Is it e or is it c? Experimental Tests of Varying Alpha

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Is the recent evidence for a time-varying fine structure 'constant' α to be interpreted as a varying e, c, $\hbar$, or a combination thereof? We consider the simplest varying electric charge and varying speed of light theories (VSL) and prove that for the same type of dark matter they predict opposite senses of variation in α over cosmological times. We also show that unlike varying e theories, VSL theories do not predict violations of the weak equivalence principle (WEP). Varying e theories which explain astronomical inferences of varying α predict WEP violations only an order of magnitude smaller than existing Eötvös experiment limits but could be decisively tested by STEP. We finally exhibit a set of atomic-clock and related experiments for which *all* (hyperbolic) varying α theories predict non-null results. They provide independent tests of the recent astronomical evidence.

The possibility that the fine structure constant $\alpha \equiv e^2/\hbar c$ might be a dynamical variable has attracted considerable attention following the observations of Webb et al [1–3]. These observations use a powerful new many-multiplet technique to extract a larger fraction of the information encoded in quasar (QSO) absorption spectra at medium redshift than traditional doublet studies. They study energy differences between relativistic transitions to many different ground states and compare observations with laboratory measurements and many-body computations. The continuing trend of these results is that the value of α was *lower* in the past, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ for $z \approx 0.5 - 3.5$.

It is clear that such a discovery, if correct, has important implications for the foundations of physics, but pinpointing the precise conclusions to be drawn is more controversial. While it is reassuring that α was lower rather than higher in the past, so that QED remains perturbative, the full implications for unification are still unclear [4,5], and we shall not discuss them further here. These results also raise the question: which of e, $\hbar$ and c might be responsible for any observed change in α and what operational meaning should be attributed to such a determination? Undoubtedly, in the sense of [6], one has to make an operationally "meaningless" choice of which dimensional constant is to become a dynamical variable. Yet, in practice this choice is never arbitrary; it is clearly dictated by simplicity once the detailed dynamics of the theory have been established. Here, we argue that the dynamics have unambiguous observational implications: a combination of experiment and simplicity therefore selects one member of a dimensionless combination (α) of dimensional constants (e, $\hbar$ and c) to which we should preferentially ascribe its space-time variation. We will present a number of clear experimental tests which can distinguish rival theories of α variation which are expressed through explicit change in e or c. Existing theories will be used as examples.

Several theoretical contexts for the Webb et al results have been explored. Sandvik, Magueijo and Barrow [7] have recently proposed a varying electric charge theory (BSBM), inspired by an earlier construction of Bekenstein [8]. A supersymmetric version of this theory was created in [9]. Various dilatonic alternatives, in which all coupling constants vary as a function of a single field, may also be considered (including dilaton couplings to the cosmological constant [9]). Other candidates to explain variations in α are the varying speed of light (VSL) theories [10–15], which also offer an alternative to inflation for solving cosmological problems. As archetypal examples we take the BSBM theory [7] and the VSL theory presented in [15]. By introducing an appropriate change of units we can turn VSL into a constant c theory, but the dynamics will then look unnecessarily complicated; likewise BSBM can be rephrased as a constant e, varying c theory, with a concomitant increase in complexity. This is why we say that BSBM is a varying e theory while the theory in [15] is a VSL theory: dynamics fixes the choice. Crucially, the dynamics also have unambiguous observational implications. We will show that with standard dark matter VSL predicts an *increasing* α, as a function of cosmological time. By contrast, BSBM predicts a *decreasing* α, a conclusion which can only be reversed by a different choice of dark matter composition, as explained in [7]. This is a striking difference, but pending the determination of the nature of the dark matter one can use both BSBM and VSL to fit the Webb et al results. The same remark applies to other cosmological tests, such as constraints arising from the cosmic microwave background (CMB) and Big Bang Nucleosynthesis (BBN) [24,25].

However, BSBM and VSL theories also make different predictions regarding *spatial* variations in α near massive objects. Due to these variations all changing-α theories predict a 'fifth force' effect [7–9,26,27], but we will see that the exact details can distinguish between BSBM and VSL. In BSBM theory the fifth force induces an anomalous acceleration which, unlike gravity, depends on the material composition of the test particle and so violates the weak equivalence principle (WEP). The VSL theories, on the other hand, are consistent with the WEP, as first noted by Moffat [27].

The exact level of WEP violation predicted by BSBM depends upon an unsolved problem in nuclear and hadronic physics: how much of the mass-energy of nuclei is of electrostatic nature? As yet, there is no reliable answer to this question [28] but we can still estimate the magnitude of WEP violation, which reveals that the BSBM theory is marginally consistent with current Eötvös experiments. However, the next generation of WEP tests, such as the STEP project [29], will easily be sensitive enough to detect violations of the WEP as predicted by BSBM even by the most conservative estimates. Should violations be observed, it should be seen as a success for varying e theories. If not, then we must narrow our interest to VSL theories in order to accommodate observational signals of varying α. Thus, space experiments such as STEP can provide an independent experimental test of any astronomical evidence for varying α, and decide between a varying e or c interpretation.

We start by describing briefly the two theories to be used as exemplars. In the BSBM varying α theory, the quantities c and $\hbar$ are taken to be constant, while e varies as a function of a real scalar field ψ, with $e = e_0 e^{\psi}$. As shown in [19], it is possible to rewrite this theory in such a way that ψ only couples to the free electromagnetic lagrangian $\mathcal{L}_{em}$. The field tensor $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ and the covariant derivatives $D_\mu = \partial_\mu + ie_0 a_\mu$ then do not contain ψ, and the action takes the form:

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_g + \mathcal{L}_{mat} + \mathcal{L}_\psi + \mathcal{L}_{em} e^{-2\psi} \right), \quad (1)$$

where $\mathcal{L}_\psi = -\frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi$, $\mathcal{L}_{em} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$, and $\mathcal{L}_{mat}$ (the lagrangian of all matter fields apart from $\mathcal{L}_{em}$) does not depend on ψ. The gravitational lagrangian is the usual $\mathcal{L}_g = \frac{1}{16\pi G}R$, with R the curvature scalar.

In contrast, the covariant VSL theory proposed in [15] assumes that c varies, and builds the simplest dynamics on this premise, which is equivalent to a choice of a system of units. It assumes that $c = c_0 e^{\chi}$ (with χ another real scalar field) and that the *full* matter Lagrangian $\mathcal{L}_M$ does not contain χ. Up to a free parameter, q, this assumption fixes how all matter couplings scale with c; in particular, one has for all interactions i associated with gauge charges e_i that $\alpha_i \propto e_i \propto \hbar c \propto c^q$. The action is [23]

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_g + \mathcal{L}_\chi + \mathcal{L}_M e^{b\chi} \right), \quad (2)$$

with $\mathcal{L}_\chi = -\frac{\omega}{2}\partial_\mu\chi\partial^\mu\chi$, and $\mathcal{L}_g$ is as given above. It was shown in [15] that only when $b + q \neq 0$ can these theories be conformally mapped into dilaton theories, and into Brans-Dicke theories only when $q = 0$. This theory has an obvious novelty when compared to BSBM: all α_i are variable. However, recent cosmological variations in non-electromagnetic α_i are beyond the reach of current direct astrophysical observations. Hence for the purpose of this Letter we shall ignore their consequences.

Varying the action with respect to the metric leads to straightforward generalizations of Einstein's equations [15,7]. Variation with respect to the new scalar fields leads to dynamical equations for α. For small variations, $\delta\alpha/\alpha \ll 1$, these are:

$$\Box \frac{\delta\alpha}{\alpha} = \frac{4}{\omega}\mathcal{L}_{em} \quad (3)$$

for BSBM, and

$$\Box \frac{\delta\alpha}{\alpha} = -\frac{bq}{\omega}\mathcal{L}_M \quad (4)$$

for VSL. In both cases the right-hand side is zero for relativistic matter, predicting negligible variations in α during the radiation-dominated cosmological epoch. Two striking differences appear in the matter epoch, when the RHS becomes non-negligible, in both the coupling parameters and the driving source $\mathcal{L}$. The requirement that the fields χ and ψ have a positive definite energy forces $\omega > 0$. This fixes the sign of the coupling for BSBM ($4/\omega$) but not for VSL ($-bq/\omega$). The source $\mathcal{L}$ is also different for each theory and is parameterized by different ratios determined by the dark matter: $\zeta = \mathcal{L}_{em}/\rho$ for BSBM, and $\xi = \mathcal{L}_M/\rho$ for VSL.

The value of ζ for baryonic and dark matter has been disputed [26,9,7]. It is the difference between the percentage of mass in electrostatic and magnetostatic forms. As explained in [7], we can at most *estimate* this quantity for neutrons and protons, with $\zeta_n \approx \zeta_p \sim 10^{-4}$. We may expect that for baryonic matter $\zeta \sim 10^{-4}$, with composition-dependent variations of the same order. The value of ζ for the dark matter, for all we know, could be anything between -1 and 1. Superconducting cosmic strings, or magnetic monopoles, display a *negative* ζ, unlike more conventional dark matter. On the other hand it was argued in [15] that the value of ξ (characterizing the VSL dynamics in the matter epoch) is -1 for all non-relativistic matter. This is equivalent to requiring that non-relativistic matter is dominated by its potential energy (including rest mass) rather than by its kinetic energy T. We shall use this fact in the rest of the paper although it is not essential for most of what follows.

It is clear that the only way to obtain a cosmologically increasing α in BSBM is with $\zeta < 0$, i.e with unusual dark matter, in which magnetic energy dominates over electrostatic energy. In [7] we showed that fitting the Webb et al results requires $\zeta_m/\omega = -2 \pm 1 \times 10^{-4}$, where ζ_m is weighted by the necessary fractions of dark and baryonic matter. On the other hand VSL theory fits the Webb et al results with $bq/\omega = -8 \times 10^{-4}$, for all types of dark matter. Hence, if we were to determine that $\zeta > 0$ for the dark matter in the universe, we could experimentally rule out BSBM but not VSL. This is just one way in which the question in the title of this paper could be answered.

However, pending identifying the dark matter, we may still answer this question by looking at spatial variations

in α. In all causal varying-α theories defined by a wave equation the observed redshift dependence of α requires there also to be spatial variations near compact massive bodies [18,7]. The relevant equations may be obtained by dropping the time dependence in (3) and (4). Then, a linearized spherically symmetric solution in the vicinity of an object with mass M_s and $\zeta = \zeta_s$ is

$$\frac{\delta\alpha}{\alpha} = -\frac{\zeta_s}{\omega}\frac{M_s}{\pi r} \approx 2 \times 10^{-4}\frac{\zeta_s}{\zeta_m}\frac{M_s}{\pi r} \qquad (5)$$

for BSBM

$$\frac{\delta\alpha}{\alpha} = -\frac{bq}{\omega}\frac{M_s}{4\pi r} \approx 2 \times 10^{-4}\frac{M_s}{\pi r} \; . \qquad (6)$$

for VSL. We note that the level of spatial variations in BSBM, given [2,3], depends on the nature of the dark matter (the ratio ζ_s/ζ_m), whereas for VSL it does not. In VSL, α *increases* near compact objects (with decreasing c if $q < 0$, with increasing c if $q > 0$) but in BSBM α *decreases* (since $\zeta_m < 0$ and $\zeta_s > 0$). In VSL theories, near a black hole α could become much larger than 1, so that electromagnetism would become non-perturbative with dramatic consequences for the physics of black holes. In BSBM precisely the opposite happens: electromagnetism switches off.

Spatial variations lead to a number of observable effects which sharply distinguish between VSL and BSBM. Most obviously α could be measured in absorption lines from compact objects, as explained in [15,7]. More subtly, alpha gradients induce a 'fifth force' effect. In order to compute this force one must model ζ or ξ for test bodies. In BSBM the test-particle lagrangian may be split as $\mathcal{L}_t = \mathcal{L}_m + e^{-2\psi}\mathcal{L}_{em}$. Variation with respect to the metric leads to a similar split of the stress-energy tensor, producing an energy density of the form $\rho((1-\zeta_t) + \zeta_t e^{-2\psi})$, and so a mass of $m((1-\zeta_t) + \zeta_t e^{-2\psi})$, (assuming electric fields dominate). In order to preserve their ratios of $\zeta_t = \mathcal{L}_{em}/\rho$ test particles may thus be represented by

$$\mathcal{L}(y) = -\int d\tau \, m((1-\zeta_t) - \zeta_t e^{-2\psi})[-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu]^{\frac{1}{2}}\frac{\delta(x-y)}{\sqrt{-g}} \qquad (7)$$

where over-dots are derivatives with respect to the proper time τ. This leads to equations of motion:

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + \frac{2\zeta_t e^{-2\psi}}{(1-\zeta_t) - \zeta_t e^{-2\psi}}\partial^\mu\psi = 0 \qquad (8)$$

which in the non-relativistic limit (with $\zeta_t \ll 1$) reduce to

$$\frac{d^2 x^i}{dt^2} = -\nabla_i\phi - 2\zeta_t\nabla_i\psi \; , \qquad (9)$$

where ϕ is the gravitational potential. Thus we predict an anomalous acceleration:

$$a = \frac{M_s}{r^2}\left(1 + \frac{\zeta_s\zeta_t}{\omega\pi}\right) \qquad (10)$$

Violations of the WEP occur because ζ_t is substance dependent. For two test bodies with ζ_1 and ζ_2 the Eötvös parameter is:

$$\eta \equiv \frac{2|a_1 - a_2|}{a_1 + a_2} = \frac{\zeta_s|\zeta_1 - \zeta_2|}{\omega\pi}. \qquad (11)$$

This can be written more conveniently as the product of the following 3 factors:

$$\eta = \left(\frac{\zeta_E|\zeta_1 - \zeta_2|}{\pi\zeta_p}\right)\left(\frac{\zeta_p}{\zeta_m}\right)\left(\frac{\zeta_m}{\omega}\right). \qquad (12)$$

The last factor is the coupling that determines cosmological time variations in α, and using the results [2,3] is best fitted to be $\zeta_m/\omega \approx -10^{-4}$. If we take $\zeta_n \approx \zeta_p \approx |\zeta_p - \zeta_n| = \mathcal{O}(10^{-4})$ then for typical substances the first factor is $\approx 10^{-5}$. Hence, we need $\zeta_m = \mathcal{O}(1)$ to produce $\eta = \mathcal{O}(10^{-13})$, just an order of magnitude below existing experimental bounds.

In contrast to this VSL theories predict that for all test particles

$$\mathcal{L}(y) = -\int d\tau \, me^{b\chi}[-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu]^{\frac{1}{2}}\frac{\delta(x-y)}{\sqrt{-g}}, \qquad (13)$$

where we have assumed $\xi = -1$. This leads to an anomalous acceleration of equal magnitude for all test particles, so that there are no WEP violations. This new acceleration does imply corrections to the standard tests of general relativity, such as the precession of Mercury's perihelion, light deflection and radar echo time-delay [20,18]. These were studied in [18] and impose the undemanding constraint of $b^2/\omega < 10^{-2}$ [30]. Therefore we conclude that an increase of about an order of magnitude in the experimental sensitivity to non-zero η would decide between the BSBM and VSL theories.

Webb et al [1–3] caution that their results might be due to some uninvestigated systematic effect. For this reason it is important to seek independent observational verification. Direct measurement of WEP violations at the predicted level could be seen as a direct confirmation of the source of the astronomical results. Spatial fluctuations in α could also be directly mapped from spectroscopy of lines formed in very compact objects or their accretion disks [7,18]. But more realistically we note that Earth-based atomic-clock experiments could also measure these fluctuations. Atomic clocks tick at a rate $\tau^{-1} \propto (E_e\alpha^2)/\hbar$, where E_e is the electron rest energy. Hence atomic-clock experiments able to measure gravitational redshifts will suffer from an extra effect: in BSBM theories these clocks tick slower in gravitational wells, with $\tau \propto 1/\alpha^2$, whereas in VSL $\tau \propto 1/c^{2q+1} \propto 1/\alpha^{2+1/q}$. Any hyperbolic varying-α theory explaining [2,3] should predict a similar effect.

In general, any gravitational-redshift experiment should be sensitive to a varying α. For example, the

Pound-Rebka-Snyder experiment uses the Mössbauer effect to produce a narrow resonance line from γ-ray photons emitted by radioactive isotopes. The effect has been used to observe gravitational redshifts, but the emitted photon's energy also depends upon α. For small variations in α the energy shift is $\delta E/E = C\delta\alpha/\alpha$ with C of order 1 (but not very well known). A similar effect will occur in experiments using Rydberg lines, with a shift in wavelength $\delta\lambda/\lambda = -2(\delta\alpha/\alpha)$ (for both VSL and BSBM theories). Once the photon is emitted, varying-α theories predict no extra redshift for free-flying photons (since $\mathcal{L} = 0$ for photons). However, the observed gravitational redshift of frequencies takes the form $\delta\nu/\nu = (1 + \alpha_{PPN})\delta\phi$, with a non-zero PPN parameter α_{PPN} induced *at emission*. Using (5) we find that for BSBM theory $\alpha_{PPN} = 2\zeta_s/(\pi\omega)$, with the quasar data [2,3] then implying that $\alpha_{PPN} \approx 10^{-8}$. For VSL theory care must be taken, because $\delta\lambda/\lambda$, $\delta\nu/\nu$ and $\delta E/E$ are distinct quantities. Defining α_{PPN} in terms of frequency in the conventional way and using (6) we have that $\alpha_{PPN} \approx (2 + q^{-1})bq/(4\pi\omega) \approx -(2 + q^{-1})10^{-5}$. Hence BSBM theory predicts a stronger redshift than general relativity, with corrections of order 10^{-8}. If $q \ll 1$, VSL theory predicts a weaker redshift effect with corrections of order 10^{-5}; but this conclusion is changed if $q \approx -1/2$. Both BSBM and VSL theories are consistent with the current bound of $\alpha_{PPN} < 10^{-4}$ [20]. Any causal varying-α theory should predict a non-zero correction to the relativistic redshift formulae.

In summary, we have explained how a combination of experiment and common sense may distinguish a varying c from a varying e. Using only minimal versions of such theories, we have shown how they can be distinguished by weak equivalence principle violations, by the type of dark matter required to give the variations inferred from quasar observations [2,3], and by gravitational-redshift experiments. In non-minimal varying-e and -c theories, the distinguishing observational signatures should be even more obvious. For instance, if Lorentz invariance were found to be broken, [21,22], then a varying-c theory would provide a better framework for expressing variations in α.

Finally, we note that the experiments proposed in this paper are by no means the only discriminators between varying-e and -c expressions of a varying α. In [17] the authors examined black hole thermodynamics, by changing the values of e and c in their description of black hole thermodynamics (which, however, may be too simplistic [18]). In this context they found that interpreting a varying α as varying e or c leads to opposite black-hole dynamics, with a varying-e contradicting the second law of thermodynamics. In principle one could test whether or not black hole areas always increase with time in the next generation of gravitational-wave observatories. Like the various experiments described in this paper, this is experimentally unambiguous, since the ratio of two areas is dimensionless.

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