Foundations of Physics
https://doi.org/10.1007/s10701-019-00296-6

Higgs Naturalness and Renormalized Parameters

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Received: 5 December 2018 / Accepted: 4 September 2019

Abstract
A recently popular formulation of the Higgs naturalness principle prohibits delicate cancellations between running renormalized Higgs mass parameters and EFT matching corrections, by contrast with the principle's original formulation, which prohibits delicate cancellations between the bare Higgs mass parameter and its quantum corrections. While the need for this latter cancellation is sometimes viewed as unproblematic since bare parameters are thought by some to be divergent and unphysical, renormalized parameters are finite and measurable, and the need for delicate cancellations between the renormalized Higgs mass parameter and EFT matching corrections is therefore considered by some to constitute a more salient formulation of the Higgs naturalness problem. Here, we argue that to the contrary, the need for fine tuning of the renormalized Higgs mass parameter is an eliminable, unphysical artifact of renormalization scheme, and that this severely weakens the grounds for regarding it as a problematic instance of fine tuning. In doing so, we highlight what we take to be a number of important conceptual lessons about the physical interpretation of model parameters in QFT.

Keywords Higgs · Naturalness · Fine tuning · Renormalization · Effective field theory

1 Introduction

The Higgs naturalness principle, which was used to justify the expectation that the Large Hadron Collider (LHC) would turn up hints of new physics beyond the Standard Model (SM), is often understood as the requirement that free parameters of the SM should not be fine tuned. In the case of the Higgs, the fine-tuning allegation often refers to delicate cancellations between the bare Higgs mass and its quantum corrections.

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Published online: 20 September 2019

Assuming that the SM has a physical cutoff of order $\Lambda \gg m_H$, where $m_H \approx 125\,\text{GeV}$ is the measured value of the Higgs pole mass, this cancellation is of the order of m_H^2/Λ^2. The absence of new physics up to scales above 1 TeV suggests that the naturalness principle in this sense is violated to at least some degree by the SM.

More precisely, this formulation of the Higgs fine-tuning naturalness problem rests on the observation that the leading contribution to the perturbative one-loop expansion of the (squared) Higgs pole mass m_p^2 in terms of bare SM parameters gives

$$
\begin{aligned}
m_p^2 &= m_0^2 + \delta m^2 \\
&= m_0^2 - \frac{y_{t0}^2}{8\pi^2}\Lambda_{\text{SM}}^2 + \cdots \\
&= \Lambda_{\text{SM}}^2\left(\tilde{m}_0 - \frac{y_{t0}^2}{8\pi^2}\right) + \cdots
\end{aligned}
\tag{1}
$$

where y_{t0} is the bare top quark Yukawa coupling, Λ_{SM} the SM's physical cutoff (that is, the scale at which it ceases to be empirically valid),[1] m_0^2 is the bare Higgs mass, and $\tilde{m}_0^2 \equiv \frac{\tilde{m}_0^2}{\Lambda_{\text{SM}}^2}$ is the dimensionless bare Higgs mass in units of Λ_{SM}.[2] Measurements at the LHC have further determined that $m_p^2 = (125\text{ GeV})^2$, and that $(1 \times 10^3\text{ GeV})^2 \lesssim \Lambda_{\text{SM}}^2 \lesssim (10^{19}\text{ GeV})^2$. While the lower limit $(1 \times 10^3\text{ GeV})^2$ has been set on the basis of LHC measurements, the upper limit, equal to the Planck scale, is set by theoretical expectations regarding the scales at which quantum gravitational effects can no longer be ignored. Together, these facts imply that the bare Higgs mass m_0^2 must be "fine tuned" in order to recover the measured value of the physical, pole mass of the Higgs. The minimal degree of fine tuning required to recover the measured pole mass $m_p^2 \sim \mathcal{O}(10^4)$ increases with the empirically established lower bound on Λ_{SM}^2. At the lower limit of the allowed range for Λ_{SM}, $\Lambda_{\text{SM}} = 1 \times 10^3$ GeV, the relation (1) gives $\mathcal{O}(10^4) = \mathcal{O}(10^6) - \mathcal{O}(10^6)$ while at the upper end of this range, $\Lambda_{\text{SM}} = 10^{19}$ GeV, it gives $\mathcal{O}(10^4) = \mathcal{O}(10^{38}) - \mathcal{O}(10^{38})$. That is, the "best-case" scenario, where $\Lambda_{\text{SM}} = 1 \times 10^3$ GeV, requires that m_0^2 and δm^2 cancel to one part in 10^2 to 10^3. The "worst-case" scenario, where $\Lambda_{\text{SM}} = 10^{19}$ GeV, requires that m_0^2 and δm^2 cancel to one part in 10^{34}. For many physicists, cancellation to one part in 10^2–10^3 already begins to be problematic.

There exist two conflicting attitudes toward the delicate cancellation between m_0^2 and δm^2 required to recover the measured value of m_p^2 for large values of Λ_{SM}. The first view is that this cancellation requires an unlikely coincidence between the bare Higgs mass and the bare parameters that enter into the calculation of δm^2. This point of view, advocated by Susskind, 't Hooft, and Giudice, rests on the interpretation of the bare Higgs mass and other bare parameters of the SM as fundamental parameters—i.e., the single, physically correct, underlying parametrization of the Standard Model.

[1] At leading non-trivial order perturbation theory, one can take y_{t0} to be either a bare coupling or a renormalized coupling, since the corresponding expressions only differ at higher orders in perturbation theory.

[2] See, for example, Martin [4] for a formulation of the Higgs naturalness problem along these lines.

If m_0^2, Λ, and y_{t0} are all independent parameters of the SM, then for large Λ it seems like an odd coincidence that m_0^2 and $\frac{y_{t0}^2}{8\pi^2}\Lambda^2$ should agree so precisely since these quantities are calculated from independent parameters of the model. The fine tuning problem can be understood as the need either to avoid the such a cancellation—e.g., through a small cutoff Λ—or to explain the origin of this cancellation in terms of deeper physical theories beyond the SM (BSM).

The second view is that this original formulation of the naturalness principle, as a prohibition against fine tuning of bare parameters, is not a well-motivated constraint on SM. Rationales supporting this claim differ, but are rooted in the belief that the values of bare parameters at the SM's physical cutoff are not to be taken seriously as representing real physical features of the world, but are in some sense mere artifacts of our mathematical description. For this reason, delicate cancellations between bare parameters and quantum corrections do not constitute a mysterious coincidence in urgent need of the special attention that they have received. Perhaps the first articulation of this view was by Wetterich [10]:

> fine tuning of bare parameters is not really the relevant problem: we do not need to know the exact formal relation between physical and bare parameters (which furthermore depends on the regularization scheme), and it is not important if some particular expansion method needs fine-tuning in the bare parameters or not. The relevant parameters are the physical parameters, since any predictions of a model must finally be expressed in terms of these.

More recently, a similar view has been taken up by Bianchi and Rovelli [2] in the context of the cosmological constant problem, which is structurally very similar to the Higgs naturalness problem. Citing reasons provided by Wetterich, Williams [11] also says that fine tuning of bare parameters is not really a problem. Most recently, Rosaler and Harlander [6] attempt to provide a more detailed elaboration of the assumptions underlying Wetterich's claim that fine tuning is an artifact of a particular expansion method, arguing that Wetterich's claim carries important implications for the mathematical formulation and physical interpretation of Wilsonian effective field theories (EFTs).

Within the literature on high energy effective field theory, an alternative formulation of the naturalness principle, which precludes fine tuning of *renormalized* Higgs mass parameters, has become prevalent. Ostensibly, this alternative formulation provides a way of rescuing the naturalness principle from worries about the legitimacy of formulations in terms of bare parameters. For example, Barbieri [1] writes,

> After all—one says sometimes—aren't we supposed to talk only of physical renormalized quantities, with all divergences suitably reabsorbed? … Indeed a neater way to state the naturalness problem is in terms of the renormalized running Higgs boson mass. … The quadratic divergence of the Higgs mass is not the problem *per se*, but the sign of the sensitivity of the Higgs mass to any threshold encountered at higher energy scales.

Thus, Barbieri appears to regard formulations of the naturalness problem in terms of renormalized parameters as more salient. After dismissing "bare" formulations of the naturalness principle, Wetterich [10] explains that "A second, more serious possibility of a fine-tuning problem involves the relation between physical parameters at short and long distances." He writes further,

> It is the beauty of renormalizable quantum field theories that the model can be completely parametrized by an appropriate set of quantities which can, in principle, be measured. We will call these the physical parameters. Predictions on physical quantities can then be given in terms of measured physical parameters. ... For our case of interest—spontaneous symmetry breaking in a scalar field theory—the relevant physical parameters are scalar mass terms $\mu^2(M^2)$ and self couplings $\lambda(M^2)$ normalized at some given energy scale M.

It seems clear here that Wetterich's "physical parameters" can be understood to include a running renormalized Higgs mass parameter, as in Barbieri's discussion.

Several of the more recent presentations of the Higgs naturalness problem adopt just such a formulation, which concerns the matching corrections to the running renormalized scalar mass of a light scalar field when a heavy particle is integrated out of a theory including both the light scalar and the heavy particle; see e.g., Barbieri [1], Skiba [8], Schwartz [7], Craig [3]. It is such formulations of the Higgs naturalness principle that we primarily consider here.

Our central goal in this article is to provide one strong reason for skepticism about the claim that delicate cancelations relating renormalized scalar mass parameters constitute a problematic instance of fine tuning in need of special attention. The argument rests on the fact that the need for such cancellations depends entirely on one's choice of renormalization scheme, which is purely a matter of arbitrary mathematical convention (similar in some respects to a choice of coordinate system). Unlike the presentations of the "renormalized" formulation of the Higgs naturalness principle, we explain how the delicate cancellations in the relationship between renormalized scalar mass parameters can be removed entirely through a change of renormalization scheme, without altering the physical contents of the theory.[3] The fact that the choice of renormalization scheme is merely a matter of convention, and the fact that the delicate cancellations occur only in some renormalization schemes but not others, undermine the notion that these delicate cancellations rest on a mysterious physical coincidence demanding special attention in the search for new theories beyond the Standard Model. However, we stress that our aim here is not to dismiss *all* fine tuning problems as pseudo problems, and offer examples of other fine tuning that we do regard as genuinely problematic. Our aim is only to explain why the particular cancellation between renormalized running scalar masses examined here, like the cancellation between bare scalar masses and quantum corrections, does not itself fall into the category of genuine fine tuning problems.

A running theme throughout the discussion is that care must be taken in the physical interpretation of model parameters; one must be wary of ascribing too much physical

[3] On some usages of the term "renormalization scheme," different renormalization schemes are associated with different choices of renormalization scale μ, and the difference between on-shell, minimal subtraction, off-shell momentum subtraction, etc. is a difference between different *families* of renormalization scheme. On the usage we adopt here, on-shell, minimal subtraction, off-shell momentum subtraction etc. are themselves referred to as different renormalization schemes; variation of the parameter μ within such a scheme does *not* constitute a change of renormalization scheme, but only a change of renormalization scale.

significance to the numerical values of these parameters, particularly when these values depend on arbitrary, unphysical choices of mathematical convention. We describe various grades of physical interpretability of model parameter values, emphasizing a distinctive feature of model parameters in QFT—namely that many numerically distinct values for model parameters in an EFT generate exactly physically equivalent predictions for physical quantities/observables.

The discussion is outlined as follows. In Sect. 2, we distinguish a number of different types of scenario that arise in the physical interpretation of model parameter values in physics. In Sect. 3, we emphasize a distinctive feature of the relationship between parameters and observables in QFT models, which bears heavily on our interpretation of delicate cancellations associated with the Higgs mass. In Sect. 4, we explain why worries about delicate cancellations involving renormalized running Higgs mass parameters are subject to much the same fate as worries about delicate cancellations involving the bare Higgs mass parameter—namely, that these cancellations reflect an arbitrary choice of mathematical convention and can be eliminated through a different, physically equivalent choice of convention. Section 5 is the Conclusion.

2 Physical Interpretation of Model Parameters

It is at the core of physics that the infinite number of observables and the relations among them can be parametrized by a finite (and relatively small) number of parameters. Typically, one associates these parameters with certain "physical properties" (mass, charge) of the underlying objects. QFT, however, teaches us that such associations may sometimes be misguided, as we will describe in this and the following section.

The Higgs naturalness problem is often characterized as arising from an "unlikely" cancellation between parameters, or quantities calculated from these parameters, that urgently demands explanation by BSM models. However, it is important to emphasize that in physics one always wants a deeper explanation of the parameter values in one's models, irrespective of whether there exist conspicuous similarities between values of these parameters, or whether these parameters are "natural" and "typical" of the model's parameter space. In the case of the Higgs naturalness principle, we would like BSM theories to explain not only the value of the Higgs mass, but the values of all SM parameters. What is supposed to distinguish fine tuning problems is not that they require explanation of a theory's parameters, but the special urgency with which fine tuned parameters appear to demand such explanation, and the special clues that fine tuned parameters are thought to give as to the nature of new physics. The naturalness principle encourages special focus on the explanation of fine tuned parameters over non-fine-tuned parameters, ostensibly because they provide hints of BSM physics that are not contained in other SM parameters.

Given that we expect the values of all SM parameters to be explained by some deeper theory anyway, does it make sense to think that the need to explain the values of certain parameters is somehow more urgent than the need to explain others? Although we argue here against the notion that the Higgs mass poses an especially glaring demand for explanation that other SM parameters do not, there exist less controversial cases in the history of physics where it seems clear that the choice to focus on explaining certain parameters in particular was justified. For example, it was reasonable from the vantage point of early atomic theory to inquire as to the origin of the approximate equality between the proton and neutron masses, or the equality of the electron and proton charges, which were independent free parameters in early models of the atom. Such mysterious confluences spoke to the existence of deeper, as yet unknown mechanisms that served to explain them, which were ultimately revealed by more fundamental descriptions of these particles. The smallness of the observed SM Higgs mass is thought to rest on a similarly glaring and unexplained confluence between independent model parameters. However, we argue below that the need for delicate cancellations involving renormalized scalar mass parameters does not constitute the sort of conspicuous feature of model parameters that the approximate equality of proton and neutron masses once did.

The notion that that there is something problematic or coincidental-seeming in the delicate cancellations associated with the Higgs mass rests in part on the association of concrete physical properties to the values of these parameters—why did nature choose these properties, which are ostensibly independent of one another in the framework of our existing theories, to be so close in value? Here, we underscore several general lessons about the interpretation of model parameters, both in general and in QFT specifically, that undermine this interpretation of these cancellations. From the alternative perspective outlined here, these cancellations do not reflect nature's choices, but rather our own arbitrary choices of convention in the mathematical description of nature.

In the remainder of this section and in the next, we distinguish three categories of physical interpretability of model parameters, illustrating each with an example. The first category consists of model parameters whose relationship to physical, measurable quantities—what are sometimes referred to as the model's observables—is relatively simple, enabling one to attach an intuitive physical interpretation to these parameters. The second category consists of model parameters who relationship to observables is highly complicated and non-linear, making it difficult to attach any simple or intuitive physical interpretation to the parameter values. The boundary between the first and second categories is not intended to be sharp: in general, the physical meaning of a model parameter is determined by its place in the mathematical structure of the model and its relationship to the model's observables; the intuitiveness of the parameter's physical interpretation will vary with the complexity of its relationship to observables. The third category of interpretability consists of cases in which the relationship between model parameters and observables is non-unique in that there exist multiple, and perhaps infinite, parametrizations of the model that generate exactly the same physical predictions. In this case, the numerical values of the parameters are attached to a certain arbitrary and physically irrelevant choice of convention, akin to a choice of coordinates or gauge. This third category is discussed in detail in Sect. 3.

2.1 Physically Interpretable Model Parameters: The Classical Harmonic Oscillator

Let us consider a classical harmonic oscillator. Its Lagrangian can be written as

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2. \tag{2}$$

In the following, we assume that the units for time, length, and energy have been set by systems independent from the one above. The Lagrangian thus depends on two parameters, m and ω, and one function $x(t)$, where t is the time variable. The parameter m drops out of the equations of motion, but it is still not irrelevant since it determines the energy scale of the system. If we assume it to be of dimension mass, then x is measured in units of length. Thus, the Euler–Lagrange equation

$$\ddot{x} + \omega^2 x = 0 \tag{3}$$

describes general trajectories for the system. The solution of Eq. (3) tells us that the corresponding movement is periodic, and the independent parameter ω determines the frequency.[4] The parameter ω has a clear, intuitive physical interpretation: it is the proportion of a single cycle (measured in radians) that the object traverses in a single unit of time. Its simple mathematical relationship, $\omega = \frac{2\pi}{T}$ to a measurable quantity—i.e., the period T of the oscillator—is what allows for this intuitive physical interpretation.

2.2 Opacity in the Physical Interpretation of Model Parameters: Complex Systems

The intuitive interpretation of model parameters in terms of physical observables may be much more difficult, if not impossible in cases where the relationship between the parameter and the observable is highly complicated and non-linear. This already happens in classical physics in so-called "complex systems" involving non-linear interactions, for example, possibly among many different components. This also includes so-called emergent phenomena which should in principle be determined by fundamental properties of the system, but are too intricate to be accessible through realistic calculations. In many cases, such systems can be described through effective theories which introduce a new, effective set of degrees of freedom and associated parameters. But again, the relation between the effective and the fundamental description remains obscure. Popular examples of such systems from classical physics is the formation of hurricanes, the specific form of snow-flakes, or the dynamics of sand dunes.

We list these examples merely to show that in general one should not expect to be able to associate parameters of a theory to observables in an immediate way.

3 Non-uniqueness of Parameter Values in QFT

At the current level of understanding, the deepest level of explanation of non-gravitational phenomena is given by QFT, or more specifically, the Standard Model

[4] Note that, in order to measure anything about the system, it needs to couple to something else. For example, the oscillating mass(es) could carry electric charge, which lets us access the frequency ω of the system by the electromagnetic radiation it emits.

of elementary particle physics (SM), which is based on QFT. The SM itself has 19 parameters which are not explained, just like ω is not explained by the Lagrangian in Eq. (2). Of course, the majority of physicists expects that the SM parameters will at some point be explained by a deeper theory. In this section, we examine some of the important conceptual and mathematical differences between the relationship that holds between parameters and observables in QFT and this relationship in the context of other theories like classical and quantum mechanics. These differences impose important qualifications on the sense in which parameters of a QFT can be attached to intuitive physical properties of a system.

3.1 Renormalizable QFTs Without Cutoff

There is a severe qualitative difference between a classical Lagrangian and one that is formulated within QFT. The latter is usually formulated in terms of "bare" parameters, which cannot be assigned finite numerical values if finite predictions are to be extracted from the theory and the theory is defined on a 4-dimensional spacetime continuum. This is the case, for example, if we assume the SM to be the ultimate theory of the world so that it holds to arbitrarily high energies. We hasten to add that this does not correspond to the real world, first and foremost because the SM does not describe gravitational interactions. However, for the sake of the argument, let us assume for the moment that there is a renormalizable QFT which includes all possible observable phenomena up to arbitrarily high energies.

If one attempts to calculate the values of observables using finite values for the parameters, the theory generates non-sensical infinite predictions. It was one of the most remarkable achievements of twentieth century physics to understand that this does not prevent the theory from making sensible, and in fact extremely accurate predictions for physical observables (or rather, relations among them). The crucial step was to realize that the parameters of a theory (QFT or other) may simply play the role of book-keeping devices in the sense that they allow one to relate observables to one another, without directly measuring the parameters themselves. The intricacies of QFT will make sure that any relation between the observables is well-defined, even though the "bare" parameters of the Lagrangian are not.

In practical calculations, one typically deals with these infinities by introducing a regulator in order to allow for the formal manipulation of finite rather than infinite quantities. Let us denote this regulator by ϵ and assume that $\epsilon = 0$ corresponds to the original theory. For $\epsilon \neq 0$ one can formally assign finite values to auxiliary, so-called renormalized parameters, which are obtained by subtracting divergent terms (in the limit $\epsilon \to 0$) off the actual, "bare" parameters of the Lagrangian. Since divergent terms remain divergent when adding finite terms to them, this subtraction procedure has a great deal of arbitrariness, meaning that the numerical values assigned to "renormalized" parameters are highly arbitrary, which makes it difficult to assign any physical interpretation to them.

The differences among the various subtraction terms is what results in different "renormalization schemes". Some of these schemes are defined in a purely theoretical way, meaning that one devises a (theoretical) regulator, and defines the subtraction scheme solely in terms of this regulator, without reference to physical observables. One example of this is the "minimal subtraction (MS)" scheme, where the regulator ϵ is a deviation from four-dimensional space-time, and the subtraction terms are simply the poles in ϵ. Other renormalization schemes do refer to physical quantities by requiring that Green's functions assume specific functional forms in certain kinematical regions. For example, in the on-shell scheme of QED, the two-point function of the electron is required to have a pole at the physical mass of the electron, which leads to a specific subtraction term of the parameter m_e in the Lagrangian.

As we have just argued, the choice of the subtraction term is in principle arbitrary. The fact that in practice certain renormalization schemes are preferred over others has a purely technical origin—i.e., it is rooted in the fact that some schemes are more convenient for the purposes of performing certain calculations. For example, the perturbative series may be better behaved in one renormalization scheme than in the other. However, even if the MS scheme (or rather its more popular variant, the "modified MS", or $\overline{\text{MS}}$ scheme) has become most widely used in this context, it turns out that in certain cases, for example in specific kinematical regions, other schemes may yield a seemingly better perturbative expansion. But again, these are purely technical reasons; a truly "all-order" result will not depend on the renormalization scheme. That is, it is not the case that one renormalization scheme constitutes a more accurate representation of physical phenomena than the others; all schemes are physically equivalent.

To parametrize observables in the $\overline{\text{MS}}$ scheme, for example, one is obliged to choose a value for the unphysical scale parameter μ relative to which the values of renormalized parameters are specified. Changing the arbitrary reference point μ necessitates an adjustment in some finite set of renormalized parameters $g_r(\mu)$, given by the continuum RG flow, in order to maintain the values of observables. There exist other types of renormalized parametrization, associated with on-shell schemes, off-shell momentum subtraction schemes, and regulator-independent schemes, each of which generates a one-parameter continuum of parametrizations of the QFT's observables. In all cases, the invariance of observables $O_i(g_r(\mu); \mu)$ under changes of renormalization scale μ follows from the corresponding invariance of Green's functions,

$$\mu \frac{d}{d\mu} G(x_1, \ldots, x_n; g_r(\mu), \mu) = 0. \tag{4}$$

Thus, when expressed in terms of renormalized parameters, physical amplitudes are subject to two kinds of invariance: invariance under the change of renormalization scheme ($\overline{\text{MS}}$, on-shell, etc.), and invariance under change of scale μ within a particular renormalization scheme. These changes merely constitute a change in how we choose to represent and calculate physical quantities, and do not reflect any difference in the actual physics.

Note that, by contrast with the Wilsonian RG flow, which takes place in an infinite-dimensional space of bare parameters $g(\Lambda)$, as we discuss shortly, the continuum RG flow involves only a finite set of renormalized parameters $g_r(\mu)$. No single set of values for the parameters $g_r(\mu)$, associated with any particular choice of scale μ, constitutes the "true" or "physically correct" parametrization of the theory's amplitudes; all such parametrizations constitute physically equivalent representations of the same physical state of affairs. To be sure, some parametrizations are more convenient for the purpose of ensuring more rapid convergence of perturbation expansions; however, nature itself does not recognize the difference between different orders of perturbation theory (which is an artifact of our calculations) but only the value of the expanded amplitude itself.

3.2 Effective Field Theories with Finite Cutoff

While our discussion here primarily concerns formulations of the naturalness problem in terms of renormalized parameters, it is worthwhile to draw a connection here to earlier formulations of the naturalness problem in terms of finite-cutoff bare parametrizations, and also to our analysis of this formulation in [6]. However, in doing so we digress somewhat from the main thread of our discussion, which concerns "renormalized" formulations of the naturalness problem, and the reader may skip to Sect. 4 without disrupting the flow of the argument. This digression does however provide a second useful illustration of the manner in which the correspondence between the numerical values of a QFT's parameters and that QFT's observables is many-to-one.

Recall that up to now we have assumed that the QFT under consideration, as understood from the perspective of perturbation theory, is defined without a cutoff regulator. However, mathematical difficulties associated with the continuum limit of non-perturbative QFT, combined with the knowledge that all known realistic QFTs only describe phenomena up to some finite energy scale, motivate the introduction of a cutoff, resulting in what is called an effective field theory (EFT). Such a cutoff acts as a regulator, and therefore the bare parameters *can* be assigned finite numerical values in an EFT.

In fact, multiple distinct notions of "cutoff" exist within the context of EFT, and are often conflated. It is possible to distinguish between the "physical cutoff" of an EFT, and various types of "theoretical" cutoff that do not relate directly to the physical cutoff. The physical cutoff is the empirically determined, physical scale at which an EFT ceases to give accurate predictions—corresponding for example to the mass of a heavy particle not contained in the theory or to some fundamental granularity of spacetime. The empirically determined physical cutoff of an EFT is distinct from the parameter Λ that is employed as a variable cutoff regulator in path integral or loop integral expressions. This cutoff regulator is the same Λ that parametrizes Wilsonian renormalization group (RG) flows; its value is arbitrary and chosen as a matter of convention. Yet another notion of cutoff is the upper scale at which Wilsonian RG flows (and also renormalized RG flows) cease being mathematically well-defined. This occurs, for example, in the case where the theory possesses a Landau pole; in such cases, the coupling of the theory diverges at a finite value of the parameter Λ, and the theory is simply not mathematically defined above this scale. However, the scale at which the RG flows cease to be mathematically defined is unrelated to the physical cutoff, apart from the obvious requirement that the physical cutoff of an EFT lie below this theoretical cutoff. For more detailed discussion of the difference between these various notions of cutoff see [6].

In the EFT framework, the naturalness requirement can be formulated as a prohibition against fine tuning of bare parameters, or by saying that dimensional bare parameters should be given by the appropriate power of the physical cutoff, times a number of order one. Pole masses can then be "evaluated" by adding quantum corrections to the bare parameters of the theory. For fermions and gauge bosons, symmetry requirements determine the "appropriate power" of the cutoff to vanish, so that the dependence of the quantum corrections can be at most logarithmic. For the bare Higgs mass parameter, however, no such symmetry arguments apply, and we would expect a number of order Λ^2. Assuming Λ to be of the order of the Planck mass $M_{\mathrm{Planck}} \sim 10^{19}$ GeV, this means that the quantum corrections must balance the bare Higgs mass to a level of 10^{-34} in order to arrive at the observed physical mass of 125 GeV.

However, we should not forget that the assumption of a cutoff only postpones the previous discussion to a higher energy scale; in particular, the quest for an explanation of the nature and origin of the cutoff itself arises. As we argue here and in Sect. 4, the viability of naturalness-based arguments rests to a large extent on the nature of BSM physics, and on the precise physical origins of the SM's physical cutoff. Our main point in this article is that the existence of delicate cancellations involving the Higgs mass parameters does not in and of itself imply the existence of a coincidence or fine tuning problem.

For example, as we discuss in more detail in Sect. 4, the cutoff could be due to the existence of a heavy particle with mass $M \sim \Lambda$, which is not part of the original theory. If the new theory has no cutoff, the discussion above repeats itself: its bare parameters cannot be assigned finite numerical values, and thus do not allow for a physical interpretation. The bare Higgs mass of the EFT, on the other hand, would just be an intermediate, auxiliary parameter without any intuitive physical interpretation. From this point of view, the quantum effects to the Higgs mass should not be considered as being added to the bare Higgs mass of the EFT to arrive at the pole mass, but the other way around: the quantum corrections (which are then understood with an opposite sign) are added to the Higgs pole mass. Since the former are very large, and the latter is comparatively tiny, it is not surprising that the resulting bare Higgs mass in the EFT is so extremely close to the contribution of the quantum effects. It would be the same as being surprised about the fact that an elephant with an ant on its back is almost exactly as heavy as the same elephant without the ant.[5]

Let us consider a situation where the cutoff is not directly associated with the mass of a new heavy particle, but due to some other kind of new physics, such as a granularity of space-time etc. This would still raise the question for the underlying physics, because obviously, a QFT with a cutoff cannot describe physics of the cutoff itself. For example, in solid-state physics, one can derive a quantum theory of the lattice modes, but the physics of the individual components of the lattice itself (the atoms or molecules and their interactions outside the lattice) is not described by this theory.

[5] This line of argument is further developed in (see [6]).

So far, we have argued that, by no means do bare parameters need to be considered as more fundamental in any sense than renormalized parameters or physical observables. There is one point where bare parameters stand out of the rest of the parametrizations of a QFT. Consider the custodial symmetry of the SM which manifests itself in the following simple relation:

$$\rho \equiv \frac{g_1^2 + g_2^2}{g_2^2} \frac{M_W^2}{M_Z^2} = 1, \tag{5}$$

where M_W and M_Z are the bare gauge boson masses, and g_1 and g_2 the bare couplings of SU(2) and U(1), respectively. Of course, this relation propagates also to the renormalized parameters, but it is affected by quantum corrections. However, we can apply the same view on this observation as before: the bare parameters are abstract objects (book-keeping devices) which allow us to reduce complicated relations among observables to an efficient formalism, and relations like Eq. (5) are just part of this formalism. There is no need to ascribe to any reality content of the bare parameters.

It will be useful at this point to briefly review the argument of [6] for the case of a theory with a cutoff, because the renormalized parameters in such a theory share a number of features which occur also in a continuum theory. In a Wilsonian parametrization of an EFT, where one specifies the theory by some set of bare parameters g and regulates the theory with a finite cutoff Λ, one may calculate observables O as functions $O(g; \Lambda)$ of bare parameters g, understood as the coefficients in the path integral Lagrangian. Furthermore, one may adjust the value of Λ, and perform compensating adjustments to the parameters g (which thereby acquire a Λ dependence) so that the values of all observables O_i are left exactly unchanged. From this it follows that there exists a continuous range of finite parametrizations of the theory's observables $O_i(g(\Lambda); \Lambda)$, related by the Wilsonian RG flow. The invariance of observables under such reparametrizations, $\Lambda \frac{d}{d\Lambda} \mathcal{O}(g(\Lambda), \Lambda) = 0$, follows from the associated invariance of Green's functions $G(x_1, \ldots, x_n; g(\Lambda), \Lambda)$ from which observables are calculated,

$$\Lambda \frac{d}{d\Lambda} G(x_1, \ldots, x_n; g(\Lambda), \Lambda) = 0, \tag{6}$$

which in turn follows from the invariance of the path integral under these reparametrizations, $\Lambda \frac{d}{d\Lambda} Z[J = 0; g(\Lambda), \Lambda] = 0$, which is the defining criterion for Wilsonian RG flows. It is important to keep in mind that Eq. (6) concerns an RG flow in an infinite-dimensional parameter space, while (4) describes an RG flow in a finite-dimensional parameter space.

The relation (6) illustrates in the context of finite-cutoff bare parametrizations the same lesson that (4) serves to illustrate in the context of renormalized parametrizations: that the association between parameter values and observables in QFT is many-to-one. The association between values of QFT model parameters and a given physical state of affairs is therefore non-unique, and the choice of any single parametrization is imbued with a significant element of arbitrary convention. One must therefore be wary of ascribing too much physical significance to the numerical values of these parameters, and to mathematical relations (e.g., delicate cancellations) that occur in some parametrizations but not others, which may reveal more about how we've chosen to describe a physical system than about the system itself. The implications of this freedom for the interpretation of fine tuning of bare parameters are examined in detail in Ref. [6], where, elaborating a suggestion by Wetterich [10], we argue that the fine tuned cancellation between bare Higgs mass and quantum corrections may be interpreted as artificial by product of an arbitrarily chosen parametrization, associated with an arbitrary choice of the unphysical scale parameter Λ.

3.3 Summary

Let us summarize the discussion up to this point. A common formulation of the naturalness principle is in terms of a constraint on the allowed values for the bare parameters of a QFT such as the SM, which limits the amount of fine tuning that these parameters may exhibit. We emphasize that while model parameters in classical physics and quantum mechanics often have natural, straightforward interpretations as physical features of the system being described, in QFT this is less the case due to the specific and highly non-unique relation between the parameters (bare or renormalized) and observables. In a renormalizable QFT without cutoff, the bare parameters cannot even be associated with any finite numerical values. However, also in an EFT, there is no compelling reason to associate any physical interpretation with the bare parameters. This is suggested by the fact that there exists a vast space of physically equivalent bare parametrizations associated with different values of the scale Λ that generate exactly the same values for observables such as pole masses and cross sections.[6] The large element of conventionality that enters the values of these parameters makes their physical interpretation less immediate, and for this reason one must be careful in ascribing too much physical significance to their numerical values.

The physically salient quantities in any physical theory are the observables, where we understand an "observable" to be any physical quantity measurements of which can be used to fit the parameters of the theory that does not depend on arbitrary choices of mathematical convention such as choice of gauge, coordinate axes, or renormalization scheme. An observable is thus any quantity in the model that represents some feature in the physical world whose numerical value can in principle be determined through experiment and is not tied to any particular such choice of convention.

Given the wide range of parametrizations that exist for a QFT's observables, the assumption that a single set of values is distinguished as fundamental or as physically real imposes an additional element of metaphysical interpretation that is not strictly speaking needed to recover the empirical successes of QFT, or to make the process of generating QFT predictions mathematically well-defined within the Wilsonian formalism. The assumption that there are such matters of fact is analogous in this respect to the assumption prior to relativity theory that there are matters of fact about which objects are at rest and which are in motion, since these also are not necessary to explain the phenomenology associated with relativity theory. On the view that one should take physically seriously only those elements of a theory that are necessary to generating its successful empirical predictions—which, in the case of QFT, always proceed via the

[6] See Rosaler and Harlander [6] for a more detailed defense of this claim.

calculation of correlation functions—it is possible that one may dispense altogether with the notion that any particular parametrization of a QFT or EFT is singled out by nature uniquely physical or fundamental.

4 Higgs Naturalness and Renormalized Parameters

In this section, we review the formulation of the Higgs naturalness problem in terms of renormalized running mass parameters (Sect. 4.1), and then explain why the problematic cancellations are an artifact of renormalization scheme rather than a genuinely physical effect (Sect. 4.2). Therefore, the delicate cancellations involving renormalized running mass parameters possess much the same status that Wetterich and others have ascribed to the delicate cancellations involving bare mass parameters, in that they reflect our choices of mathematical convention more than they reflect a mysterious feature of the natural world.

4.1 Fine Tuning of Renormalized Higgs Mass in EFT Matching Calculations

A formulation of the Higgs fine-tuning problem that is frequently cited in the literature on EFT underscores the need for fine tuning of renormalized parameters in matching calculations for scalar running masses in the $\overline{\text{MS}}$ scheme; see for example the work of Skiba [8], Craig [3], Schwartz [7], Williams [11]. Let us see how this allegation can be dismissed by following an analogous line of reasoning to that given for the case of bare parameters in Rosaler and Harlander [6].

Assume that there is a theory T which contains all the SM degrees of freedom, plus additional particles with masses of the order of $M \gg v$, where v is the Higgs field vacuum expectation value. The Higgs mass renormalized within this theory in the $\overline{\text{MS}}$ scheme will be denoted as $\tilde{m}_H(\mu)$, where μ is the 't Hooft mass (renormalization scale). The SM is then an EFT which can be obtained from T by integrating out the heavy particles. Renormalized parameters of the SM can be related to those of T by requiring that the Green's functions of SM fields evaluated in the two theories agree in the limit $M \to \infty$. For the Higgs mass, we write this relation as

$$m_H^2(\mu) = \tilde{m}_H^2(\mu) + \delta m_H^2(\mu), \tag{7}$$

where $m_H(\mu)$ is the $\overline{\text{MS}}$ Higgs mass renormalized within the SM, and $\delta m_H^2 \sim M^2$ is a finite, calculable quantity. This shows that, except maybe for very specific choices of μ (see the toy example below), the numerical value of the $\overline{\text{MS}}$ Higgs mass in the full theory may differ by many orders of magnitude from its value in the SM.

Moreover, from experiment we know that the Higgs pole mass M_H is of the same order of magnitude as the SM $\overline{\text{MS}}$ mass, and thus also differs from the $\overline{\text{MS}}$ mass within T by many orders of magnitude. We thus write

$$M_H^2 = \tilde{m}_H^2(\mu) + \delta\hat{m}_H^2(\mu), \tag{8}$$

where $\delta\hat{m}_H^2(\mu) \sim M^2$. This means that delicate cancellations between the large values of $\tilde{m}_H^2(\mu)$ and $\delta\hat{m}_H^2(\mu)$ are required in order to arrive at the observed small value for the Higgs pole mass.

However, recall that the $\overline{\mathrm{MS}}$ mass is defined through an auxiliary, unphysical regularization parameter (the complex space-time dimension), and in this sense should be considered unphysical. On the other hand, whether one uses the $\overline{\mathrm{MS}}$ or the pole mass in the calculation of an observable is *in principle* irrelevant; the reason to prefer one over the other is usually determined from the convergence behavior of the perturbative series. Due to this purely theoretical character of the $\overline{\mathrm{MS}}$ mass, it seems untenable to base any physical conclusion on its numerical value. Similar to the bare parameters discussed above, the $\overline{\mathrm{MS}}$ parameters should be considered as book-keeping devices that allow one to relate different observables to one another.

To make this "renormalized" formulation of the Higgs naturalness principle more concrete, consider its application within the context of a concrete model—specifically, the often-cited case of a Lagrangian that couples a light scalar field to a heavy fermion field[7]:

$$\mathcal{L}_{\psi,\phi} = i\bar{\psi}\partial\!\!\!/\psi - M\bar{\psi}\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\tilde{m}^2\phi^2 - g\bar{\psi}\psi\phi \tag{9}$$

where $\tilde{m} \ll M$ and the parameters are understood as renormalized parameters employed in the calculation of tree level amplitudes. At low energies, heavy fermions are not produced, and we can use an effective Lagrangian that describes only the dynamics of the light scalar field:

$$\mathcal{L}_\phi = \frac{1}{2}A(\partial_\mu\phi)^2 - \frac{1}{2}B\phi^2 + \frac{1}{4!}C\phi^4 + \cdots, \tag{10}$$

where A, B, C are coefficients to be determined by matching to the full theory, and the ellipsis designates non-renormalizable terms whose influence is negligible at energies much less than M. Using $\mathcal{L}_{\psi,\phi}$, the two-point function for the scalar ϕ at vanishing external momentum reads

$$\bar{G}_2 = [-\tilde{m}^2 + \bar{\Pi}(0, \tilde{m}^2)]^{-1}, \tag{11}$$

where

$$\Pi(0, \tilde{m}^2) = \frac{4g^2}{(4\pi)^2}M^2\left(1 + 3\ln\frac{\mu^2}{M^2}\right) + \mathcal{O}\left(\frac{\tilde{m}^2}{M^2}\right), \tag{12}$$

and $\tilde{m}$ is the scalar $\overline{\mathrm{MS}}$ mass of the full theory. Requiring that this agrees with the two-point function of the effective theory,

$$\bar{G}_2' = [-m^2]^{-1} + \cdots, \tag{13}$$

[7] The detailed form of the heavy field is not important to the argument here, and the same general points—which rest on the presence of matching corrections to the light scalar mass term that are quadratic in the mass of the heavy field—can be illustrated in a model where the heavy field is a scalar rather than a fermion field.

where the ellipsis denotes terms of higher perturbative order, one finds

$$m^2(\mu) \equiv B(\mu) = \tilde{m}^2(\mu) - \frac{4g^2}{(4\pi)^2} M^2 \left(1 + 3\,\ln \frac{\mu^2}{M^2}\right) \equiv \tilde{m}^2(\mu) + \delta m^2(\mu), \quad (14)$$

where μ is again the 't Hooft mass, and we have chosen the notation in analogy to Eq. (7). Except for a delicate choice of μ one indeed finds a difference between the scalar mass parameters in the full and the effective theory of order M^2. To obtain a small value for m^2, there must be a delicate cancellation between the renormalized parameter $\tilde{m}^2$ of the full theory and the matching correction δm^2. Thus, it appears on one common interpretation as if the scalar mass term $\tilde{m}^2$ and quantum corrections are mysteriously conspiring to give a small value for the scalar $\overline{\text{MS}}$ mass term m^2 in the effective theory of $\mathcal{L}_\phi$. For completeness, we remark that, at the perturbative order we are considering here, the $\overline{\text{MS}}$ and the pole mass coincide in the effective theory, and thus one can replace the left-hand side of Eq. (14) also by the scalar particle's pole mass, thus arriving at Eq. (8) for that specific example.

The heavy field pole mass M sets the physical cutoff of the effective theory $\mathcal{L}_\phi$, since it is the scale around which it ceases to be empirically reliable in a world governed by the full theory $\mathcal{L}_{\psi,\phi}$. One way to avoid the need for delicate cancellations is for M to be small. Thus, in this example, $\mathcal{L}_\phi$ is analogous to the SM, while the full theory is analogous to a more encompassing BSM theory, in which the heavy fermion is analogous to some as yet undiscovered heavy BSM particle. For more detailed presentation of this example, see Skiba [8] or Schwartz [7].

4.2 Absence of Fine Tuning in the On-Shell Scheme

Our central thesis is that delicate cancellations relating renormalized scalar mass parameters like the ones just considered need not, and probably should not, be interpreted as a mysterious coincidence or fine tuning. The reason is simple: these cancellations occur in some renormalization schemes but not others, and one's choice of renormalization scheme is a matter of arbitrary convention. The fact that the cancellations rest on any particular such choice of mathematical convention suggests that we should, at the very least, be wary of endowing them with too much physical significance.

We claim that the need for delicate cancellations relating renormalized scalar mass parameters is an artifact of renormalization scheme. Matching calculations require one to choose a renormalization scheme both for the full theory and for the low-energy effective theory. In the calculation above, the parameters of both theories were defined in the $\overline{\text{MS}}$ scheme. However, this choice of scheme is primarily a matter of calculational convenience.In principle, one could choose to renormalize either theory in any scheme that one wants.

In particular, one could choose to renormalize both the full theory and the low-energy EFT in an on-shell scheme. In this case, the value of the renormalized mass parameters in both the low-energy EFT and the full theory are the same—namely, both are equal to the light scalar pole mass. For this reason, matching in the on-shell scheme involves no large quadratic matching corrections, or any matching corrections at all, to the renormalized mass parameter. The need for delicate cancellations relating renormalized scalar mass parameters between the EFTs can be seen to reflect our choice of mathematical convention, rather than a mysterious agreement between genuinely physical quantities, which are renormalization scheme independent.[8] It can be eliminated entirely through an alternative choice of mathematical convention that does not alter in any way the physical content of either EFT.

To be sure, the switch to the on-shell scheme does not eliminate the quadratic corrections relating the scalar field bare mass and pole mass in either theory; the counter terms relating these quantities still diverge quadratically with the cutoff regulator Λ. But recall that our discussion began from a point of skepticism about the formulations of the fine tuning problem that rely on the relationship between bare parameters and pole masses, a skepticism that has been justified in different ways by the sources quoted in the introduction. Assuming that delicate cancellation between bare masses and quantum corrections is not really the problem, the fact that quadratic counterterms remain in the on-shell scheme is neither here nor there.

Admittedly, the observation that renormalized masses do not sustain matching corrections when both theories are renormalized in the on-shell scheme appears somewhat trivial once it is made. One need not perform any detailed calculations to see that it is true. It holds simply by virtue of how the on-shell scheme is defined. But the relevance of this simple fact for the naturalness principle becomes manifest only once one has stopped to explicitly frame the question—i.e., of whether the need for delicate cancellations in the relationship between renormalized mass parameters is scheme independent and in this sense physical. Perhaps somewhat strangely, presentations of the above matching calculation make little of its dependence on a particular choice of renormalization scheme, to the extent that they acknowledge this dependence at all. However, inasmuch as one cares to go beyond a "shut up and calculate" attitude toward the relations that occur in EFT calculations (as one must when evaluating naturalness-based interpretations of relations like (8)), it seems a matter of common sense that questions of scheme dependence versus independence of various quantities and relations should bear critically on their physical interpretation. In particular, questions of scheme dependence bear heavily on questions about whether similarities in the numerical values of different parameters constitute a mysterious coincidence demanding special attention or an artifact of arbitrary choices that we have made in our calculations.

We emphasize that neither our analysis of the EFT matching calculation above, nor our similar conclusions about fine tuning of the Higgs bare mass, is intended to suggest that *all* fine tuning problems, or indeed that all fine tuning problems potentially associated with the Higgs, are pseudo-problems. For example, it was hypothesized some time ago by the now-debunked theory known as Technicolor that the Higgs scalar particle is in fact a bound state of two heavier unknown particles. If the pole masses of these particles were many orders of magnitude larger than the pole mass of the Higgs, recovering the much lighter Higgs pole mass as the mass of their bound

[8] Indeed, the need for running, scale-dependent mass parameters at all is an artifact of renormalization scheme, since one can in principle always take the pole mass, whose value remains fixed and does not run, as the renormalized mass parameter.

state would require a very delicate cancellation between the pole masses of these particles and their (negative) binding energy. Were this to occur, it would constitute a genuinely problematic and mysterious instance of fine tuning, since the delicate cancellation required would be between quantities whose values do *not* depend on an arbitrary choice of parametrization scale, renormalization scheme, or other such choice of convention. Specifically, the cancellation would be between the pole masses of the constituent heavy particles and their binding energy within the composite Higgs, both quantities that are invariant under one's choice of renormalization scheme and renormalization scale. However, this case of a genuine fine tuning problem rests on specific assumptions about the form of physics beyond the Standard Model—namely, that the Higgs scalar is a composite of heavier particles—that nature is not obliged to respect.

By contrast, the example of the Yukawa model described provides a simple toy model for another type of scenario for beyond the Standard Model physics, where the Standard Model Higgs is not a composite, but rather occurs as one among a wider range of fundamental fields, including those that are heavier than our current experimental capabilities allow us to probe. In this sort of scenario, where the Standard Model is recovered as a low-energy approximation simply by integrating out the heavy fields, we have argued that the delicate cancellation between the renormalized running Higgs mass in the BSM theory and the matching corrections that arise from integrating out the heavy BSM particle is merely an artifact of renormalization scheme. The cancellation can be removed through an alternative (but less calculationally convenient) choice of convention in which the renormalized Higgs mass in both theories is taken as the 125 Gev Higgs pole mass.

5 Conclusion

Because of the need for renormalization, and because of the role played by the RG, parameters and observables are related very differently in QFT from how they are related in models from classical physics and from quantum mechanics. The vast space of available parametrizations, related by changes of renormalization scale and renormalization scheme, introduces a large element of conventionality into the numerical values of QFT parameters. As a result, one must be careful not to ascribe too much physical significance to relationships that hold only in particular parametrizations and not others. Elsewhere, we have defended this point in arguing against the notion that fine tuning of the bare Higgs mass is problematic and calls out for explanation in the same way that the equality of the proton and neutron masses once did.

Here, we have argued that similar lessons apply to fine tuning of renormalized $\overline{\text{MS}}$ scalar mass parameters. Just as the need for fine tuning of the bare Higgs mass can be interpreted as an artifact of an arbitrarily chosen parametrization associated with a particular value of Λ, so the need for fine tuning of renormalized running scalar mass parameters can be understood as an artifact of the fact that matching is performed in a particular class of renormalization schemes. In both cases, the need for delicate cancellations thought to reflect the presence of fine tuning can be eliminated by a change of parametrization.

Acknowledgements The authors gratefully acknowledge support from the DFG. This work was done within the DFG Research Unit "The Epistemology of the Large Hadron Collider" (Grant FOR 2063). We would like to thank all its members for comments and encouragement.

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