Cosmological bounds on spatial variations of physical constants

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(Received 22 March 2005; published 28 April 2005)

We derive strong bounds on any possible large-scale spatial variation in the values of physical constants whose space-time evolution is driven by a scalar field. These limits are imposed by the isotropy of the microwave background on large angular scales in theories which describe space and time variations in the fine structure constant, α, the electron-proton mass ratio, μ, and the Newtonian gravitational constant, G. Large-scale spatial fluctuations in the fine structure constant are bounded by $\delta\alpha/\alpha \lesssim 2\times 10^{-9}$ and $\delta\alpha/\alpha \lesssim 1.2\times 10^{-8}$ in the Bekenstein-Sandvik-Barrow-Magueijo and varying-speed-of-light theories, respectively, fluctuations in the electron-proton mass ratio by $\delta\mu/\mu \lesssim 9\times 10^{-5}$ in the Barrow-Magueijo theory and fluctuations in G by $\delta G/G \lesssim 3.6\times 10^{-10}$ in the Brans-Dicke theory. These derived bounds are significantly stronger than any obtainable by direct observations of astrophysical objects at the present time.

DOI: 10.1103/PhysRevD.71.083520 PACS numbers: 98.80.Es, 98.80.Bp, 98.80.Cq

I. INTRODUCTION

The recent resurgence of interest in the possible slow variation of some traditional "constants" of nature has focused almost exclusively upon their time variation. This was led inspired to a considerable extent by the capability of new astronomical instruments to measure spectral lines created by the light from distant quasar to very high precision. The quasar data analyzed in Ref. [1] using the new many-multiplet method consist of three separate samples of Keck-Hires observations which combine to give a data set of 128 objects at redshifts $0.5 < z < 3$. The many-multiplet technique finds that their absorption spectra are consistent with a shift in the value of the fine structure constant between these redshifts and the present of $\Delta\alpha/\alpha \equiv [\alpha(z) - \alpha]/\alpha = -0.57 \pm 0.10 \times 10^{-5}$, where $\alpha \equiv \alpha(0)$ is the present value of the fine structure constant. Extensive analysis has yet to find a selection effect that can explain the sense and magnitude of the relativistic line shifts underpinning these deductions. Further observational studies have been published in Ref. [2] using a different but smaller data set of 23 absorption systems in front of 23 very large telescope/ultraviolet echelle spectrograph quasars at $0.4 \leq z \leq 2.3$ and have been analyzed using an approximate form of the many-multiplet analysis techniques introduced in Ref. [1]. They obtained $\Delta\alpha/\alpha \equiv -0.06 \pm 0.06 \times 10^{-5}$, a figure that disagrees with the results of Ref. [1]. However, reanalysis is needed in order to understand the accuracy being claimed. Other observational studies of lower sensitivity have also been made using OIII emission lines of galaxies and quasars. The analysis of data sets of 42 and 165 quasars from the Sloan Digital Sky Survey (SDSS) gave the constraints $\Delta\alpha/\alpha \equiv 0.51 \pm 1.26 \times 10^{-4}$ and $\Delta\alpha/\alpha \equiv 1.2 \pm 0.7 \times 10^{-4}$ respectively for objects in the redshift range $0.16 \leq z \leq 0.8$ [3]. Observations of a single quasar absorption system at $z = 1.15$ by Quast *et al.* [4] gave $\Delta\alpha/\alpha \equiv -0.1 \pm 1.7 \times 10^{-6}$, and observations of an absorption system at $z = 1.839$ by Levshakov *et al.* [5] gave $\Delta\alpha/\alpha \equiv 2.4 \pm 3.8 \times 10^{-6}$. A preliminary analysis of constraints derived from the study of the OH microwave transition from a quasar at $z = 0.2467$, a method proposed by Darling [6], has given $\Delta\alpha/\alpha \equiv 0.51 \pm 1.26 \times 10^{-4}$, [7]. A comparison of redshifts measured using molecules and atomic hydrogen in two cloud systems by Drinkwater *et al.* [8] at $z = 0.25$ and $z = 0.68$ gave a bound of $\Delta\alpha/\alpha < 5 \times 10^{-6}$ and an upper bound on spatial variations of $\delta\alpha/\alpha < 3 \times 10^{-6}$ over 3 Gpc at these redshifts; bounds on spatial variation of similar order arise from the results of Ref. [1] because of the wide distribution of the target absorption systems on the sky.

New observational studies sensitive to small variations in the electron-proton mass ratio, $\mu \equiv m_e/m_p$, at high redshift have also been reported [9–11], along with a new restriction on a possible time variation of the Newtonian gravitation constant, G, in the solar system by the Cassini mission [12]. A range of other astronomical and geophysical constraints which might limit possible changes in α have also been reevaluated [13–15].

Theories which can handle space-time variations in α and μ self-consistently (as opposed to simply "writing in" a time-varying constant into the equations in which it is truly a constant) have only recently been developed to complement the Brans-Dicke gravity theory [16] which has been available to study the cosmological consequence of time variations in G. The BSBM theory [17,18] is a self-consistent extension of general relativity which incorporates space-time variations in the α, varying-speed-of-light (VSL) theories [19–22] provide other ways of effecting variations in α and other gauge couplings, and the new theory of Barrow and Magueijo [23] allows space-time variations of μ to be studied. All of these theories model the variation of a traditional constant by means of a scalar field which obeys a conservation equation derived from the variation of an action. The time variations of α, μ, and G that are permitted by these theories have been investigated in varying degrees of detail. They differ in one respect. The variations of the scalar fields carrying variations in α or μ do not have significant effects upon the expansion dynamics of the universe: the latter remains well described by the usual general relativistic Friedmann-Robertson-Walker (FRW) universe containing the appropriate matter source. However, in the case of Brans-Dicke theory the changes in the associated scalar field do affect the expansion dynamics of the universe and the FRW models are changed into new solutions that approach those of general relativity only in the limit that the space-time variation of G tends to zero.

In this paper we will show how a simple treatment of these four scalar-tensor theories for varying α, μ, and G allows us to predict and bound the magnitude of the *spatial* variations expected in these constants in the universe on extragalactic scales by using the observed temperature isotropy of microwave-background radiation on large angular scales. We note that, in the past, bounds on spatial variation have been discussed by Tubbs and Wolfe [24], and Pagel, [25] who addressed this question at a time when the large-scale uniformity of the universe was a far greater mystery than it is in today's postinflationary era. These papers stressed that the values of combinations of physical constants that were found to be the same to high precision when deduced from the spectra of objects were so far apart on the sky that they could not have been in causal contact during the history of the universe prior to the emission of their light. Today, we expect a high degree of coherence within the whole of the visible universe because it may have evolved from the inflation of a single causally coherent domain. However, even if that were the case, if constants like α, μ, and G are actually space-time variables and possess small quantum statistical fluctuations at the time of inflation then they may have a predictable (and even observable) spectrum of inhomogeneous variations today. A particular example is given by the chaotic inflationary universe in a Brans-Dicke theory of gravity [26,27], which gives rise to a spectrum of spatial fluctuations in the value of G as well as in the density of matter.

In Sec. II we shall describe four self-consistent theories of varying constants and in Sec. III show how we can use the isotropy of the microwave background in conjunction with the predictions of these theories to derive bounds on the allowed spatial variations in these constants, before summarizing our results in Sec. IV.

II. FOUR REPRESENTATIVE THEORIES

We will consider four representative scalar theories that are of particular interest given the current observational situation. It will be clear that these theories have an analogous structure. In each case the conservation of energy and momentum for the scalar field provides a wave equation of the form

$$\Box\varphi = \lambda f(\varphi)L(\rho, p), \tag{1}$$

where φ is a scalar field associated with the variation of some constant $\mathbb{C}$ via a relation $\mathbb{C} = f(\varphi)$, λ is a dimensionless measure of the strength of the space-time variation of $\mathbb{C}$, $f(\varphi)$ is a function determined by the definition of φ, and $L(\rho, p)$ is some linear combination of the density, ρ, and pressure, p, of the matter that is coupled to the field φ and $f(\varphi) \simeq 1$ for small φ. At a given cosmic time, for small changes in φ, this equation describes small spatial variations in $\mathbb{C}$ by a Poisson equation of the form

$$-\nabla^2\left(\frac{\delta\mathbb{C}}{\mathbb{C}}\right) \simeq \lambda L(\rho, p). \tag{2}$$

A. BSBM varying-α theory

A simple theory with time-varying α was first formulated by Bekenstein [17] as a generalization of Maxwell's equations but ignoring the consequences for the gravitational field equations. Recently, this theory has been extended [18] to include the coupling to the gravitational sector and some of its general cosmological consequences have been analyzed.

Variations in the fine structure constant are driven explicitly by variations in the electron charge, e, and the fine structure constant is given by

$$\alpha \equiv e^{2\psi},$$

where the scalar ψ field obeys an equation of motion of the form (1):

$$\Box\psi = -\frac{2}{\omega_1}e^{-2\psi}\rho_{\rm em}, \tag{3}$$

where ω_1 is a dimensional constant which couples the kinetic energy of the ψ field to gravity and $\rho_{\rm em}$ is the density of matter that carries electromagnetic charge. If we write $\zeta \equiv \rho_{\rm em}/\rho$ where ρ_m is the total matter density then

$$\zeta = \frac{E^2 - B^2}{E^2 + B^2}$$

and its sign ($-1 \leq \zeta \leq 1$) depends on whether the dominant form of (dark) matter is dominated by electrostatic (E^2) or magnetic (B^2) energy (for further discussion see Refs. [18,28–31]. For the scalar field, we have the propagation equation,

$$\Box\psi = -\frac{2}{\omega_1}e^{-2\psi}\zeta\rho_m \tag{4}$$

and for small variations in ψ and α this is well approximated by

$$\Box\psi \simeq -\frac{2}{\omega_1}\zeta\rho_m. \tag{5}$$

Some conclusions can be drawn from the study of the simple BSBM models with $\zeta < 0$, [18]. These models give a good fit to the varying α implied by the quasar data of Ref. [1]. There is just a single parameter to fit to the data and this is given by the choice

$$\left|\frac{\zeta}{\omega_1}\right| = (2 \pm 1) \times 10^{-4}. \tag{6}$$

We shall use this as a conservative bound in what follows. Tighter observational limits will only strengthen our conclusions.

B. VSL theories

In one "covariant" version of these theories [19–21] variations in α (and all other gauge couplings α_i) are driven by a scalar field χ that drives explicit variations in the speed of light and couples to all the matter fields in the Lagrangian, not just the electromagnetically coupled matter, with $\alpha_i = \exp[Q\chi]$, where Q is a numerical constant. The structure of covariant VSL is analogous to the BSBM theory and for small variations in the fine structure constant, $\exp[\chi] \sim 1$, we have

$$\Box\chi \simeq -\frac{Q}{\omega_2}\rho_m \tag{7}$$

where ω_2 is a coupling constant. The observational data of Webb *et al.* [1] are fitted by

$$\left|\frac{Q}{\omega_2}\right| = 8 \pm 4 \times 10^{-4}$$

which we use as the observational bound on the coupling. There is no variation of α in the limit that $\frac{Q}{\omega_2} \rightarrow 0$. Another edition [22] of a VSL theory has variation only in the electromagnetic coupling, α, and variations are driven only by the pressure of matter:

$$\Box\chi \simeq -4\pi G\omega p(\rho) \tag{8}$$

for some new coupling constant ω. We will just examine the covariant VSL theory, Eq. (7) in what follows.

C. BM varying-μ theory

The theory recently devised by Barrow and Magueijo [23] describes a varying electron-proton mass ratio, μ, via a changing electron mass which is driven by a scalar field, ϕ, defined by

$$m_e = m_0 e^{\phi}$$

where ϕ obeys

$$\Box\psi = -\frac{m_0(n_e - n_p)}{\omega_3}e^{\phi}.$$

Here, n_e and n_p are the electron and proton number densities, and ω_3 is a dimensional coupling constant. In the case of small variations ($e^{\phi} \sim 1$) this is well approximated by

$$\Box\psi \simeq \Box\mu \simeq -\frac{\rho_e}{\omega_3} \tag{9}$$

and observational bounds on the time variation of μ at high redshift [11] impose a weak bound of

$$G\omega_3 > 0.2.$$

The $\omega_3 \rightarrow \infty$ limit is that of constant μ.

D. Brans-Dicke gravity theory

The Brans-Dicke theory [16] generalizes Einstein's general theory of relativity to incorporate a space-time variation in the Newtonian constant G by means of a Brans-Dicke scalar field $\Phi \propto G^{-1}$ which obeys a conservation equation of the form

$$\Box\Phi = \frac{8\pi(\rho - 3p)}{3 + 2\omega_{\rm bd}}, \tag{10}$$

where ρ and p denote the total density and pressure of matter, respectively, $\omega_{\rm bd}$ is the dimensionless Brans-Dicke parameter and general relativity ($G = \text{const}$) is approached as $\omega_{\rm bd} \rightarrow \infty$. The current observational lower bounds on the allowed time variation of G give $\omega_{\rm bd} > 500$ from a variety of local gravitational tests (see [13] for a review). But the strongest constraint to date is derived from observations of the time delay of signals from the Cassini spacecraft as it passes behind the sun. These considerations led Bertotti, Iess and Tortora [12], after a complicated data analysis process, to claim that

$$\omega_{\rm bd} > 40\,000(2\sigma). \tag{11}$$

This theory differs from the three listed above in that small variations in Φ have a significant effect upon the expansion dynamics of the universe because these variations control the strength of gravity. These variations can be seen explicitly by writing down the Friedmann equation for the expansion scale factor $a(t)$ in the case of zero spatial curvature:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3\Phi}\rho - H\frac{\dot{\Phi}}{\Phi} + \frac{\omega_{\rm bd}}{6}\frac{\dot{\Phi}^2}{\Phi^2}. \tag{12}$$

For the case of dust ($p = 0$) there are simple power-law solutions:

$$a(t) = t^{[2+2\omega_{\rm bd}]/[4+3\omega_{\rm bd}]}, \tag{13}$$

$$\Phi(t) = \Phi_0 t^{2/[4+3\omega_{\rm bd}]} \tag{14}$$

and we note that we recover the usual Einstein–de Sitter cosmology with $a = t^{2/3}$ as $\omega_{\rm bd} \rightarrow \infty$. These solutions solve Eq. (10) exactly in the $p = 0$ case [32]:

$$\Box\Phi = \frac{8\pi\rho}{3 + 2\omega_{\rm bd}}. \tag{15}$$

III. LARGE-SCALE INHOMOGENEITY IN α, μ, AND G

The four theories that we have introduced have a characteristic structure in which changes in α, μ, and G are driven by different parts of the density of matter in the universe. This means that inhomogeneity in the matter content of the universe is coupled to inhomogeneities in the constants α, μ, and G. If we ignore the non-Machian mode that is not driven by the matter fields [i.e. the complementary function arising from the solution of $\Box(\text{scalar}) = 0$] because it falls off rapidly in time and becomes negligible at late times in the universe, then we can estimate the allowed large-scale spatial inhomogeneity of α, μ, and G in terms of observable quantities. The inhomogeneity in the constants requires inhomogeneity in the driving matter perturbations and their associated gravitational potential fluctuations. The magnitude of the latter is observationally constrained by the temperature isotropy of the microwave background on large angular scales and leads to an upper bound on the possible inhomogeneity in the values of constants. We shall ignore the acceleration of the universe which began recently at $z \simeq 0.3$. Its inclusion leads to a very small change in the final results, no larger than the uncertainty in other parameters.

A. BSBM inhomogeneity in α

On a hypersurface of constant comoving proper time, t, Eq. (5) gives

$$\nabla^2(\delta\psi) \simeq \frac{\zeta}{\omega_1}\delta\rho_m.$$

So, noting that $6\pi G\rho t^2 \simeq 1$, any inhomogeneity in α over a scale L is linked to inhomogeneity in the density of electromagnetically coupled matter, $\delta\rho_{\rm em}$, and/or inhomogeneity in the electromagnetic quality of the dark matter, $\delta\zeta_{\rm em}$, by[1]

$$\delta\psi \simeq \frac{\delta\alpha}{\alpha} \simeq 0.3\frac{\zeta}{\omega_1}\left(\frac{L}{t}\right)^2\left[\frac{\delta(\zeta)}{\zeta} + \frac{\delta\rho_{\rm em}}{\rho_{\rm em}}\right].$$

However, we expect that the inhomogeneity in the electromagnetically coupled matter can be written in terms of the inhomogeneity of the total matter density on scale L in terms of some biasing parameter β which will not depart too greatly from being $O(1)$, so we put

$$\frac{\delta\rho_{\rm em}}{\rho_{\rm em}} \simeq \beta\frac{\delta\rho_m}{\rho_m}.$$

Hence, we have

$$\frac{\delta\alpha}{\alpha} \simeq 0.3\frac{\zeta}{\omega_1}\left[\frac{\delta(\zeta)}{\zeta}\left(\frac{L}{t}\right)^2 + \beta\frac{\delta\rho_m}{\rho_m}\left(\frac{L}{t}\right)^2\right],$$

but we note that the gravitational potential perturbations $(\delta\Phi_N/\Phi_N)$ in the space-time metric created by $\delta\rho_m$ are given by Poisson's equation ($\nabla^2\Phi_N = 4\pi G\rho_m$) as

$$\frac{\delta\Phi_N}{\Phi_N} \simeq 0.3\frac{\delta\rho_m}{\rho_m}\left(\frac{L}{t}\right)^2$$

and these fluctuations produce temperature fluctuations in the microwave-background radiation, $\Delta T/T \simeq \delta\Phi_N/3\Phi_N \simeq 2\times 10^{-5}$, on the corresponding angular scales, [33] so that

$$\frac{\delta\Phi_N}{\Phi_N} \simeq 0.3\frac{\delta\rho_m}{\rho_m}\left(\frac{L}{t}\right)^2 \simeq 3\frac{\Delta T}{T} \simeq 6\times 10^{-5}$$

on large angular scales ($\theta > 10^0$), [34]. We will assume, in accord with observations, that the gravitational potential fluctuations are scale independent to a good approximation.[2]

Hence, if we take the best fit to $|\frac{\zeta}{\omega_1}| \simeq 2\times 10^{-4}$ from Eq. (6) the observations of Ref. [1] and we assume that the spatial variations in the electromagnetic composition of the matter in the universe are approximately proportional to the variations in the matter density, with

$$\frac{\delta(\zeta)}{\zeta} \lesssim \eta\frac{\delta\rho_{\rm em}}{\rho_{\rm em}},$$

where $\eta \sim O(1)$, then the spatial fluctuations in α and the required microwave-background temperature anisotropies in these theories are simply related by

$$\frac{\delta\alpha}{\alpha} \simeq 0.9\frac{\zeta}{\omega_1}\beta(1+\eta)\frac{\Delta T}{T} \simeq 2\times 10^{-9}\beta(1+\eta). \quad (16)$$

Hence, for the two cases of small and large spatial variations in ζ, respectively, we have

$$\frac{\delta\alpha}{\alpha} \lesssim 2\times 10^{-9}\beta \quad \text{if } \eta \ll 1,$$
$$\frac{\delta\alpha}{\alpha} \lesssim 2\times 10^{-9}\beta\eta \quad \text{if } \eta > 1.$$

These bounds on large-scale spatial variations of the fine structure constant are extremely strong and we have obtained them by assuming there is a bound on the level of time variation consistent with the observations of Ref. [1]. For comparison, the sensitivity of direct searches for variations in α which compare observations of different qua-

[1]Note that $\nabla^2\psi \to \frac{10}{3L^2}(\psi(x) - \bar{\psi}(x))$ where $\bar{\psi}$ is the average value of ψ in a spherical region of radius L as $L \to 0$ and is defined by $\bar{\psi}(\vec{x}) \equiv \frac{3}{4\pi R^3}\int_V \psi(\vec{x}+\vec{r})d^3\not{r}$.

[2]Note that the smallness of $\delta\Phi_N/\Phi_N$ is the justification for the so-called "cosmological principle" and the use of the Friedmann metric. The smallness of $\delta\rho_m/\rho_m$ is unnecessary [35].

sar absorption spectra around the sky is only about $\delta\alpha/\alpha \lesssim O(10^{-6})$. Recall that, in contrast, the time variation of the fine structure constant is far more strongly constrained by the quasar absorption-system data than by the microwave-background effects on small scales [36].

VSL inhomogeneity in α

A similar argument to that used for the BSBM case can be applied to the covariant VSL theory and leads to the result that the allowed spatial variation in α is again bounded by the temperature anisotropy in the microwave background by

$$\frac{\delta\alpha}{\alpha} \simeq 0.3\frac{Q}{\omega_2}\left[\frac{\delta\rho_m}{\rho_m}\left(\frac{L}{t}\right)^2\right] \simeq 0.9\frac{Q}{\omega_2}\frac{\Delta T}{T} \lesssim 1.2 \times 10^{-8}.$$

A similar bound could be deduced for the allowed spatial variations in all the gauge couplings, $\delta\alpha_i/\alpha_i$ in this theory. Again, this bound is far stronger than can be achieved by direct spectroscopic studies of quasars and other astrophysical systems at low redshift.

B. BM inhomogeneity in μ

If we apply this argument to possible spatial variations in the electron-proton mass ratio, μ, described above then a similar argument leads to an expression for the inhomogeneity in the electron-proton mass ratio, μ, of the form

$$\frac{\delta\mu}{\mu} \simeq \frac{0.3}{G\omega_3}\left[\frac{\delta\rho_e}{\rho_e}\left(\frac{L}{t}\right)^2\right] \simeq \frac{0.9\beta}{G\omega_3}\frac{\Delta T}{T} \lesssim 9 \times 10^{-5}\beta$$

where we have assumed that the inhomogeneity in the electron density is approximately proportional to that in the matter distribution:

$$\frac{\delta\rho_e}{\rho_e} = \beta\frac{\delta\rho_m}{\rho_m}.$$

In this case the numerical bounds on the allowed variation are much weaker than for $\delta\alpha/\alpha$. This is a reflection of the weak constraints that exist on time-varying μ in these theories [23] because of the different time evolution of α and μ during the radiation era.

C. Brans-Dicke inhomogeneity in G

The analysis of the level of inhomogeneity expected in a Brans-Dicke universe is slightly different because time variations in $\Phi \sim G^{-1}$ determine the expansion dynamics. For the "Machian" solutions (13) and (14) that are the attractors at late times we have $8\pi\rho \simeq \Phi H^2$ and so inhomogeneity in the Brans-Dicke field ($\delta\Phi \neq 0$) is linked to inhomogeneity in the gravitation constant ($\delta G \neq 0$) and in the total matter density ($\delta\rho \neq 0$) by

$$\frac{\delta G}{G} \simeq \frac{\delta\Phi}{\Phi} \simeq \frac{10.8}{3 + 2\omega_{\rm bd}}\frac{\delta\rho_m}{\rho_m}\left(\frac{L}{t}\right)^2\frac{(\omega_{\rm bd}+1)^2}{(4+3\omega_{\rm bd})^2}$$
$$\simeq \frac{10.8}{3 + 2\omega_{\rm bd}}\frac{(\omega_{\rm bd}+1)^2}{(4+3\omega_{\rm bd})^2}\frac{\Delta T}{T}.$$

Hence, for large values of $\omega_{\rm bd}$, as observations of the time variation of G require, this simplifies to

$$\frac{\delta G}{G} \simeq \frac{6}{5\omega_{\rm bd}} \times 10^{-5} \lesssim 3.6 \times 10^{-10}$$

if we use the Cassini bounds on $\omega_{\rm bd}$. If we replace the Cassini bound by an observational bound of $\omega_{\rm bd} > 1000$ from other solar-system constraints this weakens the bound by a factor of 40. In either case the bound on spatial variations is extremely strong. It results from a combination of the microwave-background anisotropy limits on the density perturbations which drive variations in G and the intrinsic weakness of the Φ coupling. Notice that, just as fluctuations in the microwave-background temperature are known far more accurately than the mean temperature itself, so the spatial fluctuations in G are limited to far greater accuracy than the value of G is known experimentally (see for example [37]).

We summarize the principal results obtained in Table I.

IV. DISCUSSION

We have shown that the characteristic structure of scalar theories for the space-time variation of supposed constants of nature enables us to use the observed bounds on the couplings in the theories obtained from observational

TABLE I. Summary of the inhomogeneity levels allowed by the microwave-background temperature isotropy in the theories discussed in this paper. In column 2 the link between the scalar field and the "constant" is defined; in column 3 observational bounds on the dimensionless scalar coupling constant appearing in the theory and discussed in the text are given; the propagation equation for the scalar field is given in column 4 and in column 5 the bounds on any spatial variation in the associated constant imposed by the bound on the coupling and the isotropy of the microwave background ($\Delta T/T \leq 2 \times 10^{-5}$) are summarized.

Theory	Scalar field	Scalar coupling	Scalar equation	Bound on inhomogeneity
BSBM	ψ: $\alpha \equiv e^{2\psi}$	$\lvert\frac{\zeta}{\omega_1}\rvert = 2^{+1}_{-1} \times 10^{-4}$	$\Box\psi = -\frac{2\rho_{\rm em}e^{-2\psi}}{\omega_1}$	$\frac{\delta\alpha}{\alpha} \simeq \frac{\zeta}{\omega_1}\beta\eta\frac{\Delta T}{T} \lesssim 2 \times 10^{-9}$
VSL	χ: $\alpha_i \equiv e^{Q\chi}$	$\lvert\frac{Q}{\omega_2}\rvert = 8^{+4}_{-4} \times 10^{-4}$	$\Box\chi = -\frac{Q\rho_m e^{-2\chi}}{\omega_2}$	$\frac{\delta\alpha}{\alpha} \simeq \frac{Q}{\omega_2}\frac{\Delta T}{T} \lesssim 1.2 \times 10^{-8}$
BM	ϕ: $\mu \propto e^{\phi}$	$\lvert G\omega_3\rvert > 0.2$	$\Box\phi = -\frac{\rho_e e^{\phi}}{\omega_3}$	$\frac{\delta\mu}{\mu} \simeq \frac{\beta}{G\omega_3}\frac{\Delta T}{T} \lesssim 9 \times 10^{-5}$
bd	Φ: $G \propto \Phi^{-1}$	$\omega_{\rm bd} > 40\,000$	$\Box\Phi = \frac{8\pi\rho}{3+2\omega_{\rm bd}}$	$\frac{\delta G}{G} \simeq \frac{3}{5\omega_{\rm bd}}\frac{\Delta T}{T} \lesssim 3.6 \times 10^{-10}$

searches for time variations in the associated constants and the microwave-background isotropy to obtain very strong bounds on any spatial fluctuations in these constants over large astronomical scales. Since the scalar fields which carry the space-time variations of constants are driven by all (or part) of the matter content of the universe there will always be density inhomogeneities present which drive inhomogeneities in the constants. However, the gravitational potential fluctuations associated with these large-scale density perturbations show up in the microwave-background temperature anisotropy on large angular scales. Their amplitudes are therefore bounded above and this bound in combination with limits on the strength of the coupling of the scalar field leads to a series of very strong bounds on possible spatial fluctuations in the associated constants. We have calculated these bounds for four representative theories which are of current interest. The basic argument is of wider application to other dilaton theories and with small modifications the bounds can be modified to include the small changes that arise because of a non-constant curvature spectrum of density perturbations. They will be strengthened if the underlying couplings in these theories are strengthened by future or present observational studies.

Detailed consideration of the behavior of fluctuations on smaller scales would lead to a different range of constraints. We note that the limits presented here apply to large scales and do not apply to the possible variations in the values of constants that can arise because of the nonlinear evolution of cosmic overdensities into clusters, galaxies and planetary systems. These are not bounded by the isotropy of the microwave background and still permit significant spatial variations to arise on small scales in the local universe [38].

ACKNOWLEDGMENTS

I would like to thank T. Clifton, J. Magueijo, D. Mota, M. Murphy, and J. K. Webb for helpful discussions.

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