistic corrections and that is enough to derive strong limitations on the time variation of several constants. The model-independent constraints achieved this way are collected in Table 7 (top part) [37]. The HFS results were also applied to obtain constraints on the variation of the magnetic moments [49, 50]. To derive results on the more fundamental quantities than the nuclear magnetic moments of few particular nuclei, we applied the Schmidt model (see, for example, ref. 51). Its importance for the interpretation of results on the variation of the fundamental constants was pointed out in ref. 52. The model-dependent results are shown in the bottom part of Table 7 [37].

Is it possible to reach a model-independent constraint on the time variation of of $m_{\rm e}/m_{\rm e}$ from atomic spectroscopy? Yes, it may be done in the following way. First, we extract a limitation on a variation of cR_∞ without any use of the hydrogen data and next we compare it to a variation of the hydrogen $1s$–$2s$ frequency (which is proportional to $cR_\infty(1 - m_{\rm e}/m_{\rm p})$). The constraint on the variation is

$$\frac{\partial \ln(m_{\rm p}/m_{\rm e})}{\partial t} = (-0.4 \pm 1.3) \times 10^{-11} \quad {\rm yr}^{-1} \tag{45}$$

which is more than three orders of magnitude weaker than the model-dependent constraint in Table 7. Stronger model-independent limitations should appear from molecular spectroscopy.

It is significant that we can eventually constrain the variability of the fundamental constants. Results on variations of such nonfundamental objects as the atomic transition frequencies should be rather doubtful since such a level of accuracy has never been achieved before and various details of the experiments may need an additional examination. Expression of such results in terms of fundamental constants allows a cross-comparison and makes the results more reliable.

12.4. Nonlaboratory searches for the variations of the constants

…but it all came different! Lewis Carroll

The laboratory limitations are not the strongest, but the most reliable. Astrophysical [53] results unfortunately contradict each other, as well as geochemical [54] constraints. Studies using both methods involve various systematic sources and the access to data is limited. One should deal with observations, not with experiments.

13. Fundamentality of the constants and the Planck scale

…and noticed that what can be seen from the old room was quite common and uninteresting, but that all the rest was as different as possible. Lewis Carroll

If physics is governed by the most fundamental constants and if the ultimate theory includes the quantum properties of space-time, then the fundamental scale is determined by the Planck units

$$\begin{aligned} M_{\rm Pl} &= \left(\frac{\hbar c}{G}\right)^{1/2} = 2.176\,45(16) \times 10^{-8}\ \text{kg} \\ &= 1.220\,90(9) \times 10^{19}\ \text{GeV}/c^2 \\ l_{\rm Pl} &= \frac{\hbar}{M_{\rm Pl}\, c} = 1.616\,24(12) \times 10^{-35}\ \text{m} \\ t_{\rm Pl} &= \frac{l_{\rm Pl}}{c} = 5.391\,21(40) \times 10^{-44}\ \text{s} \\ T_{\rm Pl} &= \frac{M_{\rm Pl} c^2}{k} = 1.416\,79(11) \times 10^{32}\ \text{K} \end{aligned} \tag{46}$$

At the scale determined by these units the laws of Nature should take the simplest form and if any observable fundamental constant is calculable, it should be calculable there.

Due to success of the renormalization approach it is commonly believed that physics from the Planck scale does not affect our "low-energy" world. That is true only in part. To be accurate, the idea of renormalization reads that all that we need from higher energy physics can be successfully measured at our low energies. Still, what we measure at our energies comes from the higher scale (such as, for example, the Planck scale, or a scale of the spontaneous violation of a larger symmetry related to the unification). Presently, we do not have any theory related to the higher energy scale. If the Planck or any other high-energy scale has no dynamics, we have not much hope of understanding the high-energy physics from our low-energy experiments. We can learn nothing from measured numbers until we are able to proceed with a theory from the scale of, let us say, the Z boson mass, to a really high energy. Any corrections beyond that are small as $(m/M_{\rm Pl})^2$ where m is a characteristic mass scale we deal with. However, if Planck-scale physics has dynamics, for example, a variation of certain parameters, that is not true anymore. First of all, if the bare constants (such as the bare electron charge e_0 and the the electron mass m_0) determined at the Planck scale can vary, we should be able to detect that. There is a chance, that the fine structure constant at the Planck scale is calculable and, for example, $\alpha_0 = 1/\pi^4$, but there is no chance that any numerical exercises, such as done in the past, will succeed in expressing the actual α in an simple way. Neither it is likely that m_0 is calculable in simple matter. Nevertheless, if the bare constants do not vary, we still can expect that the dressed constant (i.e., the actual renormalized constants, which we measure) show a certain detectable variation, which could appear via the renormalization.

How easily can we see such dynamics, induced by the renormalization? A question for α variation is whether the divergencies are cut at the Planck scale, or a certain supersymmetry enters into the game at an intermediate scale $M_{\rm SS} \ll M_{\rm Pl}$ and cut the divergencies off. The other question is whether the bare electron mass varies, and if it does whether the ratios $m_{\rm e}/M_{\rm Pl}$ and $M_{\rm SS}/M_{\rm Pl}$ vary.

In the case of ultraviolet divergencies going up to $M_{\rm Pl}$, the result is

$$\frac{1}{\alpha}\frac{\partial\alpha}{\partial t} \sim \frac{\alpha}{\pi}\frac{1}{M_{\rm Pl}}\frac{\partial M_{\rm Pl}}{\partial t} \tag{47}$$

Table 8. Fundamental constants of cosmology. The results are taken from ref. 28.

Constant	Value
$\Omega_{\mathrm{tot}} - 1$	0.02(2)
Ω_{bm}	0.044(4)
Ω_{dm}	0.22(4)
Ω_{Λ}	0.73(4)
T_{CMB}	2.725(1) K
H	$0.73(4) \times 10^{-10}$ yr^{-1}
n_γ/n_B	$1.64(5) \times 10^9$

while in the case of the supersymmetrical cut off, the variations may be of a quite reduced value

$$\frac{1}{\alpha}\frac{\partial\alpha}{\partial t} \sim \frac{\alpha}{\pi}\left(\frac{M_{\mathrm{SS}}}{M_{\mathrm{Pl}}}\right)^2 \frac{1}{M_{\mathrm{Pl}}}\frac{\partial M_{\mathrm{Pl}}}{\partial t} \tag{48}$$

Concerning the bare electron mass, the problem is much more complicated, because details of the so-called Higgs sector are unclear and connection of the Higgs parameters with more fundamental quantities are to be clarified. It may happen that dynamics at the Planck scale urges certain variations at the Higgs sector and afterwards a variation of the Higgs vacuum average v and consequently of the bare value of the electron mass m_0. We note, that such a variation may, in principle, keep the ratio m_0/M_{Pl} constant and reduce the value of the α variation via the renormalization that is mentioned above. Detection of a variation of the electron mass is even less clear. The variation of m_{e} is not the question from the experimental point of view, the question is a variation of $m_{\mathrm{e}}/m_{\mathrm{p}}$. Since the origin of the electron and the proton masses are very different it is hard to understand what can happen with their ratio.

Still with a number of problems to be solved the fundamental constants give us a chance to study the physics of a higher energy scale not available anywhere else.

14. Constants of cosmology

It's a poor sort of memory that only works backwards. Lewis Carroll

Constants of our Universe as a whole give us another questionable chance to study physics beyond our essential world. Such constants as listed in Table 8 may offer us a unique opportunity to learn about the early time of the Universe and thus perhaps about a very-high-energy physics.

In particular, these constants characterize the density of bright matter (i.e., visible matter), the dark matter (matter recognized because of its gravitational effects), and the dark energy (recognized due to its cosmological consequences and related to the Einstein's Λ-term) in the units of the critical density

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{49}$$

$$\rho_c = \frac{3}{8\pi}\frac{H^2}{G} \tag{50}$$

where H is the Hubble constant. The critical density is a separation mark between the closed ($\Omega_{\mathrm{tot}} > 1$) and open ($\Omega_{\mathrm{tot}} < 1$) Universe. The in between case ($\Omega_{\mathrm{tot}} = 1$) is the flat Universe. The present result for the total density is close to the flat value.

One more parameter, the ratio of the number of microwave background photons and baryons ($n_\gamma/n_{\rm B}$) is important for learning about the moment when the light split from the baryon matter.

Meanwhile, we have to remember about the evolution in the understanding of such important "constants" as the free fall acceleration g and the water density. The constants of our world are not necessarily the truly fundamental constants since their values might be taken by chance with the spontaneous breakdown of certain symmetries.

15. Physics at the edge

…As she couldn't answer either question, it didn't much matter which way she put it. Lewis Carroll

A question of the constancy and the fundamentality of the fundamental constants is certainly a question of new physics. Studying the problem experimentally via, for example, searching for the variability of the natural constants we address this new physics. Maybe that is not the best way to do it, however, we are extremely limited now in what we can do. Never from Newton's time, we have been so badly suited for going forward. Physics is in a deep crisis, despite that, it looks like a success. We are able to explain nearly everything we *can* deal with. Yes, we have some problems, but that is either because some objects are too complicated, or because the involved interaction is strong and we are not able to move from the Hamiltonian to the observable quantities. But that is normal. In a sense, that is not a problem of fundamental physics, but of the technology to be able to apply it, which indeed is also of great importance.

We have access to only a few problems related in different ways to the fundamentally new physics, these are

- details of the Higgs sector;
- the extension of symmetry from the Standard Model to a certain unification theory, which probably, involves supersymmetry;
- dark matter;
- dark energy;
- quantum gravity and physics at the Planck scale.

To address them, we suffer from an extreme shortage of information and we do not see a feasible way to reach more data soon. To illustrate the problem we summarize in Table 9 data, important for new physics.

16. Dreaming about new physics

And here I wish I could tell you half the things Alice used to say, beginning with her favourite phrase 'Let's pretend.' Lewis Carroll

What should scientists do with the obvious lack of data? Different people do different things. Some develop a "real sector" of physics, where certain problems are important, sometimes very important, but not "fundamentally" important. Some develop tools and technologies that are needed to go further. Some search for new physics, but the lack of information does not allow us to understand where is the best place to look for it. So this search is the kind of search for a treasure that does not necessarily exist.

Some dream; it is hard to qualify differently the theoretical studies without any connection to experiment, i.e., with reality. Dreamers have existed at all times. Sometimes doers and thinkers put the dreamers into shadow, but they existed.

Table 9. Fundamental constants of new physics. ⋆ – the experimental level is already within theoretical margins and any improvement of the limits is important; † — unification theory must predict its value at a certain high-energy scale; radiative corrections are needed to go down to low energy for a comparison with experiment; * — the exact value is not important for a moment.

Constant	Comment
m_{H}	should be within certain margins if the Higgs particle is elementary
CKM matrix	unitarity would confirm the minimal Standard Model
lepton analog of CKM	would constrain physics beyond the Standard Model;
	the structure of both matrices could give a hint to the flavor symmetry
$\lvert q_{\mathrm{e}} + q_{\mathrm{p}} \rvert$	should be zero in the case of unification theories
τ_{p}	would constrain unification theories⋆
d_{e}	would constrain unification theories⋆
d_{n}	would constrain unification theories⋆
$\sin \Theta_{\mathrm{W}}$	would constrain unification theories†
m_{ν}	would constrain physics beyond the Standard Model;
	the Majorana mass would confirm that $q_{\nu} = 0$
$\Omega_{\mathrm{tot}} - 1$	if the value is positive, the Universe is closed;
	if negative, it is open; if zero, it is flat;
	a small value is expected due to the inflation model (IM); it will constrain the IM
Ω_{dm}	important that dark matter exists*
Ω_{Λ}	important that dark energy exists*
$\partial H / \partial t$	would constrain cosmological theory

The shortage of experimental data makes us wonder whether "purely" theoretical progress is possible. Our opinion is rather negative. The creation of Einstein's special relativity is sometimes believed to be a perfect example of such a progress. However, there is certain confusion in use of the word "theoretical". A "theory" may be a model, a hypothesis, or a framework of certain calculations completely supported by experiment, i.e., a high-level kind of fitting.

The famous inconsistency of Newton's mechanics and Maxwell's theory of electromagnetism was not a "purely theoretical" problem. That was a conceptual disagreement between two "theoretical fits" of a huge amount of experimental data. The relativity principle in the former form was part of Newton's mechanics. Einstein's solution reproduced both theories: Maxwell's (exactly) and Newton's (as an approximation at $v/c \ll 1$). It was also immediately confirmed by numerous experiments. Later, Einstein tried to solve an inconsistency between two other pieces of the theoretical description of the then existent data — his fresh-backed relativity and the old-fashioned Newtonian gravity. In contrast to special relativity, it was difficult to confirm general relativity accurately and in detail. The progress in the field had been quite slow for a long period until vitalized by the appearance of new data.

The first important steps of quantum mechanics were directly inspired by various experimental data (for that time this theory was too crazy to appear as a result of a "purely theoretical" development) and its crucial statements were immediately checked experimentally. When the immediate experiment was not possible, the theoretical ideas were sometimes gloriously correct, but sometimes completely wrong. An example of the wrong ideas was an expectation that the proton should have the Dirac value of the g-factor, i.e., $g_{\mathrm{p}} = 2$ (see, for example, ref. 22). There was no other way but experiment to check if the idea was right or wrong and they had to wait until the idea was confirmed.

Sometimes this kind of study is just a waste of time, sometimes an important step to future theories. We can never know. Such important conceptions as antiparticles, the Majorana mass, the Kaluza–Klein theory, and the Yang–Mills gauge field appeared as purely theoretical constructions. The positron was discovered shortly after its prediction by Dirac. The Majorana mass perhaps will now find its application to the neutrino. The Yang–Mills gauge field theory is now a way to describe weak and strong interactions and likely other interactions that should appear due to unification. Speculation on Kaluza–Klein theories have inspired a lot of works on the unification of all interactions with gravity, but, maybe, this problem will be solved in another way.

17. Conclusions

> **"Are you animal – or vegetable – or mineral?' he said, yawning at every word.**
> **Lewis Carroll**

In this short paper, we have tried to present an overview of a problem of the fundamental constants and various related questions, including practical metrology and realizations of the units, simple atoms, macroscopic quantum phenomena, variations of the constants, and physics at the Planck scale.

In the beginning of the paper, we introduced the fundamental constants as certain universal parameters of the most basic equations. We noted afterwards that we hardly understood their origin. These parameters play a nearly mystic role. Such equations as Maxwell's or Dirac's appeared then as the top-fundamental summary of our understanding of Nature. Clearly, such equations interpret the behavior of certain objects in terms of the input parameters c, h, e, m_{e}, G, etc., which as any input parameters should come from outside the equations. The equations happen to be a benchmark between understood (the shape of the equations) and nonunderstood (the fundamental constants inside the equations). In a sense the truly fundamental constants are the least understood part of the best understood physics.

We still do not know where the fine structure constant α comes from and what its value should be; we still wonder what is the reason that gravity is so much weaker than the electromagnetic interaction. The trace of the origin of the fundamental constants is lost somewhere in Wonderland, which we can, perhaps, see through the looking glass and try to guess about the unseen part of the room...

Acknowledgements

This work was supported in part by the RFBR under grants 03-02-04029 and 03-02-16843, and by DFG under grant GZ 436 RUS 113/769/0-1. The author gratefully acknowledges stimulating discussions with L.B. Okun, Z. Berezhiani, S.I. Eidelman, V. Flambaum, L. Hollberg, O. Kancheli, D. Kleppner, C. Lämmerzahl, E. Peik, D. Pritchard, and V.A. Shelyuto.

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