

Varying fundamental *constants*: a full covariant approach and cosmological applications

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We build a minimal extension of General Relativity in which Newton’s gravitational coupling, G , the speed of light, c , and the cosmological constant, Λ , are spacetime variables. This is done while satisfying the contracted Bianchi identity as well as the local conservation of energy momentum tensor. A dynamical constraint is derived, which shows that variations of G and c are coupled to the local matter-energy physical content, while variation of Λ is coupled to the local geometry. This constraint presents a natural cosmological screening mechanism that brings new perspective concerning the current observations of a cosmological constant, Λ_0 , in cosmological observations. We also explore early universe background cosmology and show that the proposal provides alternatives to obtain an accelerated expansion, similar to those coming from Varying Speed of Light theories.

I. INTRODUCTION

Fundamental constants have played an important role in physics since their very first appearance in Newton’s theory of gravity, where he introduced the gravitational constant, G . Usually, they are directly connected to the strength of a particular interaction that happens in nature, but also can be used to define regimes for which a particular theory will remain valid, as it is the case for Planck’s constant, h , and the speed of light¹, c .

Looking at the history of physics, we have seen different examples of dimension-ful and -less constants that ended up not being constants after all. An example of the former is the acceleration due to gravity on the Earth’s surface, $g \simeq 9.8 \text{ m/s}^2$, which was thought to be a constant before Newtonian gravity, while an example of the latter is the fine structure constant, $\alpha \equiv e^2/4\pi\epsilon_0\hbar c$, e is the electric charge and ϵ_0 is permittivity of free space, which is actually a function of the energy scale being considered.

Therefore, it is not a surprise that even before the 1900’s Kelvin and Tait [2] were already considering variations of the speed of light. Later, Dirac [3] considered cosmological variations of G , opening room for other approaches that resulted in the Jordan-Brans-Dicke theories [4–6]. It is also important to mention that even the electric charge has been considered to be varying, a proposal first accounted by Bekenstein [7], after considering an α -varying theory. Therefore, we can already appreciate the relevance of inquiring about the constancy of the

fundamental constants considered today in our theories.

Epistemologically, we can observe that a fundamental constant remains to be so until we figure out a more fundamental model in which the aforementioned constant becomes actually a variable that assumes a particular value for the regime so far considered. A very clear example to illustrate this is the compactification procedure for extra dimensions in String Theory, which makes the Newtonian coupling dependent on the moduli fields [8], so G is only effectively a constant. Another one encompasses a large set of models commonly referred to Varying Speed of Light (VSL) theories [9], which are generally concerned with variations of the speed of light in the history of the universe.

In this work we will be interested in promoting G , c and Λ , the cosmological constant, to be functions of the spacetime coordinates. All these quantities are dimensionful and it is an important matter to distinguish if their variations will be physical in any sense. In fact, there has been an extensive debate over the years regarding the meaning of considering a dimensionful fundamental constant to be varying. This debate has orbited much more the VSL theories, since these are the ones that became more popular after solving some of the early universe puzzles without invoking an inflationary phase [9]. The main topic of the discussion has been the physical relevance of considering a dimensionful constant to vary, once we can always choose a different system of units in which this variable would be a constant again, so that its variation would merely be a unit system artifact. For people who are more concerned with the *conservative* side of the question, we recommend [1, 10] and, in particular, [11]; for the *liberal* counterpart, it is worth checking [9, 12, 13]. Our position on this issue will be addressed later.

Regardless of the *political* orientation on this matter, it is fundamental to consider what the experimental constraints concerning variations of the fundamental constants are. The most interesting result allowing for a significant variation is related to the fine structure constant. In the work of Webb *et al.* [14], they seemed to have found evidence for a slow increase of α in time

¹ Throughout this paper, we will refer to c as the speed of light, which is the common terminology when talking about the variation of fundamental constants. However, for this work, one should actually think of it as the “spacetime speed”, in the sense discussed by Ellis and Uzan [1]. In light of their work, here we are considering the spacetime speed, the speed that appears in the metric, and the Einstein’s speed, the speed that appears in Einstein’s equations, to be the same, *i.e.*, $c_{ST} = c_E = c(x^\alpha)$, and being spacetime variables.

for redshifts between 0.5 and 3.5. These results have been questioned and an update on this discussion can be found in [15]. For the other constants, most of the constraints come from experiments using atomic clocks, the Oklo phenomenon, Solar System observations, meteorites dating, quasar absorption spectra, stellar physics, pulsar timing, the Cosmic Microwave Background (CMB) and big bang nucleosynthesis (see [16, 17]). Although most of these experiments have left little room for considering variation of the fundamental constants, they are redshift and spatially constrained [18], not to mention that the interpretation of the results might be changed if someone had considered a model in which these constants would be varying as a prior. One thing is certain: if we keep ourselves to the early universe, varying fundamental constants has not been ruled out.

The proposal presented here is an extension of General Relativity (GR) in which G , c and Λ are allowed to vary, while preserving the two fundamental ingredients that were considered by Einstein: the underlying geometrical structure of the theory, namely the requirement of satisfying the (contracted) Bianchi identity, and the local conservation of the energy momentum tensor. It is important to acknowledge that there is a vast literature concerning models in which either G , c or Λ are varying, almost all of them at the background level. Some of them have an overlap with the approach developed here, and we will highlight some of the differences and similarities. Moreover, it is relevant to say that the model presented here has already appeared in the literature at the background level [19, 20], but some of the conclusions and discussions we will draw are original. More importantly, this paper aims to open room for a new framework that is being developed in which the dynamics associated to the variation of the fundamental constants is given by a potential for these variables [21], which can be seen as scalar fields in an action. This way we shall address one of the biggest criticisms towards some of the VSL theories so far: the lack of a well defined variational principle.

This paper is divided as follows: in section II, we briefly comment on the previous literature regarding varying fundamental constants in cosmology focusing mainly in their advantages and drawbacks; in section III we discuss and provide the framework we consider throughout this paper; section IV considers the generalized version of the Friedmann's equations and a simple application of them; section V presents a sort of *bootstrap* mechanism that provides a possible explanation of why we seem to observe a cosmological constant given this framework; section VI brings attention to future applications; finally we conclude in section VII.

II. VARYING FUNDAMENTAL CONSTANTS LITERATURE

There is an extensive literature of models that considers variation of classical (*i.e.* non-quantum) fundamen-

tal constants, mainly G and c , as well of the cosmological constant², Λ , and α . We will briefly discuss some of these models so that we can make comparisons among them and be aware of the advantages and drawbacks of each model. Since our objective is not to review, the reader should not expect an exhaustive list of them.

A. $\Lambda(t)$ models

There are different motivations to consider the cosmological constant to be varying. For the very early universe, for example, this could easily solve the flatness and horizon problems [21]. In fact, to some extent inflation does that in a more elaborated fashion, since during the slow-roll phase the stress tensor of the scalar field is given basically by the potential of the field, which acts effectively as a $\Lambda(t)$. For the late time universe, this is also interesting since it could account for backreaction effects coming from the effective energy-momentum tensor due to fluctuations at second order Einstein's equations that also influence the background cosmology (for a review on backreaction, see [22]). These effects also play a role in the early universe (see *e.g.* [23]).

Given these motivations, some of the models that have been considered so far include the time dependence of Λ by considering $\Lambda(H(t))$ [24], where $H(t)$ is the Hubble parameter; or by $\Lambda(a(t))$ [25], $a(t)$ the scale factor; also by $\Lambda(\alpha(t))$ [26], $\alpha(t)$ the varying fine structure constant; and even combination of H and a , [27]. All these models have different and interesting cosmologies, but as it will be clear later on, if we only assume that Λ is varying without considering also G and/or c to be varying as well, and having the dynamics solely given by Einstein's equations, thus either Bianchi identity or the local conservation laws will be violated.

B. Bimetric models

In general, bimetric models make use of two different metrics: $g_{\mu\nu}$, the gravitational field, and $\hat{g}_{\mu\nu}$, the metric that couples to matter. Usually \hat{g} depends on the gravitational field plus a new scalar [28], or vector field [29]. This implies that massless particles will have different velocities, so that special relativity will be realized differently in each of these sectors. Due to the existence of the two metrics, the coupling between geometry and matter in Einstein's equations is changed and picks a dependence on the dynamics of this new field that has been

² Note that although a bare cosmological constant should always come for free in Einstein's equations, we know that, if the vacuum energy gravitates, its effect would be given by a cosmological constant in the energy side of the equations. Since the vacuum energy is a quantum effect, we do not consider the overall Λ in the equations to be a classical constant.

introduced. Some of the consequences of such approaches are the resolution of the flatness and horizon problem in a Friedmann-Robertson-Walker (FRW) universe, also providing a graceful exit to the inflationary epoch, and even recovering a scale invariant spectrum for the fluctuations [30].

However, their successes are limited since the very existence of a different metric coupling to the matter sector explicitly violates the equivalence principle. Moreover, although the Bianchi identity is satisfied in relation to the metric g , the local conservation laws are satisfied in relation to the metric \hat{g} .

C. Jordan-Brans-Dicke framework

The very first implementation of a varying G model after Dirac's initial phenomenological proposal was made by Jordan [4]. This was an important step, since it incorporated the variation of G as a dynamical feature of the model instead of just being imposed by hand, setting up a consistent framework from which equations of motion could be derived in a consistent matter following an action principle. The action proposed by him was given by:

$$S = \int d^4x \sqrt{-g} \phi^\eta \left[R - \xi \left(\frac{\nabla\phi}{\phi} \right)^2 - \frac{\phi}{2} F^2 \right], \quad (1)$$

where η and ξ are two parameters, and F is the electromagnetic field strength. It then follows that G and α are promoted to be dynamical variables.

As it is summarized in [16], later it was realized that if $\eta \neq -1$ the atomic spectra will be space-time dependent. After fixing $\eta = 1$, the model becomes 1-parameter dependent only and represents a class of scalar-tensor theories in which only G is a dynamical variable. This idea was further explored by Brans and Dicke [5]. The most recent work that makes use of this approach is the BSBM proposal, which actually considers the variation of G and α , and it can be seen as combination of the initial proposal from Bekenstein [7] revived by Sandvik, Barrow and Magueijo [31].

D. VSL theories

Besides the proposals we have briefly overviewed above, there is a whole class of models typically called VSL theories. There is a big overlap among them and their consequences, so here we focus in two seminal papers, Moffat's early work [32] and Albrecht and Magueijo's paper [33]. It is historically fair to bring attention to the fact that Moffat's first paper was published in 1993, and his ideas were mostly neglected until the work of Albrecht and Magueijo, which finally got the proper attention to VSL models from the community.

In summary, the initial idea brought up by Moffat involved a spontaneous breaking of Lorentz invariance (associated to a first order phase transition) in the early universe, and this symmetry would be later restored. The symmetry breaking was produced after introducing a Higgs mechanism for four scalar fields. The flatness and horizon problems are solved with the phase transition, which also leads to a scale invariant power spectrum for the energy density fluctuations. It is worth mentioning that in order to solve these problems the speed of light has to drop by 10^{28} between the two phases. It preserves the underlying geometrical structure by satisfying Bianchi identity, but that leads to non-standard energy-momentum conservation laws.

Albrecht and Magueijo's proposal also considers departures from Lorentz invariance, as well as violation of the local conservation laws. The core idea is to postulate that Friedmann's equations, as known from General Relativity in a cosmological setting, should remain the same in the CMB frame, but having now c and G being also functions of the cosmological time (see discussion in section IV). This explicitly breaks covariance, and also implies that Einstein's equations, as we know them, would only be valid in the CMB frame. Although Bianchi identity is enforced, this also results in non-standard local conservation laws, which now includes source terms proportional to \dot{c}/c . Within such a prescription, Albrecht and Magueijo are able to also solve the horizon and flatness problems, as well as alleviating the cosmological constant problem. Moreover, they can also account for the large entropy inside the horizon nowadays. Unfortunately, it lacks a dynamical equation for $c(t)$, so that one has to impose its dynamics by hand.

For a more extensive review and discussion of different models, please see [9, 34, 35].

III. THE FRAMEWORK OF THE PROPOSAL

We have seen above that among the different ideas that have been considered so far regarding the variations of the fundamental constants, most of them lack a constraint for the variations of the different *constants* coming from theoretical grounds, which usually leads to violation of the conservation laws or of the very geometrical structure in which these theories are considered. Here, we aim to revive an approach that preserves the underlying geometrical structure and the known local conservation laws.

The proposal is very simple. We start off by considering the (contracted) Bianchi identity:

$$\nabla^\mu G_{\mu\nu} = 0, \quad (2)$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is known as the Einstein tensor, and $R_{\mu\nu}$ and R the Ricci tensor and scalar, respectively. This is a formal identity after assuming a torsion-free connection and the metricity condition, $\nabla_\rho g_{\mu\nu} = 0$,

being valid for any (pseudo)-metric manifold [36]. We also consider the minimally coupled local conservation law:

$$\nabla^\mu T_{\mu\nu} = 0, \quad (3)$$

where $T_{\mu\nu}$ is the energy-momentum tensor (EMT). It is no surprise for all the readers that are familiar with General Relativity that those two equations can be written together in a self-consistent way through Einstein's equations, namely,

$$G_{\mu\nu} = \frac{8\pi G_0}{c_0^4} T_{\mu\nu} + \Lambda_0 g_{\mu\nu}, \quad (4)$$

where we have allowed the presence of a cosmological constant³, Λ_0 . Besides, the proportionality constant multiplying the EMT tensor is recovered after one considers the Newtonian limit [36]. It is important to note that Einstein's equations are supposed to rule the dynamics of the spacetime structure in the presence of matter/energy content. Even though the local experiments at the time did not indicate the existence of any space and/or time variation concerning the so called fundamental constants, as the speed of light, c_0 , and the Newton's gravitational constant, G_0 , it was a conservative call to consider that those quantities would be necessarily constants for all times and regions of the universe. Therefore, our proposal is to consider the more general case, in which the constants are promoted to be spacetime variables, while keeping intact the underlying geometrical structure of General Relativity, namely, we preserve the validity of (2) and the local conservation laws (3).

In order to do so, let us take a step back and write the most general relation between Einstein's tensor and the energy-momentum tensor to be:

$$G^{\mu\nu} = \chi(x^\rho) T^{\mu\nu} + g^{\mu\nu} \Lambda(x^\rho), \quad (5)$$

where we have now allowed a coordinate dependence on the cosmological constant-like term, represented by (x^ρ) , as well for the coupling of the EMT, instead of assuming that α and Λ are constants, as it was done for equation (4). Of course, since we still would like to have a well defined geometric structure, let us plug in the above equation back into (2), resulting in the following constraint:

$$T^{\mu\nu} \partial_\mu \chi(x^\rho) + g^{\mu\nu} \partial_\mu \Lambda(x^\rho) = 0, \quad (6)$$

after using local conservation of the EMT, eq. (3). Note that a trivial solution would be to consider both α and Λ being constants, as usual.

In order to investigate the Newtonian limit, we remind ourselves that locally Λ -effects are negligible (this is an empirical statement) and one should recover Newtonian

gravity. Therefore, after foliating our spacetime with constant time slices labeled by t , one should recover:

$$\chi(x_0^\rho) = \frac{8\pi G_0}{c_0^4}, \quad (7)$$

where x_0^α represent our local configuration on the spacetime. Now, being conservative, we can use this as an *ansatz* for $\chi(x^\alpha)$ as a whole, so that we postulate the following functional dependence:

$$\chi(x^\alpha) = \frac{8\pi G(x^\alpha)}{c^4(x^\alpha)}, \quad (8)$$

in which G and c are variables over time and space. Therefore, the constraint equation above reduces to⁴:

$$\left[\frac{1}{G} \partial_\mu G(x^\alpha) - \frac{4}{c} \partial_\mu c(x^\alpha) \right] \frac{8\pi G(x^\alpha)}{c^4(x^\alpha)} T^{\mu\nu}(x^\alpha) + [\partial_\mu \Lambda(x^\alpha)] g^{\mu\nu}(x^\alpha) = 0, \quad (9)$$

which tells us how c , G and Λ can vary altogether without violating Bianchi identity, therefore preserving the underlying geometrical framework intact, while at the same time preserving the same local conservation laws we are familiar with. The above equation is referred to as *the general constraint* (GC). The cosmological version of it has already appeared in the literature [19, 20]. One could also derive the second order constraint after operating with ∇^ν on the equation above.

Some important comments about this constraint follows:

1. Note that the variations of G and c are directly correlated with the local matter/energy distribution while the variation of Λ is correlated to the local geometry. This highlights an intrinsic difference between the variation of c and G in comparison with the variation of Λ , which will result in interesting implications for their dynamics.
2. Once that we do not expect that Λ should have much of an influence for local physics given current observations, one could locally expect to have the following constraint being satisfied:

$$T^{\mu\nu} \left(\partial_\nu G - \frac{4G}{c} \partial_\nu c \right) \simeq 0, \quad (10)$$

which implies:

$$G(x^\alpha) = \frac{G_0}{c_0^4} c^4(x^\alpha), \quad (11)$$

⁴ One could also have considered a more general constraint coming from Bianchi identity: $(8\pi G/c^4) \nabla_\mu T^{\mu\nu} = T^{\mu\nu} \partial_\mu G(x, t) - (4G/c) T^{\mu\nu} \partial_\mu c(x, t) + (c^4/8\pi) g^{\mu\nu} \partial_\mu \Lambda(x, t)$, without assuming right away the typical minimally coupled conservation law to be held. This would imply a violation of the local conservation laws that could be further explored in future works. I thank Jerome Quintin for this remark.

³ From now on the subscript 0 denotes the quantity is a constant.

assuming $G(x_0^\alpha) = G_0$ for $c(x_0^\alpha) = c_0$. This ties together the variation of Newton's gravitational coupling to the variation of the speed of light. Now, since we have G/c^4 appearing in Einstein's equations, one can see that it could be changed to G_0/c_0^4 locally, recovering the physics we are familiar with, as long as variations of c are small. In other words, for negligible variations of Λ and c locally, Lorentz symmetry is effectively restored⁵.

3. Note that vacuum solutions, given by $T_{\mu\nu} = 0$, imply Λ to be a spacetime constant, while, at the same time, dropping any constraint concerning the variation of G and c . Of course, after imposing G and c to be constants as well, one can recover the usual Minkowski, de Sitter and anti-de-Sitter cosmological spacetimes. This is a relevant observation also for non-cosmological vacuum solutions, *e.g.*, Schwarzschild's metric, since it means that the speed of light and the Newton's coupling could be varying in principle, albeit this would imply a phenomenological approach once the dynamics of c and G would have to be imposed by hand.
4. In standard GR, Einstein's equations for a traceless EMT theory implies:

$$R + 4\Lambda_0 = 0. \quad (12)$$

Since Λ_0 is constant, this also implies that the curvature would have to be constant, as it is the case for electromagnetism, for instance. However, in the current formalism, we see that one could actually have a conformal field theory (traceless EMT) without necessarily having constant curvature, given the fact that the above equations are generalized to:

$$R(x^\rho) + 4\Lambda(x^\rho) = 0. \quad (13)$$

In the next sections, we start to explore some immediate consequences of the variation of c , G and Λ having the general constraint imposed at all times.

⁵ By Lorentz symmetry we are referring to the local group of transformations that keep the infinitesimal (when considering curved spacetimes) line element, given by $ds^2 = -c^2(t, \vec{x}) dt^2 + d\vec{x}^2$, invariant. One could find odd the fact g_{00} is spacetime dependent, but it is important to remember that Lorentz transformations are infinitesimal ones: $dt = \gamma(dt' + v/c^2 dx')$, $d\vec{x} = \gamma(d\vec{x}' + \vec{v} dt')$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, so that the spacetime dependence does not play a role directly in the transformations. On the other hand, it means that for different spacetime points, the speed associated to the Lorentz symmetry will be different. This is, of course, a naive way to think about a generalization of the Lorentz group, and hopefully we will be able to address this issue in future work.

IV. COSMOLOGICAL BACKGROUND SOLUTION

After we have discussed about the most immediate dynamical implications that one could expect to have from the general constraint, we can now analyze the implications of having G , c and Λ varying in Einstein's equations. In this section, we will restrict ourselves to the background cosmology.

We start off with a FRW-like *ansatz*⁶:

$$g_{\mu\nu} = \text{diag}(-c^2(t), a^2(t), a^2(t), a^2(t)). \quad (14)$$

The reader should now be concerned with the fact that we could reparametrize the time coordinate so that the time dependence of the speed of light would disappear from the above *ansatz*. However, this would be misleading. Let us understand why that is the case.

When we observe the CMB to be isotropic (and homogeneous after assuming the Copernican Principle), we can use the CMB to define a whole class of coordinate systems, which differ from one another by the function $c(t)$ in the *ansatz* above. In standard cosmology, this function is meaningless since the speed of light is constant and we are allowed to consider time-reparametrizations, so we may as well fix it to be equal to a constant (in particular to be equal to 1 in natural units). However, for our given framework in which the speed of light is varying, even though we are still allowed to consider time-reparametrizations (given the covariance of the model), it is not possible to find a time coordinate in which the speed of light would not be varying (since its variation is not a coordinate artifact). Thus, time-reparametrizations can only hide the effect of a varying- c in the metric, for example by finding a new time coordinate, call it t' , such that $c(t)dt = c_0 dt'$. If one does that, we interpret it as finding a new frame, to be called CMB *coflowing* frame, which is the frame that makes homogeneity and isotropy explicit, but hides the effects of a varying- c in the metric⁷. We do not want to use this preferential frame in the same way that we do not want to use an inhomogeneous metric to describe background standard cosmology, since it would be very hard to make homogeneity explicit again. Hence, for us, in a general sense, the speed of light has changed with cosmological time, where the cosmological time is set up by the CMB evolution (which correlates this time coordinate directly to the CMB temperature) and we keep this time dependence in our metric *ansatz* so that the effects related to a varying- c remain explicit everywhere else.

⁶ Spatial curvature can be thought of as embedded in the \vec{x} coordinates.

⁷ The nomenclature comes in analogy to the terminology *comoving*, which refers to a spatial reparametrization of the physical coordinates, x_p , to comoving coordinates, x_c , by $dx_p = a(t) dx_c$. The comoving spatial coordinates move along with the Hubble flow, remaining fixed.

That being said, we consider a perfect fluid EMT defined by:

$$T^{\mu\nu} = \frac{1}{c^2} (\varepsilon + p) U^\mu U^\nu + p g^{\mu\nu}, \quad (15)$$

where ε is the energy density, p is the pressure, and U^μ is the 4-velocity, satisfying $g_{\mu\nu} U^\mu U^\nu = -c^2$, and, of course, $c = c(t)$ in our work⁸.

Then, considering a perfect fluid, Einstein's equations can be reduced to generalized Friedmann equations given by:

$$H^2(t) = \frac{8\pi G(t)}{3c^2(t)} \varepsilon(t) - \frac{\Lambda(t)c^2(t)}{3} - \frac{kc^2(t)}{a^2(t)} \quad (16)$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G(t)}{3c^2(t)} [3p(t) + \varepsilon(t)] + H(t) \frac{\dot{c}(t)}{c(t)} - \frac{\Lambda(t)c^2(t)}{3}, \quad (17)$$

where $H(t) = \dot{a}(t)/a(t)$ and “ $\dot{}$ ” stands for time derivatives. These same equations have already been considered in [19, 20]. Again, a few comments are worth mentioning here:

1. The first Friedmann equation has the same form as the usual one, but now with G , c and Λ being time variables.
2. The second Friedmann equation has an extra term, proportional to $\dot{c}(t)$, that disappears for $c = \text{constant}$. However, now it becomes clear that the background can expand in an accelerated fashion by demanding that:

$$-\frac{4\pi G(t)}{3c^2(t)} [3p(t) + \varepsilon(t)] + H(t) \frac{\dot{c}(t)}{c(t)} - \frac{\Lambda(t)c^2(t)}{3} > 0, \quad (18)$$

which implies that even if $\Lambda = 0$ we still can recover acceleration with non-exotic matter ($3p + \varepsilon > 0$) given that $\dot{c}(t) \neq 0$.

3. Usually, in standard cosmology, we have a , p and ε as variables together with three equations: an equation of state and two Friedmann ones. Here, the situation is trickier: we have c , G , Λ , ε , p and a as variables. However, we have two Friedmann equations, another one from the general constraint, and an equation of state, giving four equations total⁹. Therefore, it is expected to have some hypothesis imposed by hand concerning the time dependence of a pair of the set $\{G, c, \Lambda\}$. Besides, in the

same sense that the equation of state represents the underlying thermodynamic fluid being considered, one should now expect that some underlying theory could provide the hypothesis for the variation of this set of variables. In particular, if $\Lambda = 0$, one needs only one more equation [21].

4. The second Friedmann equation has a curvature correction due to the variation of the speed of light. If we had used the CMB coflowing frame, that term would be gone and we would have the following equations after considering $c(t)dt = c_0 dt'$:

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi c_0^2 G(t')}{3c^4(t')} \varepsilon(t') - \frac{\Lambda(t')}{3} - \frac{k}{a^2(t')} \quad (19)$$

$$\frac{a''(t')}{a(t')} = -\frac{4\pi c_0^2 G(t')}{3c^2(t')} [3p(t') + \varepsilon(t')] - \frac{\Lambda(t')}{3}, \quad (20)$$

where “ $'$ ” denotes derivatives w.r.t. t' . These equations look the same as what we have in standard cosmology while allowing c , G and Λ to vary.

5. Another implication is the following: if we are not considering cosmic scales, we can, in fact, choose a local coflowing coordinate system (as one would be doing when considering black hole solutions or physics on Earth, for instance) that is not isotropic or homogeneous, that is, not connected to the CMB. However, these (local) frames would consist in a subclass of all the possible frames that could be considered in principle (since for a varying- c theory, coflowing frames define a preferred class of frames). Since the CMB defines another class of frames, the ones which are homogeneous and isotropic, having among them a single *one* which is also coflowing, that implies this local subclass of coflowing frames is not always compatible with the class defined by the CMB in the sense presented by Padmanabhan [37]. He argues that it is odd the fact that operationally the CMB defines a preferred rest frame that does not seem to select any preferred class of frames in sub-cosmic level physics, which would be key if someone was to solve Einstein's equations exactly and then to consider the average of the metric on cosmological scales. In our framework, since now we do have a distinction between the CMB comoving frame class and the coflowing one, we see that the CMB would actually select a particular subclass of local frames: the ones that are not coflowing, but rather comoving, therefore addressing the problem raised by Padmanabhan. Given this clarification, one should be concerned to what sort of effect we might have been observing that could be due to the incompatibility between all these frames, since we would be locally solving Einstein's equations with

⁸ This sort of generalization has been known as minimal coupling [9]. Following the arguments above about local variations of c being negligible, this seems to be a good *ansatz* for the moment.

⁹ I thank Renato Costa for pointing this out to me.

coflowing frames¹⁰ and later expecting that these solutions would recover a comoving non-coflowing frame when averaged over large scales. Although cosmology has been quite successful as a whole, we still have two big elephants in the room: dark matter and dark energy. In the current framework, if these effects are actually due to the spacetime variation of G and c at the perturbation level, it is clear that a frame redefinition could in principle hide these effects.

Finally, we can also derive the continuity equation, namely the 0-th component of $\nabla_\mu T^{\mu\nu} = 0$, which gives:

$$\dot{\varepsilon} + 3H(p + \varepsilon) = 0. \quad (21)$$

This can also be derived after taking the time derivative of (16) and substituting it into (17), and then imposing the general constraint.

A. Λ -less acceleration

From a UV perspective, it is very reasonable to consider that G and c would be varying in the very early universe. Therefore, we consider the case in which $\Lambda = 0$, and investigate some of consequences of the variation of the other *constants*.

We have seen already that the general constraint implies for this case:

$$G(t) = \frac{G_0}{c_0^4} c^4(t), \quad (22)$$

after considering G and c to be homogeneous. Then, assuming a constant equation of state, $\omega = p/\varepsilon$, and considering (16) into (17) for $k = 0$, we have:

$$\frac{\dot{c}}{c} > \frac{1 + 3\omega}{2} \frac{\dot{a}}{a}. \quad (23)$$

If this condition is satisfied, we are guaranteed to have acceleration. If, for instance, we seek for a polynomial solution for the scale factor, $a(t) \sim t^n$, $n > 1$, this demands the following dynamics for the speed of light $c(t) \sim t^m$, $m > \frac{1+3\omega}{2}n$.

Needless to say that it would be much better if we could provide a dynamical theory in which c and G would have their variations given by an equation of motion, instead of considering any sort of hypothesis by hand. This will be presented in [21], where an action prescription will be introduced and early universe puzzles will be dealt with, providing an alternative paradigm to inflation.

V. A NATURAL SCREENING MECHANISM, OR WHY WE OBSERVE A COSMOLOGICAL CONSTANT

A screening mechanism basically works in a way that some degrees of freedom of a model are not accessible for a particular physical scale, they are “hidden”. For instance, in the context of cosmology, two mechanisms are well known, they are called *chameleons* [38] and *symmetrons* [39]. In short, these mechanisms rely on a field, typically scalar, that has its mass (chameleon) or its effective potential symmetry (symmetron) being dependent on the local matter density. The importance of such mechanisms comes from the fact that most of scalar fields coming from fundamental theories, *e.g.* String Theory, would imply strong violations of the equivalence principle. However, such violations have not been observed in the Solar System [40]. Therefore, if someone hopes that such fundamental fields could be responsible for any cosmological observations, as dark energy for instance, such mechanisms would play an important role to prevent these violations.

Within the proposal of this paper, it is also possible to find something that resembles a screening mechanism using a *bootstrap* approach after considering the GC. From eq. (9), we have at the background level:

$$\left(\partial_t G - \frac{4G}{c} \partial_t c \right) \varepsilon - \frac{c^4}{8\pi} \partial_t \Lambda = 0. \quad (24)$$

Now, we can think about this equation in two different scales: when/where ε is high, and when/where ε is low. Therefore, we could split this equation relative to these two scales,

$$\begin{cases} \dot{G} - \frac{4G}{c} \dot{c} = \varepsilon_{\text{high}}^{-1} \left(\frac{c^4}{8\pi} \dot{\Lambda} \right) \simeq 0 & , \quad \varepsilon \text{ high} \\ \frac{c^4}{8\pi} \dot{\Lambda} = \varepsilon_{\text{low}} \left(\dot{G} - \frac{4G}{c} \dot{c} \right) \simeq 0 & , \quad \varepsilon \text{ low.} \end{cases} \quad (25)$$

Therefore, we observe that when considering scales in which the energy density is high, two things happen: i) the variation of G is fixed to the variation of c , which reproduces what we have discussed above for our local physics; ii) it relaxes the temporal variation of Λ , since now its time-dependence can be “more” arbitrary, once it is suppressed by $\varepsilon_{\text{high}}^{-1}$. On the other hand, when ε is low, we also observe two things: i) the variation of c and G are not so tied together; ii) more importantly, because the variations of G and c for those scales are suppressed, the constraint tells us that effectively $\dot{\Lambda} \sim 0$, once its variation is suppressed by $\varepsilon_{\text{low}}/c^4$.

Note that we have related to these scales a sense of time (*when*) and space (*where*). For the former, we can imagine comparisons between the background energy density in the early universe and nowadays, being the first much higher than the second, what could help explaining why the variation of Λ seems to be non-existent today, even though could have been much more prominent in the early universe.

¹⁰ Since there is no reason to make explicit any cosmological variations of the speed of light when solving Einstein’s equations locally.

Regarding the sense of space, although we are assuming a homogeneous description, this can also be considered a 0th order description of inhomogeneous regions. In doing so, we take the average energy density of these regions. If we keep ourselves to galactic scales the energy density will be much higher than if we average over cosmological scales¹¹. Thus, even though this is a first approximation, it already tells us that local energy density might also suppress how Λ might be varying. Of course, a full perturbation theory is needed to be conclusive.

Hence, this embedded natural screening mechanism brings new perspective to the fact that we observe a cosmological constant in cosmological observations, and still leaves room for its local variations. This can be of great interest concerning the recent observations of [41] that have created tension for the Λ_0 CDM model concerning the constancy of Λ_0 .

A small comment is necessary at this point. The general constraint is an equation as valid as the Einstein's equations, since in this framework both equations follow from the requirement to have the Bianchi identity being satisfied and preserving the standard local conservation laws. Therefore, our dynamical analyses that result from the general constraint is automatically incorporated in the dynamics that one finds coming from Einstein's equations. In other words, it is legitimate to derive preliminary conclusions after looking only to the constraint given all the equations are consistent among themselves.

VI. PROSPECTS

In some sense, the framework presented here had already been briefly considered in the literature, but many points were not particularly addressed. Now that we have revived the model and shown some different consequences of it, we have laid the ground for further work regarding its own limitations and applications.

At the formalism level, we have emphasized the necessity of having the dynamics of the fundamental constants being given by an action principle/equation of motion. This is being developed [21] specifically in the framework discussed here, in which we consider the background cosmology for $\Lambda = 0$. Having an action in someone's hands, this opens room for considering different dynamics for the fields associated to c and G by choosing different potentials, in a similar fashion to what is done for inflation. This also allows considering early universe puzzles and their resolutions without making use of an *ad hoc* field, such as the inflaton, since there is a field associated to c and G (they are "the same" given the general constraint for $\Lambda = 0$) comes for free. Evidently, this can also be ap-

plied for late time cosmology, in particular investigating dark energy.

Another clear direction of work is to understand the cosmological perturbations in this approach. We know that given that the equations are self-consistent, we can consider a back of the envelope calculation just looking at the general constraint at linear order in perturbations:

$$\begin{aligned} & \delta T_{\mu\nu} \left(\partial^\mu G - \frac{4G}{c} \partial^\mu c \right) + \\ & + T_{\mu\nu} \left(\partial^\mu \delta G - \frac{4G}{c} \partial^\mu \delta c + \frac{4G}{c^2} \partial^\mu c \delta c - \frac{4}{c} \partial^\mu c \delta G \right) + \\ & + \frac{c^4}{8\pi} g_{\mu\nu} \left(\partial^\mu \delta \Lambda + \frac{4}{c} \partial^\mu \Lambda \delta c \right) + \frac{c^4}{8\pi} \delta^\mu \Lambda \delta g_{\mu\nu} = 0. \end{aligned} \quad (26)$$

Here it is clear that variations of c and G are connected to perturbations in the local matter density. Although we need to have a proper treatment after defining gauge invariant variables, naively it is expected that effects similar to cosmic accelerated expansion could be a local effect due to such variations.

VII. CONCLUSIONS

We have started the construction of a framework in which the fundamental constants, G , c and Λ , can be spacetime variables, as long as their variations are constrained in relation to the local geometry and local stress tensor. This framework does not provide a dynamics for the variation of these constants, which leaves room for an upgrade in which an action and equation of motion for these variables could also be obtained, instead of having their dynamics imposed by hand. When we have the full framework built, this will provide a self-consistent and covariant approach for treating the early universe puzzles after considering fundamental constants as variables.

Regardless, we can already appreciate within this formalism an explanation of why we observe a cosmological constant today, realizing cosmic accelerations by considering variations of the speed of light and a better understanding of the interpretation of having a varying- c theory which is also covariant. We also have presented extensive discussions of the implications of choosing the CMB class of frames without assuming the constancy of the fundamental constants in it. We hope that future works will be able to bring other applications of the full framework, not only at the background level, but for the perturbations as well.

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¹¹ This is the case since our homogeneous description of these scales takes a small volume (galactic) compared to a large one (cosmological) in order to consider the average energy density.

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