Studies in History and Philosophy of Modern Physics 44 (2013) 329–337

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Emergence of spacetime in stochastic gravity

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ARTICLE INFO

Article history:
Received 2 October 2011
Received in revised form
3 January 2013
Accepted 5 April 2013
Available online 3 June 2013

Keywords:
Quantum gravity
Stochastic gravity
Emergence
Spacetime

ABSTRACT

I focus on the stochastic gravity program, a program that conceptualizes spacetime as the hydrodynamic limit of the correlation hierarchy of an underlying quantum theory, that is, a theory of the microscopic theory of gravity. This approach is relatively obscure, and so I begin by outlining the stochastic gravity program in enough detail to make clear the basic sense in which, on this approach, spacetime emerges from more fundamental physical structures. The theory, insofar as it is a univocal theory, is quite clear in its basic features, and so issues of philosophical interpretation can be readily isolated.

The most obvious reason to investigate the theory as a model for the emergence of spacetime structure is how close it is to the stage at which the behavior that we recognize as spacetime actually emerges from the micro gravitational system. Approaches that begin with fully quantum gravity (insofar as there is such a thing) treat a system that is conceptually quite far removed from the stage at which emergence is relevant. The stochastic approach however begins by identifying the point at which spacetime emerges as a phenomena of interest.

I begin with an analysis of the emergence question generally and ask how best we should understand it, especially from the point of view of thinking of spacetime as emergent. A nice feature of the stochastic program is how clear the question of emergence is on this approach. In part this is because of its similarity by design to the kinetic theory of gases and solid state physics. And so many of the analyses of the emergence of macroscopic variables in the thermodynamic limit can be repurposed to understand how an apparently continuous metrical space emerges from the behavior of a non-spatial system.

A serious interpretive problem looms however. The problem is that there is no clear connection between features of the kinetic theory of gravity, as a quantum theory, and any final theory of gravity. In the third part of the paper I will argue that as far as questions of emergence are concerned, we need not begin with a final, underlying theory, and I attempt to identify general issues connected to the emergence of spacetime that can be addressed in isolation from our certainty about that final theory. I will argue that this is a common way in which we treat our other, after all, *provisional* theories. We begin with the theories we have and ask about their implications without assuming that they are final theories, and yet also without explicitly downplaying the significance of the results we derive. Moreover I will attempt to show that, whatever character a (or the) final theory of micro gravity has, spacetime as an emergent structure in that theory is likely to be similar in important respects to the way it manifests in the stochastic gravity program. Briefly this is precisely because of the metaphysical neutrality of the kinetic theory. I will expand, in this section, on the nature of the emergence of the spacetime structure in the context of the stochastic gravity program and explain how the emergence is tied *not* to the particular model of interactions appealed to, but rather to the generic features of quantum fields with correlated fluctuations at all orders.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

0. Introduction

I focus on the stochastic gravity program, a program that conceptualizes spacetime as the hydrodynamic limit of the correlation hierarchy of an underlying quantum theory, that is, a theory of the microscopic theory of gravity. This approach is not well known to philosophers, and so I begin by outlining the stochastic gravity program in enough detail to make clear the basic sense in which, on this approach, spacetime emerges from more fundamental physical structures. The theory, insofar as it is a univocal theory, is quite clear in its basic features, and so issues of philosophical interpretation can be readily isolated.

The most obvious reason to investigate the theory as a model for the emergence of spacetime structure is that its target regime is the regime at which the behavior that we recognize as spacetime

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http://dx.doi.org/10.1016/j.shpsb.2013.04.001

actually emerges from the quantum gravitational system. Approaches that begin with a complete theory of the microscopic structure of spacetime and matter, including but not limited to quantized gravity theory (insofar as there are any such), treat a system that is conceptually quite far removed from the stage at which emergence is relevant. The stochastic approach however begins by identifying the point at which spacetime emerges as a phenomena of interest.

I begin with a brief analysis of emergence generally and ask how best we should understand it, especially from the point of view of thinking of spacetime as emergent. A nice feature of the stochastic program is how clear the question of emergence is on this approach. In part this is because of its similarity by design to the kinetic theory of gases and solid state physics. As a result many of the analyses of the emergence of macroscopic variables in the thermodynamic limit can be re-purposed to understand how an apparently continuous metrical space emerges from the behavior of a non-spatial system.

A serious interpretive problem looms however. The problem is that there is no clear connection between features of the kinetic theory approach to gravity and any final theory of gravity. In the third part of the paper I will argue that as far as questions of emergence are concerned, we need not begin with a final, underlying theory, and I attempt to identify general issues connected to the emergence of spacetime that can be addressed in isolation from our certainty about that final theory. I will argue that this is a common way in which we treat our other, after all, *provisional* theories. We begin with the theories we have and ask about their implications without assuming that they are final theories, and yet also without explicitly downplaying the significance of the results we derive. Moreover I will attempt to show that, whatever character a (or the) final theory of micro gravity has, spacetime as an emergent structure in that theory is likely to be similar in important respects to the way it manifests in the stochastic gravity program. Briefly this is precisely because of the metaphysical neutrality of the kinetic theory. I will expand, in this section, on the nature of the emergence of the spacetime structure in the context of the stochastic gravity program and explain how its emergence is tied *not* to the particular model of interactions appealed to, but rather to the generic features of quantum fields with correlated fluctuations at all orders.

1. Emergence as collective behavior

Essentially all of our ontologies are expressions of the collective actions of underlying elements or base properties or what have you. Indeed something we ought to have learned by now is that all we really know about ontology comes from our experience with higher order properties or collective actions of lower level objects or processes or what have you.[1]

We do, to be sure, have examples of entities that are, from the point of view of the theories that describe them, meant to be fundamental. And I'm thinking here of electrons and neutrinos, for example. But even those examples are in-apt because we deal only very indirectly with them in our experimental practices, and in our theoretical practices they are not, after all, particularly stably construed as fundamental but rather appear themselves as very low energy modes of some more fundamental, unified objects which are the target of an as yet not fully clarified theory, strings perhaps.

There has been in the history of inquiry into nature, a steady drive toward discovering the underlying entities that come together to compose, or constitute, or produce, the objects of our experience, and an enquiry into the parts of those pieces, and into the constituents of those parts. In each case, it seems, we are looking for the fundamental underlying entities. In each case, it seems, we have failed to find fundamental entities. Now clearly we have *not* learned from the failure of this progression to produce fundamental entities that there are, in fact, no fundamental entities. Far from it. Such a strong induction over relatively few instances (the instances here are the layers of descent that we have uncovered) would be completely inappropriate. But we should have learned something else that has been said in some measure before, in isolated cases, but not yet proclaimed in full generality, or with full clarity. *The ultimate constituents of things are not directly relevant for understanding the way those things are constituted by their non-ultimate constituents*. It really does not matter whether at the end of inquiry or even at the next stage of inquiry, we find basic, brute constituents of the objects of our present stage of inquiry. From a methodological point of view, from an explanatory point of view, from an epistemological point of view, what we see is that at any but the very bottom layer the ontology is a product of the (collective) behavior of the level below—and there is no need, at any given level, to recur to more than one level down in order to understand, as well as we have ever understood anything in the sciences, the ontology at that given level. Only changes in the phenomenological account of the next level are relevant to the explanation of the ontology of a given level. The various levels of description are methodologically, explanatorily, and epistemologically insulated from each other. At least this has been our experience so far, and *this* I submit we do have strong reason to expect to continue.

What then should we take to be an emergent entity or property of interest? A number of folk are in agreement that emergence does not require the failure of reducibility of the entity to underlying entities, or novel physical processes that operate only once the emergent entity has emerged, or really any interesting metaphysical novelty at all. Rather emergence has to do with the right way to characterize and understand the way various systems function. Humphreys (1997) offers six non-necessary features, more a family resemblance concept, but Butterfield's (2011) more minimalist characterization is quite clear. He takes “emergence to mean: properties or behaviour of a system which are novel and robust relative to some appropriate comparison class.” (921) Perhaps best for my purposes, however, is a definition that comes from the philosophy of mind. Clark's characterization is powerful and flexible: “A phenomenon is emergent if it is best understood by attention to the changing values of a collective variable.” (112) And that is all I shall mean by the expression. Obvious examples of collective variables are the temperature and pressure of a gas—releated to the mean kinetic energy of its constituents. We know that the gas is composed of very many particles (or at bottom quantum field configurations (or even deeper than the bottom string or loop or foam states (or even deeper still …))) but at the scale at which it makes sense to talk about the gas, it is itself best understood in terms of its temperature and pressure. The gas, as a thermodynamic entity, and its behaviors as the behaviors of such an entity, are best understood in terms variables describing the collective behavior of its constituents.

I will now try to distinguish clearly between two very different senses of “emergence of spacetime”. The first is misleading, while the second will tend to direct our attention more toward issues of theory construction than toward matters of interpretation and metaphysics broadly construed.

1.1. Emergence of spacetime

The first sense of the emergence of spacetime is one in which we naively assume that the spacetime of our experience is a nice

[1] Here and in what follows I take “lower level” to indicate more microscopic, more fine-grained, deeper in the sense of theoretical structure. While in line with most philosophical use, this use is unfortunately out of line with many physicists' use, where “lower” generally coincides with lower in energy scale. I hope no confusion will result from this minor terminological diversity.

Lorentzian manifold with various fields on it. Then we try to understand how it could be that this world emerges from one where there is really nothing that is much like spacetime: a world where there are no continuous trajectories, no definite geometry, no "local beables", etc. But we know already that this is a deeply misleading way to describe what goes on in contemporary physics. It is now a long time since we abandoned such a view of the sciences. Instead we now characterize our experimental encounters by building models of the data, and it is about those that we theorize.[2] In any case we do not directly confront continuous trajectories, definite geometry, or "local beables". All of that is brought in much later in the day. We should dismiss this view of how we are to understand the question of the emergence of spacetime. And I believe most of us do.

The other sense is attentive to how we come to use notions appropriate to the spacetime of general relativity in the first place. It is with emergence in this sense that I will be concerned. If we understand the emergence of spacetime in this sense, however, then it is not clear that there are any particularly well-posed metaphysical questions to ask about the emergence of spacetime in quantum gravity, in the absence of an *actual theory* that counts as a quantum theory of gravity. For what we would be looking for is an answer to the question, "How is it that the world of our experience is suited to a description in terms of continuous spacetime structures, while in fact the true structure of the world is like such and such?" or perhaps "How do the true degrees of freedom of the universe restrict themselves in such a way that, here at low energies (and what are for us low temperatures), we are aware of not those degrees of freedom directly, but rather the degrees of freedom appropriate to a continuous spacetime?" But neither question is well-posed when we don't really know what the true degrees of freedom are, or what the structure is that at the low energy limit manifests as spacetime.

On the other hand, we can ask a third question that is appropriate to this second sense of emergence. And that question is, "How can we understand the way the apparently classical character of spacetime is related to *whatever* underlying quantum system it may be a low energy manifestation of?" The answer to that question, I submit, is to be found by studying the program in stochastic quantum gravity.

What is most interesting about emergence in the case of spacetime is that we don't have a reasonably stable theory of the constituents in the way we do in, for example, philosophy of mind or solid state physics, etc. In the theory of mind we do know that minds emerge as properties of creatures with sufficiently complex brains involved in sufficiently complex interactions with the world. Why this happens is not entirely clear, and we have something urgent to do: Show how this happens, or at least illuminate it. Similarly we know that molecules and molecular forces between them constitute the macroscopic objects of our experience. And we can learn a lot by investigating the relation between objects and their constituents. In each case we have reasonably stable theories of the underlying constituents. But in the case of Quantum Gravity, we have General Relativity and we have Quantum Mechanics. The former appears to be non-fundamental and the latter appears not to be a theory of anything at all, so much as a constraint on theories.[3] So in order to see how spacetime emerges from the more basic system described by a Quantum Gravity theory, we must be indirect and proceed by analogy. We need to find a way to understand the spacetime treated in General Relativity as a description in terms of the action of a collective variable even before we know what the underlying variables are.

I will now defend the controversial claim that there is nothing particularly interesting about the notion of emergence in micro gravity. In fact I will claim that the situation is perfectly banal. It has been clear for nearly two hundred years that the notions we inherited from the ancients (Aristotle, Euclid, Proclus) about the necessary structure of geometry—as a study of figure, independent of what that figure contains—are completely inadequate to our growing understanding of the facts in spacetime. In particular the discovery that there are many possible geometries demands an explanation of why, given all these possible geometries, our world has the geometry it does. Part of this understanding comes from the recognition that geometry is in a mutual conditioning relationship with matter, a recognition that begins with Riemann's (1873) investigation of the hypotheses at the bottom of geometry. Riemann articulates clearly why there must be something distinct from space(time) that is itself causally[4] responsible for the geometric structure of spacetime itself. What Riemann shows is that a continuous order structure must acquire its measure relations extrinsically. As he tells us, in addressing the question of whether our geometric hypotheses remain valid in the "infinitely small", "[e]ither therefore the reality which underlies space must form a discrete manifoldness, or we must seek the ground of its metric relations outside it, in binding forces which act upon it."(11) In general relativity we have an implementation of the latter possibility. Whether or not we accept the various claims from different quarters around the turn of this century that spacetime points cannot themselves be fundamental, or pre-given, but are rather differentiated only via the action of the metric, we all should accept that spacetime (in particular its metric structure) is not a fundamental, non-contingent feature of the world of our experience, but rather is bound up with the nature of matter. Whether we call this "emergence" or not, we are not faced with a major *new* conceptual upheaval brought about by the development of micro gravity. Rather we are continuing our slow coming to grips with the fact of spacetime's emergent character.

The central question about emergence in Quantum Gravity seems to be this: how can understanding the emergence of spacetime *as a feature* of micro gravity help us to construct a viable theory?

We are not asking an interestingly metaphysical question because we don't have anything like the resources that would be necessary in order to answer it. Certainly one can always ask metaphysical questions about individual proposals for a micro theory of gravity. But in the case of micro gravity as such, and the emergence of spacetime as such, such a question is ill-posed until we know what are the true micro gravitational constituents of the universe. The really interesting thing to focus on at this point is how our given epistemic situation can fruitfully be used to improve that situation and do so by expanding our explanatory resources.

1.2. *What is the emergence of spacetime?*

What can we hope to learn by considering the emergence question for micro gravity? And what, exactly is that question?

What questions are of importance in understanding what it is for spacetime to be an emergent feature of the world?

[2] Or some such equivalent picture. It doesn't really matter for this what exact model of scientific theories one adopts. All that matters is to recognize what, I take it, is obvious in contemporary philosophy of science: that our theories are not reflective of our encounters with the world directly but are, rather, reflective of our experimental practices.

[3] Or so one might argue in the light of information theoretic accounts of quantum mechanics.

[4] Probably "causally" is the wrong word here since causation is probably a spacetime kind of notion.

First there are two kinds of thing we could be broadly interested in: (1) Theory relations and (2) Relations between various levels of description.

But does (1) even make sense? The idea might be to show that some theory reduces to another by showing that an alteration in one or another parameter in a formulation of one theory results in a formulation of the other. I take it that something like this is imagined when, say, Newtonian mechanics is reduced to special relativity: the parameter representing the speed of light in special relativity is allowed to become infinite, and then the mathematical form of the resulting equations is that of Newtonian mechanics. But one would not want to call this a case of one theory emerging from another, or even a case where the theory of one kind of system is related to the theory of another kind of system where one system is emergent from the other. I suppose we could call this a case of one theory emerging from another—and Newtonian physics does have some interesting features that are not part of relativity theory: principally, there is an absolute notion of simultaneity. But I don't see any sense in which this amounts to the kind of thing one would normally have in mind in discussions of emergence. Perhaps rather than just any old parameter being changed in the one theory, the theories should be related more as the statistical mechanics of gases is related to the thermodynamics of gases. We all take it that in some sense thermodynamics reduces to the statistical mechanical molecular theory. While we have not succeeded in answering every question about how to display that reduction explicitly, we do have good formal methods for deriving from the latter theory the fluctuations in the systems of interest that depart from those predicted by the former. But the former *theory* cannot really be said to emerge from the latter *theory*. Rather, by knowing in some other way how we expect the system at one level of description to be related to the system at some other level, we know how to apply the more accurate (or fine-grained) theory.

Suppose that we could get our theory of the underlying features of the system to display terms of a similar sort to the other theory. Would we want to conclude on that basis that the one is emergent from the other even in such a case? Suppose we find in the statistical mechanics of gases a term with the dimensions of pressure. There is very little reason to think that term would correspond to the pressure term in the thermodynamics of gases.

The preceding examples are meant to suggest that theories are not related by emergence but some other kind of relation. And if that is correct, then it will be more fruitful to consider emergence as directly related to properties and features of some actual system and see how one level of description is appropriate to certain properties or features for which another level of description of the same system is not.[5]

I take it that in the context of spacetime and micro gravity the expression “emergent” is being used in a somewhat more contemporary sense than it is in some other contexts. That is, rather than denoting a novel ontology or property or whatnot that cannot be characterized in terms of or reduced to the properties of another lower level ontology (that is, shown to be deducible from a theory of the lower level entities) “emergent” here means only that the behavior or properties that are present at some level are novel in the sense of being present only at that level and higher and that they are not some form of mere aggregate of behaviors or properties at the lower level. So for example we would not want to call the area of a book's pages emergent from the areas of the individual pages, while we might want to consider the surface area of each page emergent from the arrangement of (area-less) molecules that compose it. And this is meant to be neutral on the question of whether or not we can reduce the one property to the properties of lower-level entities.

In any case, here I will take “emergent” to refer quite broadly to behavior or properties, or even ontology characteristic of one level of description and not of the level of description of the constituents of the system displaying the behavior, property, or ontology. This is a usage that comports nicely with the way the physicists who are concerned with the emergence of various properties actually use the term. I certainly do not mean to suggest that philosophers should alter our conception of philosophical terms of art because we see those terms being used differently by physicists. However given our present concern with emergence in the case of quantum gravity, and given how the term is used by physicists who are concerned with emergence in micro gravity,[6] and given that there are good philosophical treatments of emergence that take it in the broad way in which I am using the concept, there seems little danger of confusion. Finally because I see no evidence whatever that there *is* emergence in the older sense, and because “emergence” so nicely captures the sense of collective behavior that is at the heart of discussions of the non-fundamental character of spacetime I will adopt this approach in what follows.[7]

2. The stochastic gravity program (and some other stuff)

Most discussions of quantum gravity begin by claiming that there is a fundamental inconsistency between classical spacetime theories and quantum matter theories. Thought of in this way it makes sense to think that we need to start from scratch in order to find a replacement theory and to understand how that theory could yield results that are familiar from our earlier theories of the large scale structure of the universe. While it has now, I believe, been definitely established that classical dynamical spacetimes are compatible with quantum matter fields (Callender & Huggett, 2001; Mattingly, 2005, 2006, 2009; Wüthrich, 2005), it seems that the basic presupposition remains that we should begin our investigation of the replacement for General Relativity at the Planck scale, or below, and attempt to build a new theory from the ground up, so to speak.

One option is to try to build the unknown system directly and then attempt to read off of *that* the properties of a spacetime with quantum matter fields on it. There are two important difficulties with such an approach however. First we do not yet have a workable theory of the fundamental system, so we have no idea how to proceed in elaborating the properties of that system that will give rise to the phenomena of spacetime. This is to say, that while there are many attempts at understanding how to generate a quantum mechanical theory of gravity, none of these attempts has resulted in the kind of mathematical structure that would allow us to directly read off the properties that count as a spacetime with quantum fields. But further we do not, even in our preliminary proposals for such theories, have a good sense for where the action of collective variables becomes important—and

[5] A helpful referee points out that Batterman (2002) might be seen as offering an account of emergence as focussed on theory relations. However there the idea is that it is a *failure* of theory relations that signals the presence of emergence. Batterman's view is probably not incompatible with the above argument.

[6] For example one hallmark of discussions of the emergence of classical from micro gravity is that the latter is Lorentz invariant while the former is not. See, e.g., Jacobson (2007), Hu (2009), and Weinfurtner, Visser, Jain, & Gardiner (2007) for considerations of how Lorentz invariance can be seen to emerge in micro gravity theories.

[7] If we confined ourselves to the older usage, this volume, in my view, would have to be empty.

especially no sense for how to say when the best description of what is going on is through the changing values of such variables.

Perhaps there are interesting things to say already about the metaphysics of possible micro gravity theories, and what it is like to eliminate spacetime as a fundamental feature of our theories, and how significant the conflicts are between the ontology of micro gravity and the background ontologies that (may or may not) derive from our history of physical theorizing over the last few centuries. I don't have much hope for interesting results coming from that line of inquiry.

There is, however, another option. I claim that the stochastic program in micro gravity (or some other similar top-down approach) is most likely to lead to progress in our understanding of the quantum aspects of spacetime. And this is because it begins with a system as given that we already know a great deal about and makes the very plausible assumption (an assumption shared by essentially all programs in micro gravity) that we are looking for a fundamental microscopic theory of gravity. This is a way to think about what is generally meant by “quantum gravity” that does not involve anything like a quantized gravitational field. Rather this way of thinking considers the spacetime of our experience to be a low-energy or low-temperature phenomenon of a quantum system that has nothing particularly to do with spatio-temporality. There may well be, in a theory of this system, variables that function in roughly the way that spatial and temporal variables function in spacetime theories, but their proper interpretation won't correspond in any interesting way to the interpretation of the spatial and temporal variables in spacetime theories. That is, while we still tend to think of spatial and temporal variables in spacetime theories as corresponding to the spatial and temporal aspects of our encounters with spacetime, the variables of the theory of the fundamental system now under discussion will not.

A recurring image used by workers in the stochastic program is derived from solid state physics. Consider the sound waves that propagate in a crystal. These can be quantized and a theory of phonons developed. But doing so does not bring us closer to understanding the molecular and atomic constituents of the crystal itself—nor is the quantum theory of the phonons in the crystal a quantum theory of the crystal itself. Similarly, we may well be able to develop a quantum theory of spacetime itself. That quantum theory may well be more like the quantum theory of sound propagation in a crystal than like the quantum theory of the constituents of the crystal, the truly fundamental target of our inquiry.

This is a suggestive image, and the analogy between the excitation modes of a solid composed of atoms frozen into position on the one hand and the modes of spacetime itself understood as the frozen state of some deeper system on the other is quite striking. There are reasons to be suspicious of the analogy of course. Most important is that the crystal and the atoms of the crystal are both embedded in the same spacetime whereas the quantum system of which spacetime is supposed to be a frozen state would not, on this analogy, have anywhere to be. It isn't immediately clear what kind of space would even make sense as the space in which these quantum entities exist. Indeed, one might well think that right here, the emergence question becomes particularly acute: How can one begin to understand a causal account of anything without there being some notion of locality to hand to underwrite that account.

However, the analogy is not the program. And as we shall see, the kind of worry alluded to just now does not arise within the program itself. Instead the analogy is supposed to motivate and orient us toward the program. Whether the program makes sense, finally, is a question not about its guiding analogy but about the structure of the program itself, its fruitfulness, and its interpretability. The question addressed by the stochastic gravity program is this: How far can we advance toward incorporating what we know about quantum mechanics together with what we know about general relativity by treating general relativity as the low-energy limit of some unknown fundamental theory? The answer begins like this:

Classical spacetime is to be thought of as an open system in the non-equilibrium statistical mechanics sense. We begin with the assumption that there is a complete micro theory of gravity. We assume that the theory is appropriately quantum mechanical, that is, that it makes sense to treat it as a description of a quantum system. Even though we do not yet have such a theory, there are features of it that can be investigated given just the assumption that it exists. For example, we expect the theory to be the theory of a closed system because it is appropriately cosmological. We may then think of spacetime itself as embedded within the total system the way we take open subsystems in statistical mechanics to be embedded in their total systems. And we may then think of spacetime as connected to the remainder system as though embedded in a heat bath. What virtues does this have?

Most importantly it allows us to make use of some powerful tools developed to deal with quantum open systems. Hu (2009), Calzetta and Hu (1988), and Hu and Verdaguer (2002) (for example and among many others) have produced some remarkable results using these tools. They and their collaborators have been able to show how to treat classical, general relativistic spacetime as an open quantum system.

The analysis of spacetime as emergent begins this way. There is a robust set of phenomena, and we account for this using classical general relativity. But we are convinced that the object treated in classical general relativity—classical general relativistic spacetime—is non-fundamental. The issue that confronts us most directly is how to move forward to gain a deeper understanding of the system we have been confronting experimentally as a classical dynamical object with quantum matter fields on it. The proposal on the table is that we think of spacetime as derived from some more fundamental but unknown system. How best are we to do that?

2.1. Transition to the stochastic program

Begin like this: Observe that General Relativity as normally written down is a classical spacetime theory with classical matter fields on the spacetime. If we wish to accommodate our conviction that matter is quantum mechanical in nature, but also wish to maintain the classical character of the spacetime, we can develop a quantum mechanical stress-energy operator for the matter and set the expectation value of that proportional to the Einstein tensor, as the source driving the spacetime dynamics. This gives us the semiclassical Einstein equation: $G_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle$. There are significant conceptual hurdles to cross already at this point. One problem is that the semiclassical Einstein equation has a term, $T_{\mu\nu}$, that involves products of operators which are not well-defined on curved spacetime. Wald (1994) is an important contribution here which shows us how to construct a consistent stress-energy operator and so address that worry. What results is a mathematically satisfiable theory. Wald's solution is essentially unique. Given a viable construction for $T_{\mu\nu}$, $G_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle$ can be given a sensible interpretation as a classical gravitational field in dynamical interaction with the expectation value of a quantum field. Even though Wald's version of semiclassical gravity is immune to many of the standard critiques of non-quantized dynamical spacetime theory with quantum matter, his theory is still not anywhere close to empirically viable. For example what we do not have in this theory is any way to take account of quantum mechanical fluctuations in the stress-energy operator.

The stochastic regime is the step beyond the semiclassical. It maintains semiclassicality in the sense of coupling a classical spacetime to a quantum matter field, but it does so in a way that takes cognizance of the fluctuations of matter, and so allows us to incorporate more information about the quantum system into the theory.

2.2. Stochastic gravity

The account in this section is drawn largely from Mattingly (2009). Suppose that the metric, $g_{\mu\nu}$, is a solution to the semiclassical Einstein equation. Now we move beyond the semiclassical theory by adding to the expectation value of the stress energy terms arising from quantum fluctuations in the associated stress energy itself and calculating perturbations in the metric. Hu and Verdaguer (2003) show how to modify the semiclassical Einstein equation to reflect that perturbation to linear order off of a background metric. I suppress some technicalities here and present a somewhat simplified expression for the modified equation

$$G_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle + \xi_{\mu\nu} \tag{1}$$

here $\xi_{\mu\nu}$ is a classical tensor field defined so that its statistical average vanishes, and so that the expectation value of the anticommutator of the difference between $T_{\mu\nu}$ and the expectation value of $T_{\mu\nu}$ at two nearby spacetime points x, y is proportional to the statistical average of ξ at x times ξ at y. Explicitly

$$\langle \xi_{\mu\nu}(x)\xi_{\sigma\rho}(y)\rangle \equiv 2\langle\{[\hat{T}_{\mu\nu}(x)-\langle\hat{T}_{\mu\nu}(x)\rangle],[\hat{T}_{\rho\sigma}(y)-\langle\hat{T}_{\rho\sigma}(y)\rangle]\}\rangle \tag{2}$$

Think of $\xi_{\mu\nu}$ as introducing noise into the system. But it is not just any old noise term. This is a term that shows us that suppressing the influence of fluctuations of the quantum system on the classical sector of the system allows that sector to manifest *as* classical. Far from regimes where quantum fluctuations are important, things can seem just the way they do in GR. This is in tight analogy to the case of fluctuations in the rate of impact of gas molecules on the walls of a container: the pressure is never constant, but at time scales much larger than the intervals between impacts it can manifest that way.

Even in regimes where the fluctuations are of significant magnitude, the theory remains a theory of a classical spacetime, and so is not a direct account of Quantum Gravity. However we do see how it is that spacetime is to be seen as one aspect of the total quantum system. The classical spacetime is plausibly thought of as a non-fundamental object whose character is simply an account of a particular way of representing the total system. To be sure we do not have significantly more insight into that total system itself, based only on this method, but we do have somewhat more insight into how a system that appears to be a classical spacetime emerges from that total system.

2.3. Implementing the stochastic program: the kinetic theory

Sorting out how to incorporate higher order fluctuations as part of an empirically adequate and well-motivated theory from these beginnings takes more work. Calzetta and Hu (1988) and Hu (2002) have outlined a complete program for quantum gravity—the kinetic theory approach—that is one way of implementing the stochastic program. The kinetic theory approach to micro gravity is an approximative, and incremental approach. At each stage of development the program gives guidance on how to produce further corrections to the previous stage. The idea is that whatever underlying microstructure spacetime possesses we can work our way down to this structure by progressive approximations. But this is not the failed program of attempting a renormalization of the Wheeler–deWitt equation. Rather than starting with a given theory and deriving approximations from it—a strategy that runs into trouble with self-interacting force carriers—the kinetic theory approach begins with the effective theory and attempts to determine what next lowest level of theory it is an approximation to. The plan for the program goes like this (Hu, 2002, 15): (1) Deduce the correlations of metric fluctuations from correlation noise in the matter field; (2) Reconstitute quantum coherence from the correlation functions; (3) See what spacetime counterparts are required by metastable structures in kinetic and hydrodynamic regimes of quantum matter theory. In carrying out this plan one develops a hierarchy of correlations that allows for an interpretation of the theory as expressing a kind of interaction between quantum fields of unknown character that have, as their collective action, the basic form of classical spacetime variables on a spacetime supporting quantum matter fields. These collective variables are then how we understand semi-classical gravitation and its associated classical spacetime.

While extremely complicated in practice, the idea in principle is quite simple. We know that the stochastic correction to the semiclassical theory in the Einstein–Langevin equation is only the first rung in the correlation hierarchy and so leaves out correlations between higher order fluctuations which make up the full theory of stochastic gravity. One can think of the kinetic theory approach as a principled way to find and incorporate these correlations by generalizing the noise term from stochastic gravity. Outlining the full procedure is beyond my scope here, but the important insight is that one can use powerful tools from statistical mechanics to understand the link between quantum coherence and fluctuations in the stress energy. The idea is to develop a master equation governing the correlations between the quantum fields that make up the stress energy terms. Whereas the correlation in the noise term in stochastic gravity is first order, the true correlations may be of any order of complexity. However these correlations will themselves obey a kind of statistical mechanics where we can think of the correlations at some finite order as an open system embedded in an environment that is itself made up of all higher order correlations (Calzetta & Hu, 2000). The plan at that point is to understand why there should be a stochastic term appearing at that order. Finally we use the insights gained at each order to understand how the next higher order is embedded in its own environment.

In the case of the kinetic theory approach to quantum gravity we can see how this plays out. Here it is merely assumed that some appropriate new theory has been found, a theory that underlies our current theory but that it is somehow unknown to us in its details. What we do know however is how our current theory percolates up from that underlying theory. We know that any good theory will have to give correctly the correlations between the quantum fluctuations of matter at every order, and we hope that by accounting for these correlations by hand some new important insights will be found into the nature of that underlying theory. There is, of course, no guarantee here. Like all theoretical endeavors, constructing a theory that accounts for the quantum nature of matter and the dynamical character of spacetime is fraught with difficulties and uncertainties. What we can say is that examples from the transition to relativity theory and to the quantum theory show that strategies of this sort *can* yield important insight into the construction of new theories when direct theoretical construction cannot.

One might well ask what kind of interpretation we are to put on the kinetic theory. Is it supposed to be itself an effective theory that, so to speak, rules its own domain and ignores whatever is going on at other levels of description? Or is it instead supposed to be an approximation scheme to stand in for some other theory we do not yet have? I think the right answer is, "neither." For there is no natural energy scale that marks the boundary of its presumptive validity, and at each stage it incorporates higher orders of fluctuation that become more and more relevant at shorter and shorter length scales. And there is also no sense that some final theory must lurk around the corner. Rather the approach is to be understood, I think, in the following way. We remain neutral on questions of how other approaches fit into the picture and we remain neutral on whether there is any end to the process. Instead we say as clearly as we can what must be true of any theory valid at the level of description the program currently occupies. And what we learn, or what we hope to learn, is how the apparently classical character of spacetime arises from structures known to be represented, at the level at which this apparently classical character does emerge, in any micro gravity theory.

And an interpretation of this sort allows an understanding of the nature of emergence that is in line with my overall approach. That is, it allows us to think of the emergence of spacetime as a description of how certain phenomena are best understood as the action of changing values of a collective variable. The appropriate collective variables are, at least so the architects of the program have it, present as descriptive features of any system where quantum systems display classical behaviors in some appropriate limit.

The kinetic theory is simply one approach one might take in attempting to illuminate the way in which the classical spacetime emerges from the fundamental quantum system. However that approach illustrates clearly that the key feature in the emergence of spacetime in micro gravity is to be found at the level of coupling between the classical spacetime description and the more general quantum description. It is at that level that we see the relation between the variables in the classical description and the features of the underlying system that give rise to them. We do not know what the correct theory of the quantum system is, but we can see how classical spacetime emerges from whatever that system turns out to be.

2.4. Implementing the stochastic program: two other approaches

While maintaining the basic idea of stochastic gravity as a bridge between the fundamental (unknown) theory and the effective (general relativistic) theory, we can think about other ways to extend the stochastic approach into regimes where the fluctuations in the matter field are significant. These approaches complement the kinetic approach, but subtly shift focus to something other than the hierarchy of correlations between interacting quantum fields and induced fluctuations in the metric.

One approach is a revival of the hydrodynamics conception of general relativity (see Hu, 1996). We try to develop appropriate equations governing the interactions between fluid elements on the assumption that what we see in our approximate physics is the transport function of some constituents of this fluid. We remain agnostic about the exact nature of these particles, and try to see what we can learn by exploiting analogies between features of spacetime physics and hydrodynamic processes—Hu's example is the analogy between conductance in a fluid and the two-point correlation function in the Einstein–Langevin equation. An intriguing idea, but one whose technical details are as yet beyond our grasp.

Another possibility is to directly consider that spacetime might be the condensed state of some analogue of a superfluid (see Hu, 2009). The idea here is to consider the universe as a whole to be a gas of quantum particles interacting in a configuration space that is now, at late times and low energies, condensed into spacetime. So the metric field would be an analogue to the two-dimensional surface of a solid object composed of zero-dimensional molecules—there are no true paths on the material between these molecules, but rather they are joined by the space in which they are all embedded. An analogy can drawn here between spacetime and its constituents and Bose–Einstein condensates and their constituents. This approach is much more speculative than the kinetic theory because we do not have a good sense for what the molecules are that manifest collectively as a spacetime with geometric relations on it. So the analogy can only take us so far. While it is not yet clear that this program will yield fruit in terms of the development of micro gravity itself, it does allow for a compelling picture of how spacetime emerges from some more fundamental system without ever requiring that we quantize geometry itself.

These two possibilities for implementing the stochastic gravity program may yet bear fruit in our efforts to construct a theory that takes due notice of the dynamical nature of spacetime and the quantum nature of matter. As things stand they are more suggestive than productive. And yet together with the kinetic theory approach they make clear that already at the point of undertaking these constructions we can see that, independent of the final constituents that may be appealed to in our micro gravity, we can understand clearly how it is that spacetime can be regarded as emergent from micro gravity. All three approaches begin with the conviction that the micro character of gravitation can be understood best—at least for now—by postulating spacetime as emergent from the collective action of entities or processes far away from the regime of sub-Planckian physics. They then probe in their different ways what connects the semi-classical regime with the regime from which it emerges.

3. The upshot

As things stand it should be clear that there is, at least for now, only one way to understand the emergence of spacetime in micro gravity. For it is clear that any proposal in micro gravity that we can accept—at least given our very broad sense of how it is possible for physics research to proceed at this point—must pass through the stochastic gravity sector. That stage will be well characterized as involving a classical spacetime, but it will also show in outline how that spacetime is embedded as a subsystem in a broader quantum system whose details need not be known before we are able to speak intelligibly about either the non-fundamentality of spacetime as an entity described by the theory or about the non-fundamentality of spacetime features we hold dear in the classical case. The idea, as will be developed more fully below, is to attempt to see classical spacetime geometry first as determined by a classical matter field, and then to see that matter field as a certain kind of average of the quantum matter distribution—this is the semiclassical theory. If we stop there, however, we lose much of the information about the full system. So then we add in terms that reflect how spacetime *as a system* is embedded as a subsystem in whatever quantum system it is that is the target of micro gravity theory. These terms are not best thought of as corrections in an approximation scheme—though they are that too—but rather as indications of how the system at one level of description is embedded in the complete system. At each stage of correction, the task for theory is to describe at the new level of description how the system at that level of description is embedded in the complete system. So unlike the case, say, of quantum electrodynamics where the trick is to compute the next term in the series, given a theory, the trick here is to determine, given one stage and showing its embedding relation, how next to proceed in understanding that stage as a system at a new level of description embedded in the complete system.

3.1. Emergence at what level?

I will not be arguing here that pursuing this approach to micro gravity is the right strategy for constructing a theory of micro gravity—though I do think this and have argued in its favor—instead I will focus on the interpretive side of things and attempt to show that whatever comes from other approaches to micro gravity, this approach suffices as the locus of our questions about the emergence of spacetime, as that concept (in the case of theories) or entity (in the case of our experience of the world) manifests in our experimental practices as a continuous Lorentzian manifold. I am not committed to the claim that the world of our experience is *not* after all a continuous Lorentzian manifold, but rather I adopt that view for the purpose of addressing the emergence question.

One of the most interesting questions we can ask about the emergence of spacetime is, "at what level of description does each type of property emerge?" How far does the classical description extend, and indeed for the various portions of the classical description (continuous metric, light cone structure, field equations, etc.) how far does each extend?

This way of looking at things is, I think, comparable to asking a similar question about the thermodynamics of gases, or solid state systems. Consider generally the strategy of statistical mechanics in regard to the development of a theory of the transition between the continuum and the discrete levels of description. We know that the emergence of thermodynamic phenomena, say, can be captured by the mere postulation of the molecular hypothesis—not any particular theory of it, but just that there is one. That is, given that we know that gases are composed of vast numbers of particles, and the limitations both of our observational capacities and our calculational capacities, we know that there will be phenomena best characterized in terms of collective variables characterizing the macroscopic state: pressure, temperature, etc. These phenomena include heat capacity, the efficiency of heat engines, etc. Of course we will need much more detail in order to get just right the various macroscopic properties appropriate to a given system. For example, without the van der Waal's force term, we cannot get just right the relation between temperature and pressure of a gas. At what point however is it appropriate to say that we understand how temperature and pressure emerge from the behavior of the underlying system? Is it necessary to have the quantum theory of gases in place ? (or string theory? or …?) Must we, if so, have solved the measurement problem for quantum mechanics before we can say that we understand how temperature and pressure emerge from the behavior of the underlying system? The answer to both of these questions must surely be "no." Unless we are prepared to abandon all non-ultimate explanations as non-explanatory, we simply cannot reserve attributions of understanding for the last stage—if there is one.

So then at what point do we understand how pressure and temperature emerge? I claim we understand *that* at the point of postulation of the molecular hypothesis itself. Does our understanding of the nature of gases improve as we develop and further clarify the situation? Of course. But the conceptual re-organization that gives us understanding of the emergence of these properties takes place when we see how pressure and temperature *could be* collective variables.

The appropriate analogue in micro gravity is this: Is it necessary to understand the fundamental theory of gravity in order to understand the emergence of various spacetime properties from other features of micro gravity? And again the answer is "no." Rather what we want is to understand how it is that classical spacetime (or its features) are manifestations of micro gravity, and we can do that, best I believe, by a careful examination of the stochastic program for micro gravity, rather than by looking at any one proposal for an ultimate theory. Further elaboration of the stochastic gravity program will shed further light, but I submit that the appropriate conceptual re-organization is produced by the recognition *that* (and *how*) spacetime can be understood as the frozen state of some quantum system or other, that it is to be understood as an open system embedded in that quantum system, and that it couples to the remainder of the system through fluctuations in the quantum matter distribution.

Given the background of the stochastic program, we can articulate clearly the question of how various features of spacetime emerge according to that program. Notice that these features emerge in a way different from our normal expectations in speaking of emergence. In part that is because we have started with a presumed, but not fleshed out background theory of micro gravity, and simply treated classical spacetime as embedded in the total system in the way that an open system is embedded in a larger system: not as an actual subpart, but rather as, itself a perspectival representation of the total system. That is, we have a total system, and we attempt to see how it is that a system with which we are familiar—classical spacetime—can be seen as embedded in that system. And what we find is that to effect this embedding we can model spacetime as an open system coupled to a background system by the exchange of an analogue of heat—in this case represented by the correlation dissipation terms.

3.2. The emergence of spacetime in any micro theory of gravity

What we mean by "spacetime" when we ask about its status as an emergent entity in various theories is, it seems to me, classical spacetime as described by the general theory of relativity. Certainly we have a lot of other examples of structures that are similar in some respects to this, but when we ask about how spacetime emerges from micro gravity theories, we must be asking about the classical spacetime of relativity theory. And so what we are really asking about are properties that are appropriately classical spacetime properties: orientability, metric relations, curvature, locality, etc. But such features are already very far removed from the features of any purported underlying constituents. If we suppose that the correct micro gravity theory is something along the lines of loop quantum gravity, we are still very far from spacetime itself even when we have calculated the quantum of area. And similar considerations apply to whatever example of fundamental theory we might suppose will be developed in the future. We simply are too far from a workable theory to make good on the possibility of making the direct transition from the items mentioned in that theory to the collective variables of the higher level system.

We already know it seems to me that spacetime cannot be fundamental in the way it was thought to be before the turn of the Twentieth Century. For it is an essential feature even of classical relativity theory that spacetime has its metric structure as a spacetime only derivatively from the presence of matter energy. So there is in that sense at least good reason to think that spacetime must emerge in some way or other from an underlying, more fundamental system.

Yet there appears to be little concrete one can say about the way some deeper theory of gravity can function in our cognitive economy given only that spacetime itself is emergent from the theory's target system. That is, little can be said about how a theory functions in advance of an actual theory about which one can ask such questions. But this should not trouble us.

For what we see in other theories that have had their basic variables replaced with deeper (or merely other) variables is that testing very often goes on as before—whether pressure is fundamental or the mere aggregate of the many impacts of some unknown particulate seems to make little difference to the question of how we go about testing. We test indirectly as always. And in the case of micro gravity at least, we will continue the time honored technique of observing pointer coincidences—really the only technique we have ever had.

It might be instructive to ask what kinds of tests we can conceive employing in any near future. These tests will, it appears, be of just the same sort we have been employing up until now. In particular these tests will not, at least as far as we now know, be able to distinguish between theories whose consequences differ only for predictions about what goes on at the Planck scale. There may well be observational difference between the various theories of micro gravity, but we are very far away from being able to test for these differences. We should not, however, hesitate to explore the philosophical consequences of the non-fundamentality of spacetime. We should move ahead but recognize that we are not in a situation where some basic theory is available for interpretation and from which we can hope to derive basic metaphysical implications. Instead what we have available is a *theoretical approach* that, while it does treat spacetime classically, does so in the context of that already being a description of an "emergent" system, or rather a system whose description is a partitioning of the quantum system in which it is embedded. The kind of philosophical analysis we can perform here is not that of traditional interpretative metaphysics, but rather is best directed toward an understanding of the role of theories in our development of the sciences, and toward understanding the methodological role that emergence plays.

What we see, if we take seriously the guiding equations of the stochastic theory, is that the classical spacetime variables are actually just what they appear to be at various stages. That is, they are the stress-energy terms at the semiclassical stage, they are those terms plus the back reaction term at the next stage, etc. All of the familiar properties of classical spacetime are simply encoded at those stages where it makes sense to regard the matter terms as free of quantum correlations, and novel properties can be expected to be encoded and discovered at those stages where we must take cognizance of quantum correlations. The promise of stochastic gravity is that we will be able to see these properties as emergent from ever deeper stages of fleshing out the correlations between the underlying quantum fields.

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