A Cosmological Tale of Two Varying Constants

John D. Barrow[1], João Magueijo[2] and Håvard B. Sandvik[2]

[1]*DAMTP, Centre for Mathematical Sciences,*
Cambridge University, Wilberforce Rd.,
Cambridge CB3 0WA, UK.
[2]*Blackett Laboratory, Imperial College,*
Prince Consort Rd., London SW7 2BZ, UK

(The Date)

We formulate a simple extension of general relativity which incorporates space-time variations in the Newtonian gravitation 'constant', G, and the fine structure 'constant', α, which generalises Brans-Dicke theory and our theory of varying α. We determine the behaviour of Friedmann universes in this theory. In the radiation and dust-dominated eras αG approaches a constant value and the rate of variation of α is equal to the magnitude of the rate of variation in G. The expansion dynamics of the universe are dominated by the variation of G but the variation of G has significant effects upon the time variation of α. Time variations in α are extinguished by the domination of the expansion by spatial curvature or quintessence fields, as in the case with no G variation.

I. INTRODUCTION

There have been many studies of the cosmological consequences of allowing some of the traditional constants of Nature to change. These include evaluations of the effects of altering the observed value of a constant to another constant value and studies of the time-evolution of 'constants' in generalisations of the general theory of relativity that allow them to become space-time variables. The most studied case is that of varying the Newtonian gravitation constant, G, through the Brans-Dicke (BD) scalar-tensor theory of gravity [1]. Recently, following Bekenstein, [2], Sandvik, Barrow and Magueijo [3], [4] have developed a theory (BSBM) which describes the space-time variation of the fine structure constant. In these, and other, studies of varying constants only a single constant is allowed to vary at one time. However, since we have no understanding of why the constants of Nature take the values that they do, whether they are logically independent, or even whether they all are truly constant, this restriction is somewhat artificial. Motivated by recent observational evidence for a time evolution of the fine structure 'constant', α, at redshifts $z \sim 1 - 3.5$, [5], [6], [7], we have unified the BD and BSBM theories to produce an exact theory which describes the simultaneous variation of α and G. This type of model also provides a framework within which to consider the consequences of changes in the scale of extra dimensions of space on apparent three-dimensional coupling constants.

In section 2 we set up the theory and evolution equations for Friedmann universes in a theory that generalises general relativity to include varying α and G. In section 3 we show how to find the cosmological solutions during the dust-dominated eras. We find an exact solution where αG is constant during the dust era while α and G^{-1} both increase with time. We then determine analytically the coupled evolution of α and G during the radiation, curvature, and vacuum-energy dominated eras of cosmological expansion. From here we go on to check the solutions numerically and we show how in Universes like our own, with actual initial values for α and G the asymptotic behaviour is never reached. Instead we find constant α and G in the radiation era, slow growth of α and slow decrease in G in the dust epoch, constant values for both in curvature dominated universe, and constant α and decreasing G in Λ dominated epoch. Generally we find that the overall evolution of the expansion scale factor of the universe is dictated by the G variation and assumes the form found in the Brans-Dicke theory to a very good approximation irrespective of the α variation. The evolution of α is influenced by the G variation but does not differ much from that found in the BSBM cosmologies where only α varies.

II. FIELD EQUATIONS

We introduce the structure of the BSBM theory for varying α as another matter field in Brans-Dicke theory. The resulting theories has two scalar fields: the BD field ϕ propagating variations in G, and the field ψ propagating variations in α. The action for this theory becomes

$$S = \int d^4x \sqrt{-g} \left(R\phi + \frac{16\pi}{c^4}\mathcal{L} - \omega_{BD}\frac{\phi_{,\mu}\phi^{,\mu}}{\phi} \right) \tag{1}$$

where

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_{em} \exp(-2\psi) + \mathcal{L}_\psi, \tag{2}$$

and

$$\mathcal{L}_\psi = -\frac{\omega}{2} \psi_{,\mu} \psi^{,\mu}.$$

The field equations for the theory, specialised to the case of a homogeneous and isotropic Friedmann space-time metric containing dust and radiation perfect fluids are:

$$3\frac{\dot{a}^2}{a^2} = \frac{8\pi}{\phi} \left(\rho_m(1 + |\zeta| \exp(-2\psi)) + \rho_r \exp(-2\psi) + \rho_\psi \right) - 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\omega_{BD}}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{k}{a^2} \tag{3}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{8\pi}{3 + 2\omega_{BD}}(\rho_m - 2\rho_\psi) \tag{4}$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega}\exp(-2\psi)\zeta\rho_m \tag{5}$$

$$\dot{\rho_m} + 3H\rho_m = 0 \tag{6}$$

$$\dot{\rho_r} + 4H\rho_r = 2\dot{\psi}\rho_r \tag{7}$$

where $\rho_\psi = \frac{\omega}{2}\dot{\psi}^2$ is the kinetic energy density for the ψ fluid, with ω the coupling setting the relevant energy scale for the ψ-field. ζ is defined as the ratio $\mathcal{L}_{em}/\rho_m$ averaged over all types of matter in the universe. The fine structure 'constant' is given by ($\hbar = c = 1$)

$$\alpha \equiv \alpha_0 \exp(2\psi), \tag{8}$$

where α_0 is the present day value of the fine structure 'constant'. The present-day value of G is set equal to unity.

We shall confine our attention to the case with $\zeta < 0$ where the magnetic energy dominates the electric field energy of the matter coupling to electric charge in the universe. This places particular constraints upon the nature of the cold dark matter dominating the universe today. From our earlier studies, [3]- [4], we know that this case provides a slow variation with α increasing logarithmically in time during the dust era but staying constant during any subsequent curvature or cosmological constant dominated era. Also, in a universe with a matter-radiation balance like our own, α remains constant during the radiation era except close to the initial singularity. Negative ζ models are well behaved and correspond to the dark matter in the universe being dominated by magnetic coupling, (for example superconducting cosmic strings contribute $\zeta = -1$). The expansion scale factor evolution is not affected by variations in α to leading order. By contrast, the choice $\zeta > 0$ creates major changes to cosmological evolution. It does not lead to slow increase of α with time during the dust era, as observations suggest, and the evolution of the expansion scale factor is affected to leading order (see for example refs. [9,10] who discuss related theories for the variation of α with $\zeta > 0$ and hence $\dot{\alpha} < 0$ cosmological behaviour in the dust era in contrast to our discussions in [3]- [4] and below). In what follows we shall investigate how the $\zeta < 0$ evolution of the fine structure constant couples to variation of G in the Brans-Dicke theory.

The constant ω_{BD} is the Brans-Dicke parameter and ω is the analogous parameter for the coupling of the ψ field driving variations in α. We have used the facts that dust is pressureless, $p = \rho_r/3$ for a sea of radiation and $p = \rho_\psi$ for a fluid with kinetic energy only. Equation (3) can be recast for numerical solutions

$$\frac{\dot{a}}{a} = -\frac{1}{2}\frac{\dot{\phi}}{\phi} \pm \frac{1}{2}\sqrt{\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{4}{3}\left(\frac{8\pi\rho}{\phi} + \frac{\omega_{BD}}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right) - 4\frac{k}{a^2}} \tag{9}$$

and eqn. (7) integrates to give $\rho_\gamma \exp(-2\psi) \propto a^{-4}$. Note that this is the combination that appears in the generalised Friedmann equation, (3). In ref. [4] we showed how to deduce the solutions of these equations when G is constant. Here, we will extend this analysis to the new situation where both α and G vary in time.

III. DUST ERA EVOLUTION

From our study of the Friedmann models in BSBM theory we know that, to a very good approximation, the α variations do not significantly affect the evolution of the expansion scale factor $a(t)$. The effects of varying G in

Brans-Dicke theories is different. No matter how slow the variation in G, a correction will occurs to the power of the time-variation of the expansion scale factor. In the dust era we assume the asymptotic solution for the Brans-Dicke (BD) flat dust model holds to high accuracy. This is an exact solution of (3) for $\zeta = k = \psi = \rho_\gamma = 0$ and is the late-time attractor of the general flat BD dust solution (see refs. [11,12], [13]) which differs only as $t \to 0$, where the solution becomes dominated by the kinetic energy of the ϕ field and approaches the BD vacuum solution. Thus, to leading order the expansion dynamics and ϕ evolution are described at late times by the exact Brans-Dicke dust solution with $k = 0$:

$$a(t) \propto t^{(2-n)/3}; \phi = \phi_0 t^n \tag{10}$$

$$\rho = Ma^{-3} \qquad\qquad \dot{M} \equiv 0 \tag{11}$$

$$\phi_0 \equiv \frac{8\pi M}{n(3 + 2\omega_{BD})} \tag{12}$$

where n is related to the Brans-Dicke parameter by

$$n \equiv 2/(4 + 3\omega_{BD}), \tag{13}$$

and M is the present density of the universe in Planck units, $M \sim 10^{-123}$.

What is the asymptotic solution for α during the dust era? The relevant equation is (5), which can now be rewritten as

$$\frac{d}{dt}(t^{2-n}\dot{\psi}) = N \exp(-2\psi) \tag{14}$$

where

$$N \equiv -\frac{2\zeta}{\omega}\rho_m a^3 = -\frac{2\zeta M}{\omega} > 0 \qquad\qquad \dot{N} \equiv 0,$$

and $-\zeta/\omega \approx 10^{-4}$ is the best fit of this parameter ratio to the observations of Webb et. al. [5]- [7][1].

Unlike the case with constant G, there is an exact solution (for ω_{BD} positive and finite)

$$\psi(t) = \frac{n}{2}\ln(t) + \frac{1}{2}\ln N - \frac{1}{2}\ln(\frac{n}{2} - \frac{n^2}{2}) \tag{15}$$

so we have, using this solution for ψ to solve for ϕ in (4):

$$\alpha(t) = \alpha_0 \exp(2\psi) \;\; = \alpha_0 \frac{2Nt^n}{n(1-n)} \tag{16}$$

Hence, there is a simple relationship between $\alpha(t)$ and $G(t)$:

$$\phi = G^{-1} = \frac{2\pi\omega(1-n)}{-\zeta(3 + 2\omega_{BD})}\frac{\alpha}{\alpha_0} = \frac{2\pi\omega(2 + 3\omega_{BD})}{-\zeta(3 + 2\omega_{BD})(4 + 3\omega_{BD})}\frac{\alpha}{\alpha_0} \propto t^n, \tag{17}$$

so αG is always a constant. Note that for large values of ω_{BD} we have a simple relation between the values of G and α:

$$G\frac{\alpha}{\alpha_0} \approx \frac{-\zeta\omega_{BD}}{\pi\omega} > 0 \tag{18}$$

As expected, α increases whilst G falls as $t \to \infty$ in a flat universe. It is interesting to note that the asymptotic value of $G\alpha$ is uniquely determined by the parameters in the model with no arbitrary constants.

Although the asymptotic behaviour is now determined, the question of whether this can be reached on a cosmological timescale depends strongly on the choice of initial conditions and needs to be investigated numerically. We can quickly conclude that the asymptotic regime is not reached in our universe. Presently we have $\alpha \approx 1/137$, and in our units the

---

[1]The value used for $-\zeta/\omega$ is the value fitted for the BSBM theory with constant G. However, since n is so close to zero it should not be significantly different numerically in the case with varying G

numerical value of M is extremely small, $\sim 10^{-123}$. Obviously the actual value of α is then many orders of magnitude larger than given by the solution in eq.(15) and we are thus nowhere near the asymptotic regime. Consequently, in order to find the behaviour of α and G we turn to numerical solutions of the equations. We evolve the Friedmann equations through time with initial conditions chosen so as to yield the present day values of G and α. For α we find a behaviour very similar to the BSBM theory, with a slow growth giving a relative change of the order 10^{-4} throughout the dust epoch. G goes through a decrease of order 10^{-3} during the same period. The numerical results are shown in Figures 1 and 2.
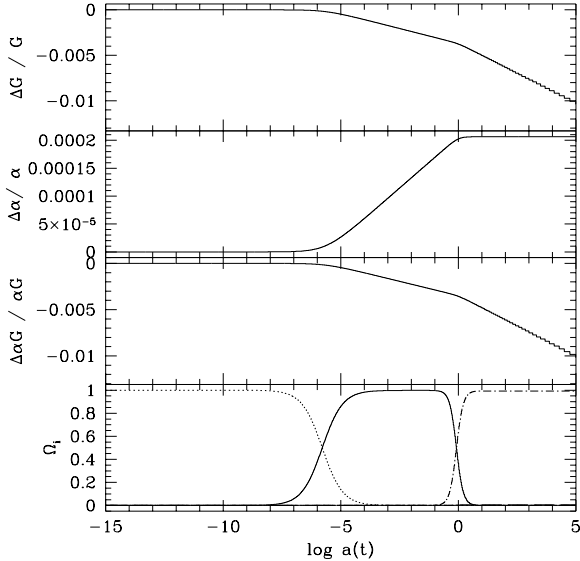

FIG. 1. Evolution of the relative shift in the values of the two 'constants' in a realistic flat cosmology with vacuum energy and approximately accurate initial values for the fields. We start from a radiation-dominated universe where both α and G stay constant. Thereafter we move into dust domination where α changes slowly, while G goes through a small decrease. As the universe becomes dominated by the vacuum energy, α goes to a constant, while G goes on decreasing indefinitely as in ordinary Brans Dicke theory. Values used for the couplings are the minimum allowed value of $\omega_{BD} = 3500$ and we take the best fit value of $\zeta/\omega = -10^{-4}$ from BSBM theory.

IV. RADIATION ERA EVOLUTION

The evolution in the radiation era is slightly more complicated because of the contribution of the ρ_ψ term to the right-hand side of the ϕ evolution equation. This means that we do not have the usual late-time asymptotic behaviour of constant ϕ to accompany the $a = t^{1/2}$ scale factor as in BD radiation universes. If we assume

$$a = t^{1/2}$$

then we have

$$\frac{d}{dt}(\dot{\phi}t^{3/2}) = R - \lambda t^{3/2}\dot{\psi}^2 \tag{19}$$

where

$$\lambda \equiv \frac{8\pi\omega}{3 + 2\omega_{BD}}, \tag{20}$$

$$R \equiv \frac{8\pi M}{3 + 2\omega_{BD}} = -\frac{4\pi\omega N}{\zeta(3 + 2\omega_{BD})} \tag{21}$$

The R term is negligible when the kinetic energy of the ψ field dominates the matter density during the radiation era. Likewise, the λ term can be neglected when the matter density dominates the ψ kinetic energy. We also have
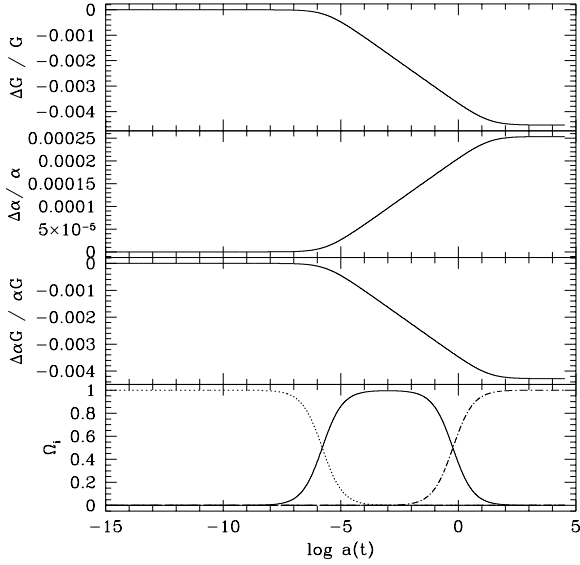
FIG. 2. Evolution of the relative shift in the values of the two 'constants' in an open universe through radiation, dust and curvature-dominated epochs. Initial values for the fields are set so as to give realistic values at present time. Again α and G are constants in the radiation dominated era, whilst α increases and G decreases through matter domination. As curvature starts to take over the expansion, both α and G tends to constants. Values for the couplings are the minimum allowed value of $\omega_{BD} = 3500$ and the best fit value of $\zeta/\omega = -10^{-4}$ from the BSBM theory.

$$\frac{d}{dt}(\dot{\psi}t^{3/2}) = N\exp[-2\psi] \tag{22}$$

as in the case with constant G. This has the exact solution

$$\psi = \frac{1}{2}\ln(8N) + \frac{1}{4}\ln(t) \tag{23}$$

as before, so $\dot{\psi}^2 = (16t^2)^{-1}$. If we substitute this in (19)

$$\frac{d}{dt}(\dot{\phi}t^{3/2}) = R - \frac{\lambda}{16}t^{-1/2} \tag{24}$$

so

$$\phi = \phi_0 + 2Rt^{1/2} - \frac{\lambda}{8}\ln(t) + Ct^{-1/2} \tag{25}$$

where C and ϕ_0 are constants. If the universe expands for long enough to reach the asymptotic regime then we have (as $R > 0$)

$$\psi \approx \frac{1}{4}\ln(t) \tag{26}$$

$$\phi = G^{-1} \approx -\frac{8\pi\omega N}{\zeta(3 + 2\omega_{BD})}t^{1/2} \tag{27}$$

so, from eqns. (18) and (8),

$$G\frac{\alpha}{\alpha_0} \approx \frac{-\zeta(3 + 2\omega_{BD})}{8\pi\omega N} \approx \frac{-\zeta\omega_{BD}}{4\pi\omega N} \tag{28}$$

for large ω_{BD}. Thus we still have the nice asymptotic behaviour of αG in the radiation-dominated epoch. However, we again need to compare with numerical results to determine whether these asymptotic solutions can indeed be realised in the Universe. As in the case of dust, the same simple reality check can now be performed on the solution (23). As in the case of constant G we are nowhere near this particular solution in our Universe. α would need to be

several orders of magnitude smaller if it was to satisfy the solution, and as in the BSBM theory we expect instead a constant value of α in the rad epoch. This assumption is indeed confirmed by the numerical solutions shown in Figures (2) and (1).

Another possible problem for the analytic solutions above would arise if the kinetic energy of the ψ field dominates the matter density during radiation domination. We regard this situation as unrealistic and it cannot be realised asymptotically.

V. CURVATURE ERA EVOLUTION

During a curvature-dominated phase of an open universe the expansion scale factor tends to that of the Milne vacuum universe, which is an exact solution of general relativity and of Brans-Dicke theory (with constant ϕ) with

$$a = t \tag{29}$$

Using this in the propagation equations for ϕ and ψ, we find that, as $t \to \infty$, so leading order

$$\psi = \psi_* - \frac{N \exp[-2\psi_0]}{t}$$
$$\phi = \phi_* + \frac{4\pi\omega N}{\zeta(3 + 2\omega_{BD})t}$$

with ψ_* and ϕ_* constants, so both α and G tend to constant values as $t \to \infty$. In a universe that passes directly from dust domination to curvature domination these constant values will be very close to the asymptotic attractors for the dust era of evolution found above in eqn. (18) providing the dust epoch has lasted long enough for the attractor to be reached.

The behaviour of G, α and $G\alpha$ in a universe like our own but which eventually becomes dominated by negative curvature is shown in Figure (2).

VI. COSMOLOGICAL 'CONSTANT' ERA EVOLUTION

In flat Brans-Dicke cosmologies a solution of the Friedmann equation with cosmic vacuum energy $(p_v = -\rho_v)$ is

$$a = t^{\omega_{BD} + \frac{1}{2}} \tag{30}$$
$$\phi = \phi_o t^2 \tag{31}$$
$$\phi_0 \equiv \frac{32\pi\rho_v}{(5 + 6\omega_{BD})(3 + 2\omega_{BD})} \tag{32}$$

and ρ_v is constant. This is not the general solution but it is the attractor for the general $p_v = -\rho_v$ solution at late times [17]. It is a power-law inflation model [18]. Note that in Brans-Dicke theory, unlike in general relativity, a $p_v = -\rho_v$ stress behaves differently in the Friedmann equation to an explicit constant Λ term [17]. It is the former that describes the stress contributed by a stationary scalar field with a constant potential. Every term in the BD Friedmann equations falls as t^{-2} for this solution. It is unusual in that it appears to predict that if the universe has just begun accelerating (as observations imply, [19,20]) then G should vary rapidly in the solar system. However, this argument assumes that the vacuum stress is dominant everywhere, right down to the solar system scale, which in reality it is not.

If we substitute this solution for $a(t)$ (but not ϕ) in the ψ and ϕ evolution equations, (4) and (5) then we get, since

$$p_v = -\rho_v = const,$$

that

$$\ddot{\phi} + \frac{3(2\omega_{BD} + 1\ )}{2t}\dot{\phi} = \frac{8\pi}{3 + 2\omega_{BD}}(4\rho_v - 2\rho_\psi) \approx \frac{-8\pi}{3 + 2\omega_{BD}}(4\rho_v - \omega\dot{\psi}^2)$$
$$\ddot{\psi} + \frac{3(2\omega_{BD} + 1\ )}{2t}\dot{\psi} = -\frac{2}{\omega}e^{-2\psi}\zeta\rho_m \approx 0$$

6

So, at late times

$$\dot{\psi} = Aa^{-3} = Dt^{-3(\omega_{BD}+\frac{1}{2})}$$
$$\psi = Et^{-3\omega_{BD}-\frac{1}{2}} + F \rightarrow F$$

$$\ddot{\phi} + \frac{3(2\omega_{BD}+1\ )}{2t}\dot{\phi} = \frac{-8\pi\omega}{3+2\omega_{BD}}(4\rho_v\ - Qa^{-6}) \rightarrow \frac{-32\pi\omega\rho_v}{3+2\omega_{BD}},$$

so

$$\phi = A + Bt^2 + Ct^{-3\omega_{BD}-\frac{1}{2}} \rightarrow \phi_0 t^2, \tag{33}$$

and, as expected, we get the same growing behaviour as in pure BD. When the universe becomes vacuum-energy dominated α tends to a constant value but $\alpha G \propto G \propto t^{-2}$ continues to fall. This behaviour is confirmed by numerical solutions shown in Figure (1). Using eqn. (18), we see that if t_v is the time when a vacuum-dominated era succeeds a sufficiently long dust-dominated era in a flat universe, then at $t \geq t_v$ in the vacuum-dominated era we expect

$$\alpha(t)G(t) = \frac{-\zeta\omega_{BD}}{\pi\omega}\left(\frac{t_v}{t}\right)^2. \tag{34}$$

Hence, today, we would have

$$\alpha(t_0)G(t_0) = \frac{-\zeta\omega_{BD}}{\pi\omega(1+z_v)^{\frac{4}{1+2\omega_{BD}}}}. \tag{35}$$

We see that, as in the situation where G is constant, the effect of a vacuum energy or quintessence field is to turn off variations in α when it takes over the expansion of the universe, [3].

VII. DISCUSSION

We have formulated a simple gravity theory which extends general relativity, by the addition of two scalar fields, to include time variation of G and α. Previously, the study of the cosmological variation of physical 'constants' has confined attention to varying one constant only or to discussing the effects of altering the values of physical constants without a self-consistent theory for their dynamical variation [24], [23]. The structure of unified gauge theories and particle physics theories with extra dimensions has given some indication as to the self consistency conditions required if traditional constants are allowed to vary, [21,22].

We have found that the expansion of the universe is affected by varying G to first order and the evolution of the expansion scale factor follows the behaviour found in Brans-Dicke cosmologies to leading order without being significantly affected by variations in α. The variations in α are affected by the variations in G through their influence on the expansion rate. This is significant in the dust-dominated era of cosmic expansion, which is known to exhibit a special mathematical behaviour in the absence of G variation, [4]. The effect of any G time variation simplifies the α variation and allows an exact solution to be found with $\alpha \propto t^n$, where $n \equiv 2/(4+3\omega_{BD})$ is determined by the Brans-Dicke parameter ω_{BD}.

In both the radiation and dust dominated eras, there are asymptotic solutions in which the product $G\alpha$ remains constant and its value is determined uniquely by the coupling constants of the theory. However in universes like our own with the values of α and G near present values these asymptotic regimes are not reached throughout the life of the universe. Typically our present values for α are much larger than the values required by the asymptotic solution.

In a curvature-dominated or quintessence-dominated era the variation in α ceases, just as in the situation with no G variation [3]. This is an important feature of all models with varying α in theories of the BSBM sort because it naturally reconciles evidence of variations in α at redshifts $z \sim 1-3$, with local ($z = 0.1$) constraints from the Oklo natural reactor if the universal expansion began to accelerate at $z \sim 0.7$, as current observations imply.

Finally, we reiterate that the conclusions drawn above apply only to varying-α theories with negative ζ. The exact solutions given in eqs. (15),(16),and (23) for the evolution of $\alpha(t)$ during the radiation and dust eras no longer exist when $\zeta > 0$ and hence $N < 0$. During the curvature and cosmological constant-dominated eras the evolution of $a(t)$

is significantly changed by the variations of ψ and the assumptions (29) and (30) for the scale factor evolution are no longer valid.

The study performed here provides a simple cosmological model in which the variation of two 'constants' can be studied exactly. A number of extensions are possible. The variations of weak and strong couplings can be included and the constraints imposed by any scheme grand unification can be imposed [21].

[1] C. Brans and R.H. Dicke, Phys. Rev. 124, 924 (1961)

[2] J.D. Bekenstein, Phys. Rev. D 25, 1527 (1982).

[3] H. Sandvik, J.D. Barrow and J. Magueijo, Phys.Rev.Lett. 88 (2002) 031302

[4] J.D. Barrow, H. Sandvik and J. Magueijo, Phys.Rev. D65 (2002) 063504

[5] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater & J.D. Barrow, Phys. Rev. Lett. 82, 884 (1999).

[6] J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska and A.M. Wolfe, Phys. Rev. Lett. 87, 091301 (2001).

[7] M.T. Murphy, J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater & J.D. Barrow, et. al. astro-ph/0012419,

[8] P.D. Scharre and C. Will, gr-qc/0109044

[9] K. Olive and M. Pospelov, hep-ph/0110377

[10] G.R. Dvali and M. Zaldarriaga, Phys.Rev.Lett. 88 (2002) 091303

[11] H. Nariai, Prog. Theo. Phys. 42, 544 (1969).

[12] L. Gurevich, A.M. Finkelstein & V. A. Ruban, Astrophys. Space Sci. 22, 231 (1973).

[13] J.D. Barrow, in Proc. Erice Summer School *Current Topics in Astrofundamental Physics*: *Primordial Cosmology*, pp 269-305, ed. N. Sanchez (1997), gr-qc/9711084; J.D. Barrow, Mon. Not. R. astr. Soc., 282, 1397 (1996).

[14] J.D. Prestage, R.L. Tjoelker & L. Maleki, Phys. Rev. Lett. 74, 18 (1995), A. Shylakhter, Nature 264, 340 (1976).Y. Fujii et al, Nucl. Phys. B, 573, 377 (2000); T. Damour and F.J. Dyson, Nucl. Phys. B, 480, 37 (1996). M.J. Drinkwater, J.K. Webb, J.D. Barrow & V.V. Flambaum. MNRAS. 295, 457 (1998).

[15] A.R. Liddle, J.D. Barrow and A. Mazumdar, Phys. Rev. D 58, 027302 (1998).

[16] T. Chiba, gr-qc/0110118

[17] J.D. Barrow and K. Maeda, Nucl. Phys. B 341, 294 (1990).

[18] D. La and P.J. Steinhardt, Phys. Rev. Lett., 62, 376 (1989); Mathiazhagen C., Johri V.B., Class. Quantum Gravity, 1, L29 (1984).

[19] S. Perlmutter et al., Ap. J. 517, 565 (1999); S. Perlmutter et al, Ap. J. 483, 565 (1997); S. Perlmutter et al, Nature 391 51 (1998); P.M. Garnavich, et al, Ap.J. Letters 493, L53 (1998); Schmidt, B.P., Ap.J. 507,46 (1998); Riess, A.G. et al 1998 AJ 116,1009.

[20] G. Efstathiou et al, 2dF collaboration, astro-ph/0109152

[21] W. Marciano, Phys. Rev. Lett. 52, 489 (1984); T. Banks, M. Dine and M.R. Douglas, Phys.Rev.Lett. 88 (2002) 131301; P. Langacker, G. Segre and M. Strassler, Phys.Lett. B528 (2002) 121-128; X. Calmet and H. Fritzsch, hep-ph/0112110

[22] M.J. Drinkwater, J.K. Webb, J.D. Barrow, V.V. Flambaum, MNRAS 295, 457 (1998).

[23] J. Moffat, Int. J. Mod. Phys. D 2, 351 (1993).

[24] S. Landau, P. Sisterna & H. Vucetich, Phys. Rev. D63, 081303(R) (2001)