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Why be Natural?

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Abstract
Naturalness, as a guiding principle for effective field theories (EFTs), requires that there be no sensitive correlations between phenomena at low- and high-energy scales. This essay considers four reasons to adopt this principle: (i) natural EFTs exhibit modest empirical success; (ii) unnatural EFTs are improbable; (iii) naturalness underwrites what Williams (Stud Hist Philos Mod Phys 51:82, 2015) calls a "central dogma" of EFTs; namely, that phenomena at widely separated scales should decouple; and (iv) naturalness underwrites a non-trivial notion of emergence. I argue that the first three are not compelling reasons, whereas the fourth is.

Keywords Naturalness · Effective field theories · Emergence

1 Introduction

In effective field theories (EFTs), naturalness is a requirement that there be no sensitive correlations between phenomena at low- and high-energy scales.[1] Instances of its failure in the Standard Model include the Hierarchy Problem, the Cosmological Constant Problem, and the Strong CP Problem. That these are taken as problems indicates the extent to which naturalness has come to be viewed as a guiding principle in the construction of EFTs. This essay considers four reasons for adopting this principle. Section 2 first reviews the steps involved in the construction of one type of EFT and how concerns with naturalness arise in this construction. Section 3 then considers three reasons to be natural: (i) natural EFTs exhibit modest empirical success, (ii) unnatural EFTs are improbable, and (iii) naturalness underwrites what Williams [2] calls a "central dogma" of EFTs; namely, that phenomena at widely separated scales should decouple. I argue that these are not compelling reasons: First, the modest empirical

[1] In this essay by an EFT I mean an effective quantum field theory. Some authors use the term effective theory in a broader sense (e.g., [1]).

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success of naturalness must be balanced by spectacular empirical failures. Second, probabilistic measures of the unlikeliness of an unnatural EFT require the specification of a probability distribution on the space of possible values of the bare parameters of the theory, and one can question the justification of both the particular form this distribution takes, and the general assumption that such a distribution is warranted in the first place. Finally, a distinction between two types of EFTs, Wilsonian and continuum, suggests that while decoupling may be a central dogma of EFTs, naturalness is not. Section 4 applies this last lesson to a fourth reason to be natural; namely, that it underwrites a non-trivial notion of emergence. Naturalness can be thought of as requiring that phenomena described by an EFT exhibit robust dynamical independence with respect to phenomena at high energies, and some authors have considered this to be a necessary characteristic of emergence. Thus to the extent that one desires to interpret EFTs as describing emergent phenomena, one should be natural.

2 How to Construct an Effective Field Theory

This section reviews the steps involved in the construction of what Georgi [3] refers to as a Wilsonian EFT. Naturalness concerns arise explicitly in this construction and the following review is meant to pin-point where this occurs. I will eventually claim (in Sect. 3.3 below) that while naturalness is a part of the internal logic of a Wilsonian EFT, it is not a part of the internal logic of another type, what Georgi [3] refers to as a continuum EFT.

After Polchinski [4], the construction of a Wilsonian EFT proceeds as follows: Given a high-energy interacting theory described by an action S[ϕ,$\partial\phi$] that is a functional of a field variable, or a set of field variables, and their derivatives, the first step is to identify low-energy and high-energy degrees of freedom. This is achieved by introducing a cutoff Λ, and then dividing the field variables into high and low momenta parts $\phi = \phi_H + \phi_L$ with respect to Λ (alternatively, one can identify "heavy" fields and "light" fields with respect to Λ). As Polchinski [4, p. 3] notes, the cutoff Λ is intended to represent the characteristic scale of the system; i.e., it is to be interpreted physically.[2] This physical interpretation of the cutoff is a key difference between Wilsonian EFTs and continuum EFTs, as will be made clear in Sect. 3.3. Moreover, the presence of a physical characteristic scale, arguably, is suggestive of naturalness, as we shall shortly see.

The next step in the construction of a Wilsonian EFT is to integrate the high-energy degrees of freedom out of the action. Formally, this is expressed as a path integral over ϕ_H:

$$e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_H e^{iS[\phi_H,\phi_L]} \tag{1}$$

[2] Schwarz [5, p. 418] points out that the Wilsonian approach to EFTs and renormalization has its origins in applications to condensed matter systems with explicit characteristic scales (e.g., atomic lattice spacing). One consequence of this, according to Schwarz [5, p. 411], is that "much of our intuition for fine-tuning and naturalness comes from condensed matter physics".

where $S_\Lambda[\phi_L, \partial\phi_L]$, referred to as the Wilsonian effective action, is a functional that depends only on the low-energy degrees of freedom ϕ_L and their derivatives. If the interaction is assumed to be weak, one can construct S_Λ as a perturbative expansion in effective coupling constants g_i:

$$S_\Lambda = S_0 + \int d^D x \sum_i g_i \mathcal{O}_i. \tag{2}$$

In this expression, the 0th-order term S_0 is identified as the free action (containing no coupling constants), D is the spacetime dimension, and the sum is over all local operators $\mathcal{O}_i[\phi_L, \partial\phi_L]$ consistent with the symmetries of the theory, each local operator consisting of powers of ϕ_L and/or its derivatives. The effective coupling constants g_i exhibit two important characteristics: First, they encode the dynamics of the high-energy degrees of freedom ϕ_H; and second, they are sufficiently small, under the weak interaction assumption. Thus expression (2) should be interpreted as describing a low-energy interacting theory obtained from an original weakly-interacting high-energy theory by a process in which high-energy degrees of freedom are removed from the original action and encoded in low-energy dynamics in the form of the effective couplings.[3]

One can now perform dimensional analysis on the effective action S_Λ to determine how its terms behave at energies $E \ll \Lambda$ that represent the scale of experimental interest. In units in which $\hbar = c = 1$, one has $[mass] = [length]^{-1}$ (thus, for instance $d^D x$ has dimension $-D$, and derivatives ∂_μ have dimension 1), and S_Λ is dimensionless. Now consider the ith term $\int d^D x g_i \mathcal{O}_i$ in S_Λ. The dimension of $\int d^D x \mathcal{O}_i$ is $\delta_\mathrm{i} - D$, where the dimension δ_i of the ith operator $\mathcal{O}_i$ is determined by the dimension of the low-energy field ϕ_L and the number of times it, and/or its derivatives, appear in $\mathcal{O}_i$.[4] Since the ith term is dimensionless, this entails that the effective coupling g_i has dimension $D - \delta_i$. Now, to the extent that the cutoff Λ is interpreted as a physical characteristic scale of the system, and the effective coupling g_i is supposed to encode the dynamics of the system at this characteristic scale, presumably it should be on the order of the scale. Hence, presumably, $g_i \sim \Lambda^{D-\delta_i}$. Thus we may write it as,

$$g_i = \lambda_i \Lambda^{D-\delta_i}, \tag{3}$$

with dimensionless coupling λ_i. Under the assumption that g_i is of the order of Λ, this implies λ_i is of the order 1, which, as we will see below, is one way to characterize

[3] This suggests that a Wilsonian effective action is only well-defined in "top-down" cases in which a high-energy theory exists. To the contrary, a Wilsonian effective action can also be constructed via a "bottom-up" process in which one writes down an expansion of the form of (2), including in it all local operators consistent with what one takes to be low-energy symmetries, and then suppresses these terms by powers of an appropriate physical cutoff Λ. Weinberg [6, p. 329] has argued that the result of using such an action to calculate S-matrix elements "… will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles".

[4] In a weakly interacting theory, the 0th order term S_0 in the expansion (2) dominates the other terms, since it contains no couplings which are assumed to be very small. S_0 only contains factors of ϕ_L and its derivatives, and it is dimensionless; and this suffices to determine the dimension of ϕ_L. See Footnote 7 below for a concrete example.

the naturalness criterion. This suggests that naturalness is part of the internal logic of a Wilsonian EFT, to the extent that the latter is described by a physical characteristic scale. I'll have a bit more to say about this in Sect. 3.3 below.[5]

At the energy scale E of experimental interest, the ith operator $\mathcal{O}_i$ is of the order E^{δ_i}, and hence the ith term is of the order $\lambda_i(E/\Lambda)^{\delta_i - D}$. One can now categorize the terms in (2) at the scale E into one of three types: An irrelevant term is characterized by $\delta_i > D$, hence it decreases as $E \to 0$; a relevant term is characterized by $\delta_i < D$, hence it increases as $E \to 0$; and a marginal term is characterized by $\delta_i = D$, hence it remains constant as $E \to 0$. To the extent that the g_i encode high-energy effects, irrelevant terms indicate an insensitivity to these effects at low energies, whereas relevant and marginal terms indicate a sensitivity to them. Thus if a Wilsonian EFT is viewed as a low-energy version of a full theory that is insensitive to the latter, irrelevant terms might be considered ideal, whereas relevant and marginal terms might be worrisome.

As an example, consider a weakly self-interacting scalar field Φ for $D = 4$ with a symmetry $\Phi \to -\Phi$ for simplicity.[6] The effective action is given by a sum of all possible terms involving powers of the low momenta field Φ_L and/or its derivatives that are consistent with the symmetry:

$$
\begin{aligned}
S_\Lambda[\Phi_L] = \tfrac{1}{2}\int d^4x(\partial_\mu\Phi_L)^2 + \int d^4x\Big[\lambda_{-2}\Lambda^4 + \lambda_0\Lambda^2\Phi_L^2 + \lambda_2\Phi_L^4 + \lambda_4\Lambda^{-2}\Phi_L^6 + \cdots\Big] \\
+ \int d^4x\Big[\textstyle\sum_{n>0}\lambda'_n\Lambda^{-n}(\partial_\mu\Phi_L)^2\Phi_L^n + \sum_{n\geq 0}\lambda''_n\Lambda^{-(n+4)}(\partial_\mu\Phi_L)^4\Phi_L^n + \cdots\Big]
\end{aligned}
\tag{4}
$$

where n is even (due to the symmetry), and the first term is the free action. The latter entails that the dimension of Φ_L is 1.[7] There are thus two relevant terms: an additive term in which no field variable appears, with coupling $\lambda_{-2}\Lambda^4$ that is quartically dependent on Λ; and a mass term containing the product of two fields, with coupling $\lambda_0\Lambda^2$ that is quadratically dependent on Λ. If a Wilsonian EFT is thought of as a low-energy restriction of a full theory that is insensitive to the high-energy degrees of freedom of the latter, then these terms may appear worrisome: a slight change in the high-energy theory will produce a large (quartic or quadratic dependent) change in the low-energy theory.

This sensitivity manifests itself in other ways, too. For instance, the relation between the "physical" mass coupling (i.e., what is measured in experiments) and the dimensionless coupling is $m^2_{\text{phys}} = \lambda_0\Lambda^2$. The physical mass is the mass of the scalar field at energies $E \ll \Lambda$; thus the dimensionless parameter λ_0 cannot be of order 1. The sensitivity of the mass term in (4) to high energy effects is thus encoded in a dimensionless parameter that is not order 1. Moreover, if one includes higher order corrections to the mass term, one finds that they are proportional to Λ^2; namely, $m^2_{\text{phys}} = m^2_{\text{bare}} + \kappa\Lambda^2$, for constant κ, and this requires a fine-tuning of the (non-renormalized) bare mass m_{bare} to guarantee the small measured value for m_{phys} (in the context of the Higgs scalar

[5] At this point, one thing that should be made clear is that, while a physical cutoff Λ suggests naturalness, it does not entail it. In other words, even though naturalness is part of the internal logic of a Wilsonian EFT, this does not mean that Wilsonian EFTs are necessarily characterized by it.

[6] This example is discussed in Duncan [7, p. 547] and Williams [2, p. 84].

[7] The first term is dimensionless. The d^4x part of it has dimension -4 and the ∂^2_μ part has dimension 2. If the dimension of Φ_L is δ, then $0 = -4 + 2 + 2\delta$, hence $\delta = 1$.

mass, see, e.g., [8, 9]). Thus the sensitivity of the mass term to high energy effects is also encoded in the necessity of fine-tuning the corresponding bare parameter.

3 Why be Natural?

The criterion of naturalness is supposed to make the concerns at the end of the preceding section explicit. After Williams [2, p. 82], I will take naturalness be the requirement that there should be no unduly large sensitive correlations between low- and high-energy phenomena in the context of an EFT. Other formulations of naturalness that appear in the physics literature include[8]:

(a) There should be no parameters with quadratic (or higher power) dependence on the cutoff.
(b) There should be no dimensionless parameters that are not of order 1, unless they are protected by a symmetry.
(c) There should be no bare parameters that require fine-tuning.

The symmetry formulation (b) is motivated by the fact that fermion masses in the Standard Model are small relative to the appropriate cutoff, and hence the corresponding dimensionless parameters are not of order 1; but because of the form of the mass term in a fermion theory, setting a fermion mass to zero restores a chiral symmetry to the theory.[9] This symmetry affects the form of the higher-order corrections to the mass term with the result that no fine-tuning of the bare mass is needed to be consistent with the small value of the physical mass. There can be other ways of avoiding such fine-tuning besides an explicit symmetry; hence, in general, one might maintain that apparent violations of naturalness signal the presence of new physics (i.e., new symmetries or interactions) that occurs between the low-energy scale E and the cutoff Λ. In other words, when low-energy phenomena appear to be sensitive to high-energy phenomena, we should look for new physics, the effect of which is to remove the apparent sensitivity.

8 These are discussed in Williams [2] who argues that they all have the sensitivity prohibition in common, and it is the latter, as opposed to the former, that should be identified with naturalness. Briefly, according to Williams, formulation (a) risks viewing violations of naturalness as dependent on the choice of regularization scheme one adopts. Formulation (b) is sufficient but not necessary for a sensitivity prohibition: there are explanations of unnatural parameters that do not involve a violation of a symmetry condition (e.g., the technicolor proposed solution to the Hierarchy problem). And finally, formulation (c) is also sufficient but not necessary for a sensitivity prohibition: there are fine-tuning problems that have nothing to do with undue sensitivity between widely separated scales (e.g., the flatness and horizon problems in cosmology, and appeals to the low entropy state of the early universe). Moreover, to the extent that the fine-tuning of formulation (c) "… always concerns relevant operators not protected by any symmetry" [2, p. 88], examples of violations of naturalness that are not associated with relevant operators fall outside its purview (e.g., the Strong CP problem).

9 This is ultimately a result of the fact that fermions are typically represented mathematically by spinor (as opposed to scalar or tensor) fields, and the former have a "built-in" chiral symmetry. The symmetry formulation of naturalness was introduced by 't Hooft [10].

3.1 Modest Empirical Success

Why insist that low-energy phenomena must not be sensitive to high-energy phenomena? One reason is that naturalness has had modest empirical success in the context of effective quantum field theories.[10] Indeed, most parameters in the Standard Model are natural, and the failure of a parameter to be natural has on three occasions signaled the presence of new physics; namely, the existence of the charm quark, the positron, and the ρ-meson.[11]

(a) Gaillard and Lee [11] predicted the mass of the charm quark to be ~ 1.2 GeV on the basis of the smallness of the difference in masses of the neutral kaons K^0 and $\bar{K}^0$ ($\sim 7 \times 10^{-15}$), and the appropriate cutoff for kaon physics; namely, $\Lambda < 2$ GeV. The existence of a new interaction mediated by the charm quark at an energy less than 2 GeV explained the apparent sensitivity of the kaon mass difference to high-energy effects.

(b) In the positron case, one observes that the electron radius r should satisfy $r > \alpha/m_e$, where α is the fine constant and m_e the electron mass; but this is much larger than what is observed; hence the electron self-energy correspondingly is smaller than what is observed. If the appropriate cutoff is on the order of $1/r$, this suggests new physics no later than 70 MeV, and this is born out by the existence of the positron.

(c) In the ρ-meson case, the observed smallness of the difference in masses of the charged and neutral pions with respect to the appropriate cutoff suggests a new interaction no later than ~ 850 MeV, and this is born out by the existence of the ρ-meson with mass 770 MeV.

These are "modest" successes to the extent that only (a) counts as a prediction, whereas (b) and (c) should be taken to be postdictions. Moreover, these successes must be balanced by three spectacular failures:

1. The first takes the form of The Hierarchy Problem, which is the failure of the Higgs mass in the Standard Model to be natural. Indirect observations at the Large Hadron Collider have assigned the Higgs mass a value of 125 GeV. For the Standard Model, the Planck mass $M_{\text{Pl}} \sim 10^{19}$ GeV is taken as a cutoff; hence, the corresponding dimensionless parameter for the Higgs mass is given by $\lambda_0 = m^2_{\text{Higgs}}/M^2_{\text{Pl}} \sim 10^4/10^{38} = 10^{-34}$, which is a very small number, certainly not of order 1. Alternatively, in order for the physical Higgs mass to be 125 GeV, given that higher-order corrections are on the order of M^2_{Pl}, the bare Higgs mass must be fine tuned to one part in 10^{-34}.
2. Another failure of naturalness is exhibited by the cosmological constant Λ_{C} (not to be confused with a generic cutoff Λ), if it is understood as an additive term in the effective action for general relativity. Observational constraints put the value of Λ_{C} at $< 10^{-47}$ GeV^4, and if the Planck mass is again taken as a cutoff, the

[10] Admittedly, naturalness as a constraint on effective theories in general has had much more empirical success than its restricted application to effective quantum field theories (see, e.g., [1]). (Thanks to Sebastian Rivet for making this point.).

[11] Details of these examples may be found in Giudice [9, pp. 168–169].

corresponding dimensionless parameter is on the order of $\Lambda_C^4/M_{Pl}^4 = 10^{-47}/10^{76} = 10^{-10}$.

3. Finally, the Strong CP Problem is the failure of a parameter θ associated with a CP-violating term in the Lagrangian for quantum chromodynamics to be of order 1. One can show[12] that $\theta < 10^{-10}$. This would be consistent with ‘t Hooft's notion of naturalness if setting θ to zero restored a CP symmetry, but this is not the case.[13]

In all three cases naturalness might be upheld if there was new physics that would explain the sensitivity of the low-energy phenomena. In the Higgs case, new physics has been proposed most prominently in the form of supersymmetry, in the cosmological constant case, new physics has been proposed in the forms of various candidates for dark matter, and in the CP-violating parameter case, new physics has been proposed in the form of gauge bosons known as axions. These and related exotica have yet to be observed. Of course this doesn't necessarily mean they, or something similar, won't be discovered in the future, but the current experimental constraints on such exotica make the detection of most of them highly unlikely.

3.2 Unnatural EFTs are Improbable

A second reason to be natural makes an appeal to probabilistic reasoning. Finely-tuned values of the parameters of a theory, we are told, are highly unlikely. Thus, insofar as an unnatural EFT requires finely-tuned values of one or more of its bare parameters in order to be empirically adequate (i.e., in order that the values of the corresponding physical, renormalized parameters agree with experiments), an unnatural empirically adequate EFT is highly unlikely. This argument can be unpacked in the following way:

(1) An unnatural, empirically adequate EFT must have finely-tuned bare parameters.
(2) A theory with finely-tuned parameters is unlikely to be true.
(3) Therefore, an unnatural, empirically adequate EFT is unlikely to be true.

Recall from Sect. 2 that an example of premise (1) is the Higgs sector of the Standard Model for which the bare Higgs mass must be fine-tuned to one part in 10^{-34}, in order for the physical Higgs mass to be consistent with observation. One might attempt to justify premise (2) by adopting a probability distribution on a space of possible parameter values, and then demonstrating that this distribution favors non-finely-tuned parameters. The main point of this section is to observe that both aspects

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[12] See, e.g., Dine [8, p. 48]. This upper limit is based on experimental limits on the electric dipole moment of the neutron, which can be derived as a function of θ.

[13] The CP-violating term takes the form $(\theta/16\pi^2)G_{\mu\nu}\tilde{G}_{\mu\nu}Z$, where $G_{\mu\nu}$ is the QCD field strength and $\tilde{G}_{\mu\nu}$ is its dual [8]. In the Wilsonian approach, $G_{\mu\nu}\tilde{G}_{\mu\nu}Z$ is not a relevant operator, and this might be puzzling if one associated the failure of naturalness only with relevant operators. This puzzle can be resolved by noting that the association of the failure of naturalness with relevant operators is made in the context of a weakly-interacting perturbative analysis of the low-energy effective action, as described in Sect. 2. The θ-term in QCD represents a non-perturbative, non-local (in fact topological) contribution to the low-energy effective action that falls outside the analytical framework of Wilsonian EFTs; yet, arguably, it is unnatural in the sense adopted in this essay; i.e., it represents a sensitive correlation between low- and high-energy scales. (In this case, it is in general a correlation between low-energy observables and the global topology of the system. Evidently, we should take it seriously to the extent that we think, for instance, instanton solutions to the QCD equations of motion should be taken seriously.) (Thanks to an anonymous referee for raising these concerns.).

of this justification have been called into question: Hossenfelder [12] argues that attempts to justify a particular form of such a distribution risk begging the question of the unlikeliness of fine-tuned parameters, whereas concerns raised by Norton [13, 14] suggest that the justification of a probability distribution in the first place, regardless of its form, is questionable.

Thus, I will ultimately argue that an appeal to a probability measure of fine-tuning in the context of naturalness to justify premise (2) fails. Before I rehearse the relevant arguments, let me first consider the related but distinct question of whether a probability measure of fine-tuning is *well-motivated*. Historically, probability measures of naturalness arose in the context of measures of the sensitivity of low-energy parameters to high-energy parameters.[14] As originally formulated by Barbieri and Giudice [16], a sensitivity measure involves specifying a set of low-energy and high-energy parameters (with the former functions of the latter), and then defining a sensitivity parameter Δ in terms of the maximum value of derivatives of the low-energy parameters with respect to the high-energy parameters. As Williams [2, p. 90] notes, this is supposed to measure how the low-energy parameters react to changes in the high-energy parameters. A typical sensitivity parameter for the weak sector of the Standard Model takes the following form[15]

$$\Delta[a_i] = \partial \ln m_Z^2/\partial \ln a_i^2, \qquad \Delta \equiv \max_i \Delta[a_i] \tag{5}$$

where a_i are a set of high-energy parameters and m_Z is the (low-energy) mass of the Z boson. One can now impose a tolerance level for naturalness, so quantified, by choosing a particular value Δ_{max} and requiring $\Delta < \Delta_{max}$. As various authors point out, the problem with this method of quantifying naturalness is that it involves a high degree of subjectivity [17, p. 7, 18, p. 365, 2, p. 90]. First, there is an arbitrary aspect to the choice of low-energy and high-energy parameters within a given theory (as well as how these parameters are parameterized), and this choice can determine whether the theory is labeled natural or unnatural. Second, the definition of the sensitivity parameters $\Delta[a_i]$ varies among authors, with alternatives including $\Delta[a_i] = \partial\ln m_Z^2/\partial\ln a_i$ and $\Delta[a_i] = \partial\ln m_Z/\partial\ln a_i^2$ [18, p. 367]. How one defines these parameters again determines whether or not a given theory is labeled natural. Finally, Craig [17, p. 7] reports that the choice of tolerance level Δ_{max} has changed over the years from ~ 10 to 1000. These and other problems lead Craig to claim that "… it is clear that measures of tuning have no intrinsic meaning" (p. 7).[16]

As Grinbaum [15, p. 621] notes, the move to a probabilistic interpretation of sensitivity measures was initiated by Anderson and Castano [19, p. 302], who suggested that (5) be divided by some measure of "average sensitivity". This requires the introduction of a probability distribution on the space of parameters, and, as Anderson and Castano [19, p. 303] admit, the choice of this distribution reflects "theoretical prejudice about what constitutes a natural value of the Lagrangian [i.e., bare] parameter". This theo-

[14] Grinbaum [15] provides a detailed history of such measures.

[15] See, e.g., Barbieri and Giudice [16, p. 64], Craig [17, p. 6], Feng [18, p. 365].

[16] Craig [17, p. 7] also reports that sensitivity measures can make intuitively incorrect judgments, labelling theories in which small energy scales are set by dynamical processes as unnatural.

retical prejudice is expressed by Anderson and Castano as: "observable properties of a system should not be unusually unstable against minute variations of the fundamental parameters" (p. 307). Hossenfelder [12, p. 10] notes that this admission effectively turns a probabilistic justification of naturalness into a circular argument. In particular, if we answer the question "Why be natural?" by referring to the unlikeliness of a fine-tuned value of a parameter with respect to a particular probability distribution, this raises the further question, "Why that particular probability distribution?" And if we answer this latter question by referring to its naturalness; i.e., that it makes fine-tuned values of parameters unlikely (or more explicitly, that it assigns a low probability to the values of parameters that reflect "unusual" instability against minute variations), we've entered a vicious circle of justification.[17]

Norton [13, 14] raises a more fundamental concern with any attempt to define a fine-tuning measure in terms of a probability distribution. According to Norton, a fine-tuning measure is characterized by a claim that has completely neutral support from current evidence, insofar as current theories take the value of a bare parameter as a brute fact. A probability distribution is required to be additive, and, according to Norton, additivity represents the complementarity between favorable and unfavorable evidence, and "… leaves no place in the representation for neutrality" [14, p. 504].[18] Norton's concern then is that adopting a probability distribution as a measure of the likeness of a fine-tuned value of a parameter risks conflating neutral evidence (i.e., data that is neutral with respect to the value of the parameter) with disfavoring evidence. For Norton, then, to justify the use of a probability distribution in the construction of a fine-tuning measure first requires providing an account how the current evidence is not neutral with respect to a particular fine-tuned value. Thus both Hossenfelder and Norton claim that fine-tuning measures require justification. For Hossenfelder, this involves justifying a particular probability distribution over others; for Norton, this involves justifying the use of a probability distribution in the first place.

3.3 The "Central Dogma"

Williams [2] suggests a third reason to be natural: "… the reason that failures of naturalness are problematic is that they violate a 'central dogma' of the effective field theory approach: that phenomena at widely separated scales should decouple." This section will argue that decoupling, understood in the EFT context, does not entail naturalness; hence, a failure of naturalness does not entail a failure of decoupling. Moreover, while it seems reasonable to view decoupling as a central dogma of EFTs, it is less clear that naturalness should be viewed in a similar way. I will attempt to

[17] Hossenfelder [12, p. 10] notes that Anderson and Castano [19, p. 302] are explicit in their admission of "an element of arbitrariness to the construction" of their probabilistic measure. One should also point out that Anderson and Castano's objective is not a justification of naturalness; rather, it is an attempt to construct a quantifiable measure of it. They are not concerned with answering the question "Why be natural?"; rather, they are concerned with the question "Given we should be natural, how can we quantitatively distinguish the natural theories from the unnatural theories?".

[18] Given n mutually exclusive and exhaustive outcomes $A_1,\ldots,A_n$, and the conditional probability $P(A_i|B)$ of outcome A_i given background evidence B, (finite) additivity requires $\sum_{i=1}^{n} P(A_i|B) = 1$. Norton interprets this as meaning that B can favor one outcome or set of outcomes only if it disfavors others.

establish these claims by considering two distinct types of EFTs. The first type are Wilsonian EFTs which were described in Sect. 2. The second type is what Georgi [3] calls “continuum” EFTs. My claim is that, while decoupling is common to both types, naturalness is more in the spirit of Wilsonian EFTs and less so for continuum EFTs. Thus while decoupling might be considered part of the internal logic of EFTs, naturalness should not.[19]

In slightly more detail, I will argue that both Wilsonian and continuum EFTs are characterized by what I will call “heuristic decoupling”. This involves the removal of high-energy degrees of freedom from a theory, and the encoding of their effects in low-energy dynamics. This notion of decoupling should be made distinct from a notion that I will call “precise decoupling”, under which the encoding of high-energy degrees of freedom in low-energy dynamics is done by means of a mass-dependent renormalization scheme. The resulting decoupling is “precise” in the sense of being an ingredient in a precise formal theorem (the Decoupling theorem below). But this encoding can be done in other ways, too. In particular, it can be done by means of a mass-independent renormalization scheme. This still results in decoupling, but not “precise” decoupling (in the sense of being an ingredient in the Decoupling theorem). I will use the term “heuristic decoupling” to refer to decoupling in general, without reference to the particular type of renormalization scheme. Thus, in the first instance precise decoupling entails, but is not necessarily entailed by, heuristic decoupling; and, as I shall claim, decoupling, in either the precise or heuristic sense, does not entail naturalness.

The two types of EFTs are based on a distinction between two types of renormalization scheme.[20] A renormalization scheme involves a method of regularizing divergent integrals, and a method of absorbing the corresponding infinities in a systematic way. Wilsonian EFTs employ mass-dependent renormalization schemes that use the cutoff Λ to regularize divergent integrals, and then absorb the divergent parts into renormalized parameters. One result of this process is that the latter are mass-dependent, hence the name. Continuum EFTs employ mass-independent renormalization schemes that use dimensional regularization to tame divergent integrals. One result of this choice is that renormalized parameters on this approach are mass-independent.

Ultimately, the choice of renormalization scheme has no empirical significance: the values of physical parameters come out to be the same regardless of scheme. Thus Wilsonian and continuum EFTs are empirically indistinguishable. However, the choice of renormalization scheme does affect the way high-energy degrees of freedom are encoded in low-energy phenomena; hence, Wilsonian and continuum EFTs are conceptually distinct in how they treat high-energy effects. For instance, a mass-dependent renormalization scheme is a necessary ingredient in the proof of Appelquist and Carazzone’s [23] Decoupling Theorem:

[19] Franklin [20, p. 23] similarly claims that the "effectiveness" of EFTs is not due to naturalness, but rather to an invariance of the low-energy dynamics with respect to changes in the state at high energies, consistent with the high-energy dynamics. According to Franklin, this type of "autonomy from microstates" is due to (effective) renormalizability in the context of EFTs. This assessment conforms to the discussion in the text above, given that "autonomy from microstates" can be identified with what I subsequently refer to as "heuristic decoupling".

[20] The following exposition is based on Bain [21, pp. 236–239, 22] and references therein.

Decoupling Theorem *In a perturbatively renormalizable theory with two widely separated mass scales, there is always a mass-dependent renormalization scheme by means of which the effects of the heavy masses can be encoded in the parameters of an effective theory in which only the light masses appear.*

On the other hand, the Decoupling Theorem fails under a mass-independent renormalization scheme. Thus, Wilsonian EFTs, but not continuum EFTs, may exhibit "precise" decoupling in the sense of satisfying the requirements of the Decoupling Theorem. On the other hand, one can identify a more general "heuristic" sense of decoupling that underwrites not just the Decoupling Theorem and mass-dependent renormalization schemes, but also EFTs that employ mass-independent renormalization schemes, too. This heuristic sense involves the removal of high-energy degrees of freedom, and the encoding of their effects in low-energy dynamics. Precise decoupling is one way of achieving this. Heuristic decoupling in general is characterized by a general sensitivity of low-energy phenomena to high-energy phenomena, but just to the extent that the latter are encoded in the former.

As I'll argue below, both Wilsonian and continuum EFTs exhibit heuristic decoupling and its related general sense of sensitivity of low-energy phenomena to high-energy phenomena. Naturalness, on the other hand, is a constraint on this sensitivity: it requires that the dependence of the effective couplings on high-energy effects cannot be too large; i.e., it requires that the general sensitivity associated with heuristic decoupling not be unduly large. Thus a failure of naturalness does not signify a failure of heuristic decoupling. Indeed, the examples in Sect. 3.1 of EFTs that fail to be natural all exhibit heuristic decoupling in the sense that they are given by effective actions that are functionals only of the light fields and are such that the effective couplings of the light fields encode the effects of the heavy fields. Their failure to be natural involves a failure of the sensitivity expressed by this encoding to be small enough.[21]

In a Wilsonian EFT, the Decoupling Theorem guarantees precise decoupling if there is a high-energy theory that is perturbatively renormalizable; and even when this doesn't hold, heuristic decoupling is guaranteed in practice via the steps outlined in Sect. 2 in which an effective action (2) is constructed by including all local operator terms consistent with symmetries. Part of this procedure involves assuming a fixed cutoff Λ that informs the order of the effective couplings; in particular, one assumes the couplings g_i both encode high-energy effects and are sufficiently small. This then suggests they are of the order of positive powers of $1/\Lambda$, and this suggests the naturalness criterion, for instance in the form of a requirement that dimensionless couplings be of order 1. Thus both heuristic decoupling and naturalness seem part of the internal logic of Wilsonian EFTs. I will now claim that, while heuristic decoupling also seems to be a part of the internal logic of a continuum EFT, naturalness is not.

One disadvantage of mass-independent renormalization schemes is that heavy field terms appear in a dimensionally regularized action, an indication of the failure of the Decoupling Theorem. Georgi's [3, pp. 227–228] notion of a continuum EFT is meant to address this.[22] To construct a continuum EFT, one starts with a dimensionally

[21] Franklin [20, p. 15] similarly stresses the significance of unnatural but (effectively) renormalizable EFTs.

[22] Assumedly, the term "continuum EFT" refers to the absence of a physical cutoff in a mass-independent scheme.

regularized action $S = \mathrm{S}[\phi_L] + S_{\mathrm{H}}[\phi_L, \phi_H]$ that is a functional of a set of light and heavy fields at some energy scale μ. However, unlike the Wilsonian approach, the next step does not involve a formal integration over the heavy fields. Rather, in the continuum approach, one evolves the action to lower energies, $\mu \to \mu - d\mu$, via the renormalization group. When the energy scale gets below the mass M of a heavy field, the action is replaced with an effective action $S_{eff} = \mathrm{S}[\phi_L] + \delta\mathrm{S}[\phi_L]$ that is a functional only of the light field plus a correction $\delta\mathrm{S}[\phi_L]$. The correction can be expressed as an expansion in effective couplings g_i,

$$S_{eff} = \mathrm{S}[\phi_L] + \int d^D x \sum_i g_i \mathcal{O}_i[\phi_L], \tag{6}$$

which exactly mimics the expansion (2) in the Wilsonian case. In the continuum case, the g_i are calculated by a matching condition, which is supposed to guarantee that high-energy observables match low-energy observables across the threshold characterized by the heavy mass.[23] This matching condition takes the place of the path integral in the Wilsonian approach, and is meant to achieve the same goal; namely, to encode the effects of the heavy field in the light field interactions. In both approaches, this encoding is performed on the effective couplings, but the exact form of the encoding, which depends on the type of renormalization scheme one adopts, differs. The end result, however, is the same measured values for the couplings.

Thus heuristic decoupling, in the sense of encoding the effects of high energy degrees of freedom in the low-energy dynamics, occurs in both Wilsonian and continuum EFTs. In fact, the matching condition in the latter is called "decoupling by hand" [3, p. 225]. Thus it seems reasonable to view heuristic decoupling as a "central dogma" of EFTs: an EFT is a way of removing high-energy variables by encoding their effects in low-energy interactions. On the other hand, while naturalness seems to be part of the internal logic of Wilsonian EFTs, as suggested in Sect. 2, this is not the case for continuum EFTs.

Recall that in the Wilsonian case, we assumed the g_i were of the order of a power of the physical cutoff Λ; namely, $g_i \sim \Lambda^{D-\delta_i}$. Thus we expressed them as $g_i = \lambda_i \Lambda^{D-\delta_i}$ and assumed the dimensionless couplings λ_i were of order 1, which is one way to characterize naturalness. Thus, it appeared that naturalness is part of the internal logic of a Wilsonian EFT, at least to the extent that (a) a physical interpretation of the cutoff Λ is assumed by a Wilsonian EFT, and (b) this interpretation suggests g_i are of the order $\Lambda^{D-\delta_i}$. In the case of the expansion (6) for a continuum EFT, we no longer encode the high-energy degrees of freedom into the effective couplings g_i at a physical cutoff Λ; rather, we have a mass threshold $\mu = M$, where M is the mass of a heavy field, at which we perform this encoding. Now we can surely express the effective couplings in terms of a power of this scale; i.e., $g_i = \lambda_i M^{D-\delta_i}$. But we aren't drawn to require that they be on the order of $M^{D-\delta_i}$ (as we seem to be in the Wilsonian case). This is because we are not adopting the attitude that M represents a characteristic scale of a complete high-energy theory.[24] We have the luxury of being

[23] These observables can take the form of scattering amplitudes, for instance.

[24] The point here is that the Wilsonian cutoff Λ and the continuum mass scale $\mu = M$ are quantitatively distinct, as Schwarz [5, p. 444] notes: "The Wilsonian cutoff Λ should always be much larger than all

agnostic about such a characteristic scale in the continuum case because we calculate the g_i by imposing a matching condition across the $\mu = M$ threshold, as opposed to a formal calculation that results from integrating high-energy degrees of freedom out of a high-energy theory. In other words, we don't need to assume anything about the g_i and their relation to the high-energy degrees of freedom, beyond requiring that they be functions of some power of M. Thus, in this case, we are not led to the presumption that the dimensionless λ_i are of order 1. Rather than encoding aspects of the high-energy physics in the *order* of the effective couplings, a matching condition is better understood as guaranteeing empirical adequacy across a mass threshold.

To sum up so far, whereas heuristic decoupling seems built into the conceptual framework of both Wilsonian and continuum EFTs, naturalness is built into the conceptual framework of Wilsonian EFTs alone. Thus, while Wilsonian EFTs might be considered naturally biased, continuum EFTs perhaps are better thought of as naturally agnostic. Indeed, Georgi [3, p. 219] suggests that fine-tuning bare parameters is perfectly reasonable if it is understood as a matching condition that guarantees empirical adequacy over a given mass threshold; and so long as we understand that an EFT does not commit us to anything that goes on at energies much larger than this threshold.[25]

4 Naturalness and Emergence

I'd now like to consider a fourth reason to be natural; namely, that it underwrites a non-trivial notion of emergence. Wilsonian EFTs are sometimes described as being motivated by condensed matter physics with their physical interpretation of the cutoff (e.g., [5, p. 418]), and condensed matter physicists have long discussed notions of emergence (e.g., [24, 25]). Moreover, in condensed matter approaches to quantum gravity that employ an effective field theory framework, one finds informal references to emergence. For instance, in a review of analog gravity, Barceló et al. [26, p. 49, 62] refer to "emergent gravitational features in condensed matter systems", and "emergent spacetime symmetries"; Dziarmaga [27, p. 274] refers to "an effective electrodynamics emerging from an underlying fermionic condensed matter system", and Volovik [28, vi] refers to "emergent relativistic quantum field theory and gravity", and "emergent nontrivial spacetimes". My suggestion in this section will be that naturalness is one way to make this informal talk of emergence in the context of EFTs more precise.

I take emergence to be a characteristic of the ontology associated with a physical system, call it the emergent system, with respect to another physical system, call the latter the fundamental system. Crowther [29, p. 429] has suggested two necessary criteria for emergence, so conceived, to be applicable. The first, "Dependence", is the requirement that the emergent system be ontologically determined, in some sense ,

Footnote 24 continued
relevant physical scales. This is in contrast to the μ in the continuum picture, which should be equal to a relevant physical scale."

[25] Certainly, one can adopt a similar agnostic attitude towards Wilsonian EFTs. The point of the above discussion is that this attitude does not seem in keeping with the way high-energy effects are encoded in effective couplings under a physical interpretation of the Wilsonian cutoff Λ.

by the fundamental system. The second criterion is "Independence", which requires that the emergent system be novel with respect to the fundamental system. The task in articulating a non-trivial notion of emergence is to resolve the tension between Dependence and Independence: an emergent system must be both dependent on, and sufficiently independent of, a fundamental system. My suggestion is that phenomena described by a natural EFT accomplish this task.[26]

Bain [30, p. 28] suggests that Dependence and Independence be cashed out in terms of microphysicalism and novelty, respectively. Microphysicalism requires that an emergent system be composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws. This captures the intuition that an emergent system cannot completely float free of a fundamental system. Novelty requires that the emergent system exhibit robust dynamical independence with respect to the fundamental system. Thus, while the micro-constituents of an emergent system obey the fundamental system's laws, the emergent system itself does not; rather, it exhibits novel dynamical behavior. Moreover, this behavior should be robust in the sense that it should be insensitive to slight changes in the dynamics of the fundamental system. The failure of such robustness would indicate that the dynamical independence exhibited by the emergent system is only apparent, or accidental, as opposed to an essential feature.

Bain [30, p. 29] claims that the phenomena described by an EFT can be interpreted as emergent to the extent that they exhibit both microphysicalism and robust dynamical independence with respect to the corresponding high-energy theory. Microphysicalism applies insofar as the field variables that enter into an effective action are simply the low-energy degrees of freedom of the high-energy phenomena. In more provocative terms, the low-energy phenomena are ontologically determined by the high-energy phenomena insofar as the former are derivative of the latter. With respect to robust dynamical independence, Bain observes that the effective action is formally distinct from the high-energy action, and this entails that the corresponding equations of motion are formally distinct, too.[27] Hence, if dynamical laws are encoded in equations of motion, the low-energy phenomena obey different dynamical laws than the high-energy phenomena, and this suggests dynamical independence. But is robustness necessarily an aspect of this independence in the context of EFTs? Examples of EFTs that fail to be natural suggest otherwise. In these examples, while the low-energy effective dynamics is formally distinct from the high-energy dynamics, robustness fails in the sense that there are effective low-energy parameters (the Higgs mass, the cosmological constant, etc.) that depend sensitively on the high-energy dynamics:

[26] Authors who have suggested emergence be associated with EFTs include Franklin [20], Crowther [29], and Bain [21, 22, 30].

[27] There are examples of EFTs for which this claim is problematic. For instance, in a "top-down" EFT in which there is a clear distinction in the high-energy theory between light fields and heavy fields, and the high-energy degrees of freedom (consisting of the heavy fields and the high-energy dynamics of the light fields) are represented in the effective Lagrangian as corrections to the low-energy dynamics of the light fields, the sense in which the low-energy dynamics is independent of the high-energy dynamics is fairly weak. Dynamical independence seems better motivated by examples of "top-down" EFTs for which there is no initial clear distinction between "heavy" and "light" degrees of freedom, and examples of "bottom-up" EFTs for which the high-energy degrees of freedom remain unknown.

in Wilsonian EFTs, for instance, a slight change in the cutoff Λ will produce large changes in low-energy physical scalar masses.[28]

Of course there is an easy remedy to this difficulty in Bain's analysis. We can simply modify the claim that EFTs in general describe emergent phenomena to the claim that natural EFTs describe emergent phenomena. The distinction between heuristic decoupling and naturalness described in Sect. 3.2 suggests a way of making this more precise. Heuristic decoupling in an EFT underwrites dynamical independence tempered by dependence. It involves dynamical independence to the extent that an effective action is formally distinct from a corresponding high-energy action; hence, if dynamical laws are encoded in equations of motion, the low-energy phenomena described by an effective action obey different dynamical laws than the high-energy phenomena described by a corresponding high-energy action. But heuristic decoupling also involves a degree of dynamical dependence, insofar as the effects of high-energy phenomena are encoded in the interactions of the low-energy phenomena. Moreover, heuristic decoupling by itself does not guarantee robust dynamical independence. Only naturalness guarantees this. Again, in a natural EFT, the dependence of low-energy interactions on high-energy degrees of freedom is sufficiently small so that slight changes in the latter do not induce large changes in the former.[29]

Thus both heuristic decoupling and naturalness provide a fairly explicit and formal way of characterizing emergence and associating it with an essential theoretical framework in physics (i.e., the EFT framework). Both physicists and philosophers should therefore be interested in "being natural", if only for the potential insight this provides for cashing out an important philosophical concept. On the other hand, from a purely pragmatic point of view, given that naturalness is a way to make the notion of emergence in EFTs more precise, and given that the notion of emergence is relevant to certain areas of physics that employ EFTs, does this provide a good reason to be natural when constructing an EFT? I suggest it does in the following qualified sense. In contexts in which we have prior reasons to believe the phenomena described by an EFT are emergent, we should be natural; i.e., we should attempt to account for these phenomena with an EFT that exhibits naturalness. Conversely, if the EFT we (successfully) employ to account for a range of phenomena exhibits naturalness, then this warrants characterizing these phenomena as emergent. If our empirically successful EFT does not exhibit naturalness, then this should give us pause in characterizing the phenomena in question as emergent.

[28] Williams [2, p. 95] mounts a similar criticism of Bain [21].

[29] Franklin [20, p. 22] also suggests that EFTs can exhibit emergence: "… EFTs are emergent if they are novel and autonomous$_{\mathrm{ms}}$ with respect to higher-energy theories", where "… novelty implies that new explanations are available which are not expressible in terms of the variables of the higher-energy theory." For Franklin, autonomy$_{\mathrm{ms}}$ (i.e., autonomy with respect to microstates) is underwritten by a renormalization scheme in which high-energy degrees of freedom are encoded in low-energy effective couplings. This is what I have identified above as heuristic decoupling. Thus, insofar as Franklin takes this to underwrite Crowther's Dependence criterion for emergence, we are in agreement. On the other hand, Franklin's notion of novelty as underwritten by a particular type of explanatory power seems to make emergence an epistemic notion. My preference is for an ontic notion of emergence that characterizes physical systems, as opposed to our knowledge of physical systems. Again, novelty, for me, involves the sort of robust dynamical independence that underwrites naturalness.

5 Conclusion

Why be natural? I've argued that we should not be natural because natural EFTs are empirically warranted, or because unnatural EFTs are improbable, or even because naturalness underwrites a central dogma of EFTs. Rather, we might be natural because it helps to underwrite a non-trivial notion of emergence associated with EFTs. Admittedly, this amounts to a pragmatic (as opposed to epistemic) reason to be natural. In particular, one should be natural to the extent that, if one is committed to describing phenomena as emergent, and one is committed to using the framework of EFTs in doing so, then one should attempt to construct a natural EFT. More generally, as an empirical hypothesis with limited empirical support, one should be cautious in using naturalness as a guiding principle; and one should be cognizant of where it occurs as an assumption in theoretical frameworks (Wilsonian EFTs, for instance). But as an ontological principle, there is nothing wrong with the project of examining what the world would be like if it were true, or how current theories might be extended if it were true.

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