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# Fantastic Beasts and where (not) to find them: Local gravitational energy and energy conservation in general relativity

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ABSTRACT

This paper critically examines energy-momentum conservation and local (differential) notions of gravitational energy in General Relativity (GR). On the one hand, I argue that energy-momentum of matter is indeed locally (differentially) conserved: Physical matter energy-momentum 4-currents possess no genuine sinks/sources. On the other hand, global (integral) energy-momentum conservation is contingent on spacetime symmetries. Local gravitational energy-momentum is found to be a supererogatory notion. Various explicit proposals for local gravitational energy-momentum are investigated and found wanting. Besides pseudotensors, the proposals considered include those of Lorentz and Levi-Civita, Pitts and Baker. It is concluded that the ontological commitment we ought to have towards gravitational energy in GR mimics the natural anti-realism/eliminativism towards apparent forces in Newtonian Mechanics.



## 1. Introduction

Energy and its conservation are a pivotal part of almost all of physics. From early on, attempts to define energy for the gravitational field in GR sparked controversy. Progress in this regard was in part responsible for GR's reinvigoration as mainstream physics from the 1950s on (see Schutz, 2012; Kennefick, 2007, Ch. 11, 12). But the quest for a fully satisfactory account of gravitational energy continues. In the following, I examine whether in GR gravitational energy - the energy ascribable to spacetime itself - is a meaningful local (differential) notion: Does there exist something like gravitational energy-momentum density? A related question concerns the validity of energy-momentum conservation: Does *non-gravitational/matter* energy-momentum 4-current possess sources or sinks?

The aim of the present paper is conceptual analysis: Can or should one endorse realism about local gravitational energy in GR, drawing only on the latter's fundamental concepts? (I will steer clear of the question of whether a *higher-level* concept of gravitational energy exacts some form of realist commitment — whether, for instance, an effective gravitational energy, definable in a certain domain, counts as a "real pattern" in the sense of Dennett (1991).[1]) My objective is conceptual clarification: What can be said about local gravitational energy *within GR's fundamental ontology and ideology*?

I contest the existence of local gravitational energy in GR. It will be argued to be an eliminable concept. Nonetheless, there is considerable continuity between GR and its precursors. Locally, the energy-momentum of matter is indeed conserved, with no need for gravitational energy contributions to restore an energy balance. The difference between GR and its precursors relevant here lies solely in the fact that GR's inertial frames are only defined locally, in contrast to the globally defined ones in Classical Mechanics (CM) or Special Relativity (SR).

Those views are widespread amongst relativists. The paper will seek to vindicate them. To date, a systematic review and evaluation of the arguments in favour of them, as well as an exposition of a coherent account are pending. This I will attempt to provide.

I will proceed as follows. In **§2**, I will first (**§2.1**) explore local energy-momentum conservation in generic spacetimes: Can the vanishing covariant derivative of the energy-momentum tensor be interpreted as a local energy conservation law? What are the role and status of gravitational energy-momentum in such conservation laws? **§2.2** zooms in on symmetric spacetimes. In particular, I address the question of global energy conservation. **§3** is devoted to local representations of gravitational energy. I first (**§3.1**) criticise Lorentz and Levi-Civita's tensorial proposal and elaborate on the necessity of non-tensorial expressions for gravitational energy-

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[1] Such an attempt is found in Read (2017), to whom I respond elsewhere.

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momentum. Exemplarily, I subsequently (**§3.2**) study pseudo-tensorial approaches via the Noether Theorems and expound their main problems. **§3.3** discusses Pitts' proposal for an infinitely many component object for representing gravitational energy. Another proposal, based on the cosmological constant, is studied in **§3.4**. A summary of my conclusions is provided in **§4**.

I build upon pioneering work by Hoefer (2000). He characterises what he declares the "received view" of local representations of gravitational energy-momentum by the following three claims: Firstly, one postulates the vanishing of the covariant divergence of the energy-momentum tensor of matter, $\nabla_b T^{ab} = 0$. Secondly, since in general it does not satisfy a proper continuity equation $\partial_b T^{ab} = 0$, the vanishing covariant divergence of the energy-momentum tensor forms a conservation law proper only for the sum of material *plus* gravitational energy-momentum. That is: One posits contributions from gravitational energy, not included in $T^{ab}$. Neglecting these contributions is supposed to result in *apparent* non-conservation of energy-momentum, which is what $\partial_b T^{ab} \not\equiv 0$ is interpreted as. Thirdly, such gravitational energy contributions are then lumped into one object, the so-called "pseudo-tensor" $t^{ab}$, inferred only indirectly, such that a continuity equation of the type $\partial_b(T^{ab} + t^{ab}) = 0$ is restored.

Hoefer rejects this received view on two grounds: Firstly, the non-uniqueness of $t^{ab}$, in his opinion, undermines its well-definedness. Secondly, he takes its non-tensorial nature to obviate interpreting $t^{ab}$ as an intrinsically meaningful, well-defined quantity. With the jury still out on future progress with respect to quasi-local definitions of energy in GR (which try to associate energy-momentum (density) not with individual spacetime *points*, but only with extended, finite *regions* of spacetime), Hoefer enjoins us to relinquish both the notion of local gravitational energy-momentum and conservation of energy-momentum in GR altogether.

Hoefer's arguments are not likely to sway believers in gravitational energy. To begin with, his claim that the gravitational energy-momentum pseudotensor $t^{ab}$ is inferred "only indirectly", insinuating its *ad-hoc* character, is misleading: As we will sketch in §3.3, $t^{ab}$ arises in a *direct* way no less naturally than energy-momentum in other field theories.

Hoefer's objections, too, call for further clarification. For one, the nature of the ambiguity and non-uniqueness of pseudotensors must be fleshed out: What does it consist in? How severe is it? Is it physically significant? Furthermore, vis-à-vis Hoefer's objection to the lack of coordinate invariance, one may be tempted to bite the bullet: What is inherently wrong with non-tensors? In itself, an object's non-tensoriality need not undercut its meaningfulness: The connection coefficients, $\Gamma^a_{bc} = \frac{1}{2}g^{ad}(\partial_c g_{db} + \partial_b g_{dc} - \partial_d g_{bc})$, attest to that. They are endowed with both a physical and geometric meaning, representing inertial structure, and (in the language of fiber bundles) connecting the fibers of the vector bundle over different points of the base manifold, respectively. Even if one shares Hoefer's skepticism towards pseudotensors, are they indeed the *only* way to locally represent gravitational energy? Might there exist other approaches? The answer is yes. Bel and Robinson, for instance, have proposed a tensor which mimics the way the electromagnetic energy-momentum tensor is constructed from the Faraday tensor (see, for instance Horský & Novotný, 1969, sect. III.4). To be sure, as a candidate for gravitational energy the Bel-Robinson tensor is not immune to criticism: Neither it nor any of its powers possess the right units of energy-momentum. It would thus mandate a novel constant of nature, i.e. an additional structure, absent in GR *simpliciter*.

This leads us to Lam's recent refinement of Hoefer's critique (Lam, 2011). He adverts to the need to introduce additional structure, in order for local energy-momentum conservation to hold and gravitational energy-momentum to be well-defined.

Lam discusses the important special case of spacetimes instantiating a so-called Killing field (see §2.2). He argues that only such spacetimes allow $\nabla_b T^{ab} = 0$ to be interpreted as a well-defined local (and global) notion of energy-momentum conservation.

Lam suggests the following construal: "[…] a time-like Killing vector field can be understood as defining a global inertial frame, which can represent a global family of inertial observers all at rest with each other" (p. 5). This global inertial frame, Lam asserts, "[…] can be understood in a certain sense as a nondynamical background structure with respect to which integral nongravitational energy-momentum can be obtained. […] A fully dynamical metric field would prevent the existence of such global symmetries. […] In this sense, the nature of gravitational energy seems to be linked to the failure of certain global symmetries and, most importantly to the lack [of] non-dynamical background structures, that is to background independence" (ibid). In other words, GR's background independence (absence of non-dynamical objects), Lam argues, subverts energy-momentum.

Lam deserves credit for honing Hoefer's analysis of the pseudotensor by specifying what in his opinion their ambiguity consists in: the freedom to insert a term of the form $\partial_c U^{a[bc]}$, i.e. antisymmetric in $b$ and $c$, into the continuity equation without altering it: $\partial_b(T^{ab} + t^{ab} + \partial_c U^{a[bc]}) \equiv \partial_b(T^{ab} + t^{ab})$. The pseudotensor, Lam concludes, hence lacks uniqueness in that any $t^{ab} = \partial_c U^{a[bc]} - \frac{1}{2\kappa}G^{ab}$ equally satisfies the continuity equation. Here we have exploited the Einstein Equations, $G^{ab} = 2\kappa T^{ab}$.

He also sharpens Hoefer's principal argument against pseudotensors, their coordinate-dependence. "In particular, at any spacetime point or along any world-line, there is a coordinate system in which $[t^{ab}]$ vanishes. Since different coordinate representations are just different mathematical descriptions, relevant physical entities are usually taken to correspond to coordinate-independent entities […]. So, the coordinate […] dependence of $[t^{ab}]$ shows that there is no (unique) gravitational energy-momentum, in the sense that such a quantity cannot be in general unambiguously defined at any spacetime point" (p. 6).

Notwithstanding its improvement over Hoefer's account, Lam's own remains unsatisfactory:

For one, as we will see below, while it is true that the existence of Killing symmetries is sufficient for the validity of local energy-momentum conservation, it is not necessary-—-contrary to what Lam seems to suggest. More precisely, Lam implicitly presupposes an gratuitously strong definition of inertial reference frames for GR, one that posits the existence of Killing fields.[2]

Lam's elucidations regarding the problematic status of and link between energy-momentum conservation and gravitational energy are deficient, as well. The connection between energy-

[2] Presumably, Lam has in mind something like Earman & Friedman's definition of inertial frames explicitly determined by a time-like Killing vector (1973, pp. 353). It remains unclear, however, why a natural general-relativistic generalisation of the notion of inertial frame in Newtonian Gravity (e.g. p. 332), should employ such a strong prerequisite. (The Newtonian counterpart only involves a time-like vector field *simpliciter.*) For one, strictly speaking this restricts the existence of inertial frames a priori to stationary spacetimes. Thereby, one precludes any even remotely realistic scenarios. The motivation behind Earman and Friedman's stipulation is to preclude spacetimes, such as Gödel's, in which no globally defined reference frames exist. These they dismiss for not allowing of a "globally consistent time sense" (p. 353). Suffice it to say here that Earman and Friedman fall short of a cogent argument why the non-orientability of certain spacetimes is supposed to be linked to inertial frames specifically: Why should we assume that the problems of how to interpret such spacetimes originate in the non-existence of global inertial frames? It seems no less plausible to concoct a definition of inertial frames that applies to such non-orientable spacetimes – and dismiss the latter (should one feel so inclined, at all) as unphysical on independent grounds. By contradistinction, I'll adopt the weaker and more natural notion of general-relativistic inertial frames, identifying them with freely falling frames (e.g. DiSalle, 2009, sect. 2.9).

momentum conservation, gravitational energy, symmetries and GR's background independence remains hand-waving-—-not least, since a precise definition of background independence is notoriously tenuous (see, e.g. Pooley, 2015, esp. sect. 7; Read, unpublished).

As in Hoefer's case, a proponent of gravitational energy is likely to counter Lam's ambiguity objection to pseudotensors by asking in what ways the situation differs from other field theories. It is crucial to note that the way pseudotensors arise in GR in a *standard* way via the Noether Theorems. Lam mentions them in a footnote. But he does not expand on the connection. This would have preempted a potential misunderstanding: The form of the continuity equation both Hoefer and Lam discuss is restricted to special (unimodular) coordinates, satisfying $|g| := \det(g_{\mu\nu}) = 1$. But such a restriction is unnecessary. More importantly, it is *not* the source of the (vicious) coordinate-dependence of pseudotensorial expressions.

Furthermore, a naive reading of what Lam offers as another way to understand the pseudotensor -namely as nonlinear corrections to the linearized Einstein tensor, i.e. higher perturbative orders[3]- is problematic. It suggests that the nonlinearity of the Einstein Equations reflects the fact that gravitational energy acts as a source of the gravitational field itself. This is misguided: Firstly, a *consistent* implementation of self-energy in (linear) Newtonian Gravity creates its own problems (see Giulini, 1997). Secondly, the desire to *explain* GR's nonlinearity stems from comparing it to *linear* theories. Both physically, with the familiar linear theories being non-fundamental, and structurally, such a comparison is implausible, though: Rather, one should compare GR with likewise non-linear vectorial Yang-Mills-type theories (cf. Deser, 1970). An immediate lesson is: nonlinearity *is not* the source of GR's trouble with its field-energy.

Similarly, Lam's objection of coordinate-dependence is not persuasive. Coordinate-dependence by itself need not be problematic: As long as a coordinate-dependent structure has the same symmetry group as the one of the *given* spacetime, the coordinate dependence is benign. It becomes vicious only if the coordinate-dependent object is not invariant under the spacetime's symmetry transformations. I revert to this in §3.3. Although I concur with Lam's thought that coordinate-dependence of the type pseudotensors elicit amounts to an unphysical gauge-fixing, we must clarify whether pseudotensors are benign or vicious in the sense just mentioned.

Hoefer and Lam raise deep questions, and I am largely in agreement with their positions. The following paper intends to supplement their work, attempting to fill some of the indicated lacunae and to provide a systematic account of local energy-momentum conservation and local notions of gravitational energy within GR.

## 2. Energy-momentum conservation in local form

### 2.1. Energy-momentum conservation in generic spacetimes

In lieu of the ordinary (partial) zero-divergence for the matter energy-momentum tensor, in GR we have the zero *covariant* divergence, $\nabla_b T^{ab} = 0$. Here, $T_{ab} = -\frac{2}{\sqrt{|g|}} \frac{\delta}{\delta g^{ab}} (\sqrt{|g|}\, \mathscr{L})$ denotes the matter energy-momentum tensor, with the matter Lagrangian $\mathscr{L} = \mathscr{L}(g_{ab}; \psi, \nabla_a \psi, \nabla_{a,b}\psi, ..)$ for the (tensorially generic[4]) matter fields $\psi$.[5] Throughout, a Lagrangian approach will be assumed also for GR's matter sector. The vanishing of the covariant divergence holds independently of the Einstein Equations: It follows from diff$(\mathscr{M})$-invariance, the imposition that the dynamical matter variables $\psi$ satisfy the Euler-Lagrange Equations, and the above form of the matter Lagrangian (see e.g. op. cit., sect. 3 for technical details).

In the literature (e.g. Brading and Brown, 2002; Weinberg, 1972, p. 166; Padmanabhan, 2010, p. 213, it is sometimes stated that $\nabla_b T^{ab} = 0$ does not express local energy-momentum conservation simpliciter; rather, it is aid to denote the degree to which conservation is violated. This is supposed to quantify the extent to which energy-stress density/flux is no longer source-free/sink-free in *generic* reference frames:

$$\partial_b T^{ab} = -2\Gamma^{(a}_{bc} T^{b)c} (\not\equiv 0) \tag{1}$$

This is not the most perspicuous way of formulating the problem. It distracts from what is *special* about GR's energy-momentum: In which regard does the above situation differ from conservation of the external electric 4-current, $\nabla_a j^a_{(ext)} = 0$? After all, it *too* can be rewritten to yield apparent "sources/sinks" in generic reference frames:

$$\partial_a j^a_{(ext)} = -\Gamma^a_{ab} j^b_{(ext)} (\not\equiv 0) \tag{2}$$

Only for unimodular coordinates (modulo global re-scalings), i.e. coordinates satisfying $\sqrt{|g|} = const.$, does the "source term" of the continuity equation on the r.h.s. vanish.[6]

(The restriction to unimodular coordinates can be lifted. By considering the weight-1 vector *density* $\tilde{j}^a_{(ext)} = \sqrt{|g|}\, j^a_{(ext)}$, one obtains a 4-current that satisfies a continuity equation in *every* coordinate system:

$$\partial_a \tilde{j}^a_{(ext)} \equiv 0 \tag{3}$$

The electric charge flux $\tilde{j}^a_{(ext)}$ thus defined is locally conserved without qualification.)

To see what is troublesome about energy-momentum in GR specifically, we must be more circumspect. Consider an arbitrary time-like vector $\xi$. Along its direction, one can define the energy-momentum 4-current $j^a[\xi] = T^a_b \xi^b$. Unless $\xi$ is special (in a sense to be made precise presently), the covariant divergence of this 4-current does *not* vanish, $\nabla_a j^a[\xi] = T_{ab} \nabla^a \xi^b \neq 0$. This yields an ordinary continuity equation with *non-vanishing* source/sink terms in *any* reference frame: In *no* reference frame is $j^a[\xi]$ locally conserved. (The transition to a vector density is of no avail here.)

In short: The external electric 4-current satisfies an ordinary continuity equation for *some* coordinate systems – namely unimodular ones. (The corresponding weight-1 density does so even for

[3] E.g. Hobson et al., 2006, p. 473 for a similar claim.

[4] For how spinorial matter fits into the picture see Pitts, 2011.

[5] Despite representing gravity via variational derivatives of a gravitational Lagrangian, Einstein never systematically defined energy-momentum tensor variationally nor did he work with matter Lagrangians (see Pitts, 2016a).

[6] Note that with this fixed volume element, unimodularity introduces an absolute structure, extraneous and inimical to GR's overall non-absoluteness (cf., for instance, Anderson, 1967). The choice of unimodular coordinates is directly related to inertial frames – a concept that will occupy centre stage in the subsequent analysis. With respect to their global/integral properties, unimodular coordinates can be regarded as the closest counterparts of Lorentz/Euclidean coordinates on generically curved manifolds: They preserve a constant volume element. $\partial_a j^a_{(ext)} = -\Gamma^a_{ab} j^b_{(ext)}$ yields a continuity equation proper, only if one selects as reference frames those counterparts of *inertial reference frames in Minkowski spacetime*. But the latter cease to be distinguished in *GR*. One conclusion of my subsequent arguments is that we should take *GR's* notion of inertial structure more seriously: We need not, nor should not, import it from non-GR theories.

*every* coordinate system.) By contrast, whether the energy-momentum 4-current $j^a[\xi]$ satisfies an ordinary continuity equation (for some reference frames) depends on the choice of its direction $\xi$.

In generic ("non-symmetric", see §2.2) spacetimes, what are these distinguished directions of energy-momentum flux along which one would register the absence of sinks/sources? For inspiration, consider a free-falling observer $\xi$. In her proper (comoving) reference frame $\gamma$, we have $\xi^a|_\gamma = \delta^a_0$ and $\Gamma^a_{bc}|_\gamma = 0$. The corresponding energy-momentum 4-current $j^a[\xi]$ that $\xi$ measures *along her worldline* is source-/sinkfree: $\partial_a j^a[\xi]|_\gamma = 0$. Sources/sinks would appear for $\xi$ only, when adopting reference frames other than her proper one. The energy-momentum 4-current along the worldline of observers *not* in free-fall, i.e. non-inertial observers, is locally conserved in *no* reference frame.

How to construe these sinks/sources in local energy-momentum 4-currents for generic reference frames and along generic directions? Does it *seriously* jeopardise *real* energy-momentum conservation (in a sense to be made precise)? For an answer, I will first identify the class of $\xi$s picked out by the apparent anisotropy of local energy-momentum conservation, as well as the reference frames for which energy-momentum is uncontroversially conserved locally.

A prior side-glance to CM is instructive. There, an analogous problem arises for apparent/fictitious/inertial forces, e.g. the Coriolis or centrifugal force. They flout Newton's Third Law of action-reaction. In contrast to *genuine* ones, apparent forces do not mediate physical interactions. They are not causes nor are they caused (cf. Nerlich, 1989, sect. 5). We do not ascribe them the status of entities "out there", forces as real as, say, the Lorentz force. They are more like shadows: ontologically dependent, causally inert and explanatorily non-fundamental. We are wont to conceive of them as springing from descriptions in non-inertial coordinate systems. Ontologically, apparent forces are reduced to inertial motion, as it *appears* from non-inertial reference frames. They are artefacts of physically artificial descriptions (e.g. Maudlin, 2012, pp. 23 fn. 7).

Inertial reference frames are inherently distinguished by "natural" (as opposed to "forced") motion (cf., for instance, Brown, 2007, p. 163). This class of kinematic states, privileged as default motion, is furnished the theory's inertial structure. Bodies moving inertially do *not* call for deeper explanations of this kind of motion (Janssen, 2009; Nerlich, 1979). Only non-inertial motion does: When a body *deviates* from inertial motion we ask for causes—in the form of external forces.

"Inertially framed" accounts afford simpler explanations than a "non-inertially framed" one. They are adjusted to the spacetime geometry, thereby making explicit what merits realist commitment. Conversely, extra terms that arise in non-inertial frames are representational (or perspectival) artefacts of physically unnatural descriptions.

How do these remarks bear upon GR's local energy-momentum conservation? Firstly, they answer which directions are distinguished in generic spacetimes: directions along natural/inertial motion. In GR, this is motion along (time-like) geodesics (for systematic and historical details, see e.g. DiSalle, 2009, esp.2.9; Petkov, 2012; Knox, 2013, esp. sect. 2). Secondly, we also get an answer to what the privileged reference frames are: the inertial ones. In GR, the latter are identified as free-fall frames $\gamma$, the coordinates adjusted to them being normal coordinates. (Henceforth, I will use Fermi's.) By construction, their connection coefficients vanish, $\Gamma^a_{bc}|_\gamma = 0$.

For a geodesic/free-fall trajectory $\zeta$, the inertial frame $\gamma$ is comoving. Concomitantly, in the adapted inertial coordinates, $\zeta^a =$ *const*. In consequence, the energy-momentum 4-current along free-fall trajectories $\zeta$ is locally conserved, $\partial_a j^a[\zeta]|_\gamma = 0$.

The lesson to be drawn is this: *Apparent* violations do not evince a *real* break-down of local energy conservation. Nor do they betray that we have neglected some (presumably: gravitational) energy contributions. Rather, such "violations" are artefacts of an *unphysical* direction for the 4-current or of adopting non-inertial frames.[7] Neither should unsettle us. GR's matter energy-momentum 4-current is free of sinks/sources *no less than* in CM or SR. In these, as well, energy can *appear* not to be locally conserved, when adopting non-inertial/accelerated reference frames.

GR and pre-GR theories differ, of course, in their specific inertial structure. I will outline the ramifications these differences entail for *global* energy-momentum conservation shortly (§2.2). But first, it is apposite to discuss GR's local energy-momentum conservation from a different angle that links it to gravitational energy.

Following Einstein in his 1916 GR review paper (see Hoefer, 2000, p. 191), $\partial_b T^{ab} = -2\Gamma^{(a}_{bc}T^{b)c}$ has occasionally (e.g. Brading & Brown, 2002, p. 17) been dubbed the "response equation". Putatively, it captures the interchange of energy-momentum between gravitational and ordinary energy-momentum - gravity's back-reaction upon matter. I commented already on how focusing of the decomposition of $T^{ab}$'s vanishing covariant divergence, $0 \equiv \nabla_b T^{ab} = \partial_b T^{ab} + 2\Gamma^{(a}_{bc}T^{b)c}$, is misleading.[8]

One can reformulate the response equation interpretation (REI) more judiciously.[9] Recall that for generic spacetimes and $\xi$, the covariant divergence of the energy-momentum flux along $\xi$ does not vanish:

$$\nabla_a j^a[\xi] = T^a_b \nabla_a \xi^b (\neq 0) \tag{4}$$

According to the ameliorated REI, the non-vanishing $\nabla_a j^a[\xi]$ reflects the intertwinement of gravitational and matter energy-momentum. The presence of sinks/sources in the 4-current on the r.h.s. is attributed to the *neglect* of gravitational energy contributions. (Equivalently, consider the weight-1 density $\tilde{j}^a[\xi] := \sqrt{|g|}j^a[\xi]$. The corresponding continuity equation with source terms is: $\partial_a \tilde{j}^a[\xi] = \sqrt{|g|}T^a_b \nabla_a \xi^b$.)

What is the relationship then between the presence of gravitational energy and the presence of sinks/sources? Is the former *necessary* for the latter? That is: Is gravitational energy the only reason for the failure of the conservation of matter (non-gravitational) energy-momentum? This seems implausible: In inertial frames, the r.h.s. of $\nabla_a j^a[\xi] = T^a_b \nabla_a \xi^b$ reduces to $T^a_b \partial_a \xi^b$. The latter, however, is not straightforwardly related to gravity: One would expect gravitational degrees of freedom to be encoded in metric-dependent quantities. The fact that the 4-current $j^a[\xi]$ contains sinks/sources thus would *not* be related to gravity, either. (One may object to evaluating $\nabla_a j^a[\xi]$ in inertial reference frames, as in the latter, gravity has been "geometrised away". Rather than an

[7] Landau and Lifshitz (1975, p. 283) insist (with neither argumentation nor even explication) that any serious candidate for energy must satisfy a continuity equations in *all* reference frames. I reject this assumption: There is nothing wrong with (and hence nothing to ameliorate in) an equation that does *not* take its simplest form in non-inertial frames. Presumably, what motivates Landau and Lifshitz's reasoning is Einstein's (erroneous) interpretation of general covariance as an extension of the Relativity Principle, asserting the equivalence of *all* reference frames (cf. Norton, 1985).

[8] Due to his own interpretation of GR, Einstein had no such qualms talking about "inertial" and "gravitational" components of the decomposition of the geodesic equation (see Lehmkuhl, 2010).

[9] The improved form of the REI avoids the objections against its usual form (cf. Read, 2018).

objection, this objection just anticipates the conclusion *for* which I ultimately argue.)

With reason, the REI can, however, take the presence of gravitational energy to be *sufficient* for the presence of sinks/sources in $j^a[\xi]$. Equivalently by contraposition, $\nabla_a j^a[\xi] = 0$ should imply the *absence* of gravitational energy. This makes sense: If the matter energy-momentum 4-current contains no sources/sinks, gravitational energy does not contribute to the energy balance.

Does anything more interesting follow from this claim of the REI for gravitational energy? For an answer, first recall our earlier discussion that in generic spacetimes energy-momentum 4-currents along directions other than along inertial trajectories are physically ungrounded: We need not extend our realist commitments to such quantities. Now consider a situation (in a non-symmetric spacetime) with gravitational energy present. According to the REI (again by contraposition), it would follow that $\nabla_a j^a[\xi] \neq 0$. This is possible (in non-symmetric spacetimes) only for $\xi$s that describe *non-inertial* trajectories or a description in non-inertial reference frames. Hence, this non-conserved form of $j^a[\xi]$ is barred from our realist commitment. In short: The REI implies that gravitational energy leads to what would *appear* as a violation of local energy-momentum conservation. Our analysis of inertial motion, however, disclosed that the energy-momentum 4-currents that apparently are not locally conserved are *unphysical*.

Consequently, according to the REI -as a plausible link between gravitational energy and local energy-momentum conservation-gravitational energy is an idle wheel: *Real* energy-momentum -energy-momentum meriting realist commitment-is locally conserved in GR; gravity does not contribute to the energy balance equation.

This renders precise and corroborates Norton's conjecture that GR, as a theory that "geometrises away"[10] gravity, also compromises gravitational energy. Like apparent forces in CM, gravity is not a force arising from gravitational interaction, a *real* entity. In GR, gravity is not a force: Gravitational phenomena are manifestations of non-Minkowskian inertial structure (see Earman & Friedman, 1973, sect. 5; Norton, 2003; Nerlich, 2013).[11] GR thereby inaugurates a shift in what phenomena are in need of explanations in terms external causes (see Nerlich, 2013, Ch. 8; Dorato, 2014) Non-vanishing gravitational energy is an artefact of bestowing on directions and non-inertial reference frames a physical significance that in truth they lack.[12] (Of course, in generic spacetimes -those lacking time-like Killing fields-inertial reference frames exist only "locally": Only along a single, privileged (geodesic) time-like path can we find coordinate-systems whose time-like axes move inertially. I return to this later on in §2.2.) In short: Non-vanishing gravitational energy is the result of a spurious realist commitment. I will call this position "eliminativism about gravitational energy".

Taking seriously GR's inertial structure, I argued in this section that local energy-momentum conservation is valid in GR. Apparent violations in GR for certain reference frames and along certain directions merit no more realist commitment than violations of Newton's Third Law by apparent forces. Just as one should be an eliminativist about apparent forces in CM, one should be an eliminativist about those energy-momentum 4-currents in GR that are not conserved.

In pre-GR theories, local energy-momentum conservation gives rise to an invariant *global* conservation law, as well: The energy of matter contained in a closed space-like hypersurface remains constant across time. Does this also hold in GR?

### 2.2. Local energy conservation in symmetric spacetimes

In the previous section we considered generic spacetimes, devoid of symmetries. What changes with respect to energy-momentum conservation in a spacetime with symmetries?

In CM, it is often said (e.g. Landau & Lifshitz, 1976, §6–9) that conservation of energy and linear/angular momentum are correlated with the homogeneity of time and the homogeneity/isotropy of space, respectively, via Noether's First Theorem. (The latter remains neutral, though, about spacetime symmetries per se.[13] Rather, it establishes a link between conserved quantities and symmetries of an *action* under rigid coordinate transformations (see Sus, 2017for a lucid account). But because in CM Euclidean coordinates are inertial coordinates, one may identify rigid transformations as time/space translations.)

GR's spacetime symmetries are expressed by means of Killing vectors $\xi$, the infinitesimal generators of a spacetime's isometries. They are defined via a vanishing Lie-drag of the metric along them

$$0 = \mathfrak{L}_\xi g_{ab} = \nabla_{(a}\xi_{b)} \tag{5}$$

Killing vectors give constants of motion along a geodesic $x^a(\tau)$, with the affine parameter $\tau$:

$$\frac{d}{d\tau}(\dot{x}^a \xi_a) = 0 \tag{6}$$

For energy-momentum $T^a_{\ b}$ in particular, the existence of a time-/spacelike Killing field $\xi$ gives rise to an energy-momentum 4-current $j^a[\xi] = T^a_{\ b}\,\xi^b$ that satisfies a local conservation law,

$$\nabla_a j^a[\xi] = T^{ab}\nabla_a \xi_b = -T^{ab}\nabla_b \xi_a = -T^{ab}\nabla_a \xi_b \equiv 0 \tag{7}$$

In inertial frames $\gamma$, it even simplifies to a familiar, ordinary continuity equation, $0 \equiv \nabla_a j^a[\xi]|_\gamma = \partial_a j^a[\xi]$. (As in §2.1, the transition to the vector density yields a continuity equation in all reference frames.) The Killing field thus provides a distinguished direction along which energy-momentum has no sinks/sources. The status of conservation of an energy-momentum 4- current along a Killing field is the same as that of conservation of the external electric 4-current in Maxwellian electrodynamics.

Assuming that $T^a_{\ b}$ has compact support (or benign fall-off conditions), the 4-current $j^a[\xi]$ also gives rise to an associated invariant, *globally* conserved "charge" $Q[\xi] = \int_\Sigma d\Sigma_a j^a[\xi]$ (with the

[10] "*Geometrising* away" must not be confused with "*transformed* away". The former denotes the fact that gravitational phenomena are not attributed to external forces which deflect particles from their rectilinear inertial paths, (see, for instance, Maudlin, 2012, Ch. 6). Rather, they are reconceptualised as manifestations of a non-Newtonian/non-Minkowskian inertial structure. "Transforming away", on the other hand, suggests that one could make these effects disappear though a suitable choice of coordinates. This is not the case for GR: Consider, for instance, the wave-equation for the Faraday tensor in general-relativistic Einstein-Maxwell Theory with external current $J^a$. It contains gravity-related curvature terms (see Read et al., 2017 (ms), sect. 2,3 for details):

$$\nabla_c \nabla^c F_{ab} = F^d_{\ [b} R_{a]d} - R_{abcd} F^{cd} - \nabla_{[b} J_{a]}.$$

As tensors, these curvature-containing terms cannot be eliminated through any choice of coordinates.

[11] Cf. Dewar & Weatherall, 2018, esp. section 5 for the similar case of Newton-Cartan-Theory, a geometrised version of Newtonian Gravity.

[12] In [rDürr, 2018 ms], the case of gravitational waves, and the question whether they carry energy, is discussed. I argue that the standard arguments supposed to demonstrate that they do are not convincing. An alternative account of the spin-up of binary systems is given. (Of course, I do not deny the *reality* of gravitational waves and their effects. I only deny that they *transport* energy.)

[13] I am grateful to Brian Pitts (Cambridge) for pressing me on this point.

directed infinitesimal volume element $d\Sigma_a$): $Q[\xi]$ doesn't depend on the choice of the Cauchy hypersurface $\Sigma$ (see, for instance, Padmanabhan, 2010, Ch. 6.5 for details).

Spacetimes with Killing symmetries thus admit of both, local and global energy-momentum conservation. Minkowski space, for instance, possesses ten Killing fields, corresponding to the 10-dimensional Lie algebra of the Lorentz group. They are associated with conservation of energy, linear and angular momentum. The coordinates that are adapted to the time-/spacelike Killing vectors correlated with energy-momentum conservation are *globally* defined (as opposed to defined only at a *point* or along a *curve*, as in the GR case for Riemann or Fermi normal coordinates, respectively) inertial coordinates—the familiar Cartesian/Lorentz coordinates.

Contrast the situation with the non-symmetric spacetimes from §2.1. In the absence of Killing symmetries $\xi$, the 4-current $j^a[\xi]$ is source-/sinkfree merely for the only inherently distinguished directions available in such spacetimes: $\xi$s along *inertial/free-fall* trajectories. The coordinates adapted to the inertial frames are comoving. They are only defined along the inertial paths, not globally. For non-Killing $\xi$, the 4-current $j^a[\xi]$ does not yield a well-defined global charge: Different 3+1-decompositions of spacetime imply different charges, each such slicing in itself being but an arbitrary conventional choice. In particular, the charges are not conserved across time: For a 3+1-decomposition of the manifold into the one-parameter family of spacelike hypersurfaces $\{\Sigma_\sigma : \sigma\}$, we have:

$$\frac{d}{d\sigma}\int_{\Sigma_\sigma} d\Sigma_a j^a[\xi] \neq 0 \qquad (8)$$

Generic spacetimes lack Killing symmetries. In particular, in generic spacetimes energy-momentum fails to be conserved *globally*. This is illustrated the "singularly striking example" (Schrödinger, 1950, p. 105) of the decrease of energy in a closed bounded universe (cf. Misner, Thorne, & Wheeler, 1974, §19.4 for technical details): "In simple models the loss [of energy contained in a closed 3-vol] can be computed and equals the amount of work the pressure would have to do to increase the volume, if a piston had to be pushed back as in the case of an adiabatically expanding volume of gas." However, Schrödinger stresses, this is a *fictional* account: There is no piston nor any boundary through which energy could escape. Global energy conservation just ceases to be valid for the expanding universe.

Strictly speaking, such inertial frames, in which energy-momentum is conserved, have zero-volume. Consequently, matter energy-momentum is conserved only in systems of zero-volume: Prima facie, energy-momentum would thus fail to be conserved in *any* systems of interest. Only point-/line-/plane-/hyperplane-like things could happen in inertial frames, precluding nearly everything physicists like to study. Is this an inacceptable consequence, tantamount to a reductio?

Two considerations attenuate the objection. Firstly, many systems of interest, such as the chemical energy in my car's engine, are *sufficiently* small: Relative to the relevant scales, they occupy zero-volume. For all practical purposes these systems' energy-momenta then are conserved. (From a *Newtonian* perspective, their gravitational potential energy does not change.) Secondly, also cases of non-negligible energy-momentum non-conservation can be handled *for most practical purposes* — within a certain regime. Notice that the degree of non-conservation is well-quantifiable. Within some degree of accuracy and suitably small world-tubes around inertial paths,[14] it is thus possible to restore the *apparent* conservation. One only needs to add the "missing" bits by fiat: From a *Newtonian* perspective, these would correspond to the Newtonian gravitational potential energy (and its post-Newtonian correction) to the non-gravitational energy-balance. But from the more fundamental perspective of *GR*, this potential gravitational energy is *fictitious*: They are translations (or projections) of *GR* phenomena onto a *pre-GR* framework.[15] (This fictional "Newtonised" account ceases to be available beyond a certain degree of accuracy, and in particular, if the curvature effects are very strong even for the relevant volume scales.)

This brings us back to GR's gravitational energy. In §2.1–2, we achieved a transparent account of local energy conservation in GR. In it, gravitational energy was wholly absent. One may take this absence at face value - to the effect that in GR gravitational energy is dispensable. Some will deem this too quick. They may shrug off that absence as irrelevant to the *possibility* of meaningfully defining gravitational energy-momentum. But is there any motivation for that? Carroll (2010), for instance, impugns this. With respect to concocting notions of gravitational energy to restore the violation of global energy conservation, he writes: "the entire point of this exercise is to explain what's going on in GR to people who aren't familiar with the mathematical details". Yet, it seems fair to countenance that our analysis of energy-momentum conservation may not have been the right starting point for a search of gravitational energy.[16] Next, I will therefore squarely examine extant proposals for local representations of gravitational energy.

## 3. Local gravitational energy

### 3.1. Tensorial hopes?

Only a year after presenting GR in its full form, Einstein applied to it Noether's Second Theorem (*avant la lettre*). From the invariance under arbitrary coordinate transformations as the general symmetry of GR's action (modulo surface terms), four continuity equations ensue (see Brading, 2005) for historical and mathematical details):

$$\partial_b\left(\sqrt{|g|}\left(T_a^b + t_a{}^b\right)\right) = 0 \qquad (9)$$

Here, $t_a{}^b$ is a suitable object, called "energy-momentum pseudotensor". It corresponds to the Noether-current for the purely gravitational Lagrangian. (More on this shortly.) It is non-unique: Due to its anti-symmetry in the upper indices, inserting an arbitrary term of the form $\partial_c \mathfrak{U}_a^{[bc]}$ into the continuity equation leaves the latter unaffected.

The most prominent example of a pseudotensor is Einstein's:

$$t_a{}^b = \frac{1}{\sqrt{|g|}}\left(-\mathfrak{S}\delta_a^b + \left(\frac{\partial\mathfrak{S}}{\partial(\partial_b g_{cd})} - \partial_e\frac{\partial\mathfrak{S}}{\partial(\partial_{b,e} g_{cd})}\right)\partial_a g_{cd}\right) \qquad (10)$$

Here, $\mathfrak{S} = \sqrt{|g|}g^{ab}\Gamma^d_{a[b}\Gamma^c_{c]d}$ is the so-called the truncated/ "$\Gamma\Gamma$-"Lagrangian.

[14] Essentially, this is the case in the regime in which the PPN-formalism is applicable and in which the Equations of Motions (i.e. the field equations for matter/non-gravity) admit of a Lagrangian formulation (cf. Poisson & Will, 2016).

[15] In [reference suppressed], I clarify in the context of gravitational waves the non-trivial difference between violation of energy-momentum and energy momentum depletion via transport (cf. also Weatherall, 2016, Ch.2, fn 103).

[16] Suppose one shares this view. That is: Suppose that one disputes that considerations of energy conservation have a direct bearing on gravitational energy. Then, the simple Geroch-Malament argument against a local gravitational energy in GR that Dewar and Weatherall (2018, pp. 1) cite is blocked.

Four features of the Einstein pseudotensor stand out, underscoring the continuity with other field theories. Firstly, like other energy-momenta from relativistic field theory, it contains solely first derivatives of the field variables, $g_{ab}$. Einstein's pseudotensor is constructed fully analogously to energy-momentum in other field theories via the customary Noetherian machinery.[17] The reason is that rather than the full Einstein-Hilbert Lagrangian (plus nondynamical and boundary terms) we may utilize the $\Gamma\Gamma$-Lagrangian, itself containing only first derivatives of $g_{ab}$ (see e.g. Hobson, Lasenby, & Efstathiou, 2006, Ch. 19; Poisson, 2007; Ch. 4.1 for technical details). Secondly, Einstein's pseudotensor is index-asymmetric. This mars its utility for defining angular momentum. But the shortcoming can be amended by the Belinfante-Rosenfeld symmetrisation (see Belinfante, 1940; Rosenfeld, 1940). This technique is familiar from the likewise non-symmetric energy-momenta in hydro- or electrodynamics (The technicalities shall not detain us.). Thirdly, although the Einstein pseudotensor transforms tensorially only under *affine* transformations, the continuity equation, $\partial_b(\sqrt{|g|}(T^b_a + \ \mathrm{t}_a^{\ b})) = 0$, is valid for *every* coordinate system. Fourthly, the weak-field limit reproduces the classical potential energy, and yields reasonable "kinetic" terms for gravitational waves (see e.g. Maggiore, 2007, Ch. 1–3).

Yet, GR's general covariance makes things a little more delicate, when it comes to the symmetrisation procedure and the non-tensoriality. Leclerc, 2006, (p. 3) cautions that the Belinfante-Rosenfeld symmetrisation presupposes a distinction of certain coordinates inherently not warranted in GR: "The Belinfante procedure relies on the Noether current corresponding to global Poincaré (coordinate) transformations. Certainly, any diffeomorphism invariant action will also be globally Poincaré invariant, but there is no apparent need, a priori, to favor a certain subgroup. In our opinion, this is against the spirit of general relativity. (For instance, in [GR] with cosmological constant, the de Sitter subgroup is at least equally well justified.)" So, if one requires that a suitable candidate for gravitational energy-momentum be index-symmetric, and given that the Belinfante-Rosenfeld symmetrisation exalts certain symmetries *in an ad-hoc way*, then Einstein's pseudotensor seems just not suitable. One might counter: Don't free-fall inertial frames already privilege the Poincaré transformation? Consequently, the Poincaré group would seem already distinguished. It is unclear, however, that this distinction is relevant. Local Poincaré transformations relate only inertial frames. But in inertial frames the Einstein pseudotensor vanishes, anyway. (N.B. This argument does not apply to *non-gravitational* energy-momenta. The Belinfante-Rosenfeld symmetrisation thus does *not* elevate the Poincaré group in any ad-hoc way.)

The other feature of pseudotensors -their non-tensorial nature- looks even more suspect in light of GR's geometric, coordinate-free spirit. Doesn't non-tensoriality conflict with the invariance one would naturally demand of real objects? Before pursuing this further in §3.2, we should enquire into the necessity of pseudotensors: Could pseudotensors for local representations of gravitational energy-momentum perhaps be avoided? Several authoritative texts (e.g. Misner et al., 1974, p. 467), deny this, pointing to the Equivalence Principle: Since gravity, the argument goes, can always be made to vanish locally by adopting a free-fall reference frame, gravitational energy can always be "transformed away".

However, the argument has a flaw: It presupposes that the alleged gravitational energy-momentum depends only on *first* derivatives of the metric. Only they could be "transformed away" in suitable coordinates. Why insist on that assumption? Pauli (1981, Ch. 61), for instance, voiced his misgivings along the following lines: Gravity manifests itself as curvature (think e.g. of geodesic deviation), represented by the Riemann tensor, $R^d_{abc} = \partial_{[b}\Gamma^d_{c]a} + \Gamma^e_{a[c}\Gamma^d_{b]e}$. But the latter is built from up to *second* derivatives of the metric. Hence, one might expect any natural representation of gravitational energy-momentum likewise to be built from up to second derivatives of the metric.

From the Noetherian perspective on pseudotensorial gravitational energy (more on this in §3.2), such considerations might appear futile: Why seek an object with second derivatives, when the terms in the Einstein-Hilbert Lagrangian that contain higher derivatives make no difference to the field equations? In response, note that although a Lagrangian approach is often fertile, it is unclear whether the Lagrangian is more than a mathematical expedient, useful but not physical – comparable to, say, ghost fields in gauge quantum field theory. Suppose first that one adopts a merely instrumentalist stance towards the Lagrangian, i.e. regarding the Einstein-Hilbert Lagrangian as not physical in any direct sense. The co-existence of Lagrangians that do not differ merely by surface terms might suggest this view (espoused, for instance, by Brown and Holland (2004, pp. 7)). Then independent criteria would be needed to make plausible the physical significance of the Einstein pseudotensor. Here, Pauli's considerations *would* be pertinent – and may be viewed as *disfavoring* the Einstein pseudotensor's suitability. Suppose now that one *did* consider the Lagrangian as something physical. For instance, the role the action plays for, say, in the Feynman path integral or its connection link to black hole horizons (see e.g. Padmanabhan, 2005) might suggest such a realism. But then it *would* matter whether one considered the full Einstein-Hilbert Lagrangian, its "$\Gamma\Gamma$"-version or perhaps a completely different Lagrangian – irrespective of their contribution to the field equations (or lack thereof) upon variation. In conclusion, Pauli's objection cannot be brushed aside.

Recently, Curiel (2014) closed this loop-hole. There exists indeed no tensor with the natural desiderata for representing gravity: Apart from the Einstein tensor, no symmetric, divergence-free, homogeneous (for reasons of dimensionality) rank 2-tensor that vanishes, if the spacetime is flat ($R^d_{abc} = 0$), can be constructed from up to second derivatives of the metric.

In fact, Lorentz and Levi-Civita proposed the Einstein tensor, $\mathrm{G}_{ab} = \mathrm{R}_{ab} - \frac{1}{2}R\mathrm{g}_{ab}$ (or, for reasons of dimensionality, $-\frac{1}{2\kappa}\mathrm{G}_{ab}$) as a suitable representation of gravitational energy (for a historical account of this proposal, see Pauli, 1981, fn. 350–351; Cattani & De Maria, 1993, esp. sect. 5–11). We now turn to this proposal.

At first blush, it looks attractive. Firstly, the Einstein tensor is a bona fide tensor. Secondly, it also obeys a bona fide covariant conservation law: the contracted Bianchi identity, $\nabla_b G^{ab} \equiv 0$. The attendant total energy-momentum $(LLC)^{\mathfrak{T}^{ab}} = -\frac{1}{2\kappa}G^{ab} + \mathrm{T}^{ab}$, satisfies both an ordinary and covariant continuity equation, $\partial_b((LLC)^{\mathfrak{T}^{ab}}) = \nabla_b((LLC)^{\mathfrak{T}^{ab}}) = 0$. Thirdly, the Einstein tensor is the exact *gravitational* counterpart of the *matter* energy-momentum tensor: Whereas the latter is defined variationally as $T_{ab} = -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}(\sqrt{|g|}\,\mathscr{L}_{(m)})$, one obtains the Einstein tensor (up to a proportionality factor) by replacing the matter Lagrangian by the purely gravitational Einstein-Hilbert Lagrangian,

[17] See Schrödinger, 1950, Ch.XI; Dirac, 1975, Ch. 31, 31 for a more convenient expression.

$$G_{ab} \propto \frac{1}{\sqrt{|g|}} \frac{\delta}{\delta g^{ab}} \left( \sqrt{|g|}\, R \right) \quad (11)$$

Two objections speak against the proposal: physical implausibility and vacuity, respectively. Firstly, consider the Einstein Equations in vacuum. This, on Lorentz and Levi-Civita's proposal, would yield *vanishing* gravitational energy, $G_{ab} = 0$. But that is counterintuitive: Since the Einstein tensor is constructed from traces of the Riemann tensor, a solution of the vacuum Einstein Equations has in general non-vanishing Weyl structure. The latter encapsulates gravitational radiation (see e.g. Padmanabhan, 2010, pp. 263) for technical details). Prima facie one would expect it to possess gravitational energy − contrary to Lorentz and Levi-Civita's proposal. Equally implausibly, it purports that there are no differences between gravitational energy in the exterior of a static and, say, charged rotating black hole, respectively: In either case, gravitational energy would be zero.

Besides doubts regarding its physical plausibility, it seems mysterious and contrived that, on Lorentz and Levi-Civita's proposal, any matter energy-momentum is exactly counterbalanced by gravitational energy. (Note that in *all* possible spacetimes, the *total* energy always vanishes, $-\frac{1}{2\kappa}G_{ab} + 2\kappa T_{ab} = 0$.) It is elusive what positing such an entity would help *explain*. In his correspondence with Levi-Civita, Einstein (1917, cited in op. cit., pp. 77) made this point. In a letter to him, Levi-Civita concedes that his proposal is indeed sterile in that "[…] the energy principle would lose all its heuristic value, because no physical process (or almost none) could be excluded a priori. In fact, [in order to get any physical process] one only has to associate with it a suitable change of the [gravitational field]".

In short: Lorentz and Levi-Civita's proposal lacks physical informativeness. The charge is aggravated by the fact that the contracted Bianchi identities, $\nabla_b G^{ab} \equiv 0$, as *mathematical* identities, barely count as conservation laws in any *substantive* sense. (By contrast, $\nabla_b T^{ab} = 0$ requires a certain coupling of the metric to the matter fields. It thus hinges on physically substantive assumptions (for details, see Read, Lehmkuhl, & Brown, 2017 (ms), sect. 3).

In consequence, Curiel's theorem seems to entail that local notions of gravitational energy will invariably conjure up non-tensoriality. In the following, I will exemplarily focus on pseudotensors, the most common type of non-tensorial objects. (The critique of other non-tensorial objects resorted to in order to avoid pseudotensors, e.g. via tetrads, carries through (for details, see Szabados, 2012, sect. 3.1.4–3.1.6).

### 3.2. Pseudotensors

The Noetherian framework is the royal road to gravitational energy in GR. Field-theorists (e.g. Weinberg, 1972) may feel inclined to treat GR like any other Lagrangian theory. Regarding the technical procedure, they may well be right (see the standard field-theoretical treatments in Horský & Novotný, 1969, or Barbashov & Nesterenko, 1983; for other possible advantages of their perspective, cf. Pitts, 2016b, Pitts, 2016c). (I will ignore potential issues with diverging action integrals: The metric need not fall-off "nicely".) But interpreting the results is more subtle. Goldberg (1958, p. 319; cf. Horský & Novotný, 1969, pp. 427), for instance, observes: "Clearly, the existence of a complex with a vanishing divergence is insufficient evidence for the conservation of a physically interesting quantity." To this I now turn.

Can pseudo-tensorial expressions adequately represent gravitational energy-momentum 4-currents? I challenge this. Two problems afflict pseudotensors: One is a problematic coordinate-dependence, the other a danger of arbitrariness/ad-hocness, due to ambiguity. While the second problem can be somewhat tempered, this comes at the price of trivializing the content of local gravitational energy-momentum.

The first problem stems from a formal property of pseudotensors: They are invariant only under affine transformations. Under more general transformations, pseudotensors are coordinate-dependent. Should this disconcert us? Not necessarily: Calling to mind the Kleinian conception of geometry, Wallace (2016) recently reiterated that nothing is *inherently* baneful about coordinate-dependent objects. For pseudotensors, however, the coordinate-dependence is "vicious" (Pitts): In general, the preferred coordinate transformations do *not* pick out the characteristic invariants of the spacetime. The spacetime symmetries do not align with the pseudotensor's symmetry group. This is highlighted by the fact that pseudotensors do not transform like 4-vectors neither under purely spatial transformations, $x^\mu \to x'^\mu = (x^0, x'^i(x^j))$, "which mean nothing more than a mere renumbering of points of the three-dimensional configuration space" (Horský & Novotný, 1969, p. 431), nor under purely temporal ones, $x^\mu \to x'^\mu = (x'^0(x^0), x^i)$, encoding "a continuous change in the rate and setting of the coordinate clock" (ibid.). In this sense, pseudotensors require structure absent in a given (non-flat) spacetime.

In order both to connect the issue of pseudotensors with our thoughts from §II, and to prepare the discussion of another proposal in §3.3, it is rewarding to revisit two historical complaints about pseudotensors' coordinate-dependence (for a detailed account, see Cattani & De Maria, 1993). Soon after Einstein's proposal of his pseudotensor, Schrödinger explicitly computed it for an incompressible fluid sphere. He showed that through a suitable choice of coordinate one can make the Einstein pseudotensor vanish. Einstein responded by showing that for systems of several masses, at least the Einstein pseudotensor cannot be made to vanish *everywhere*. Reversing Schrödinger's argument, Bauer subsequently observed that for suitable coordinates Einstein's pseudotensor also allows for even flat space possessing non-vanishing gravitational energy.

What to make of these objections? Let's look at Pitts' answers to them. According to Pitts (2010), Bauer failed to adapt his coordinates to the spacetime theories. But shouldn't the choice of non-adapted coordinates be irrelevant in *a generally covariant theory*? Isn't evaluating the gravitational energy-momentum flux $\chi_b j^b[\eta] = \chi_b t^b_a \eta^a$ for some observer $\chi$ and along some direction $\eta$ merely a question of *convenience*, not of principled importance? This casts Pitts' reply into doubt. For him, everything is as it should be - once one adopts privileged, *adapted* coordinates.

In §2.1, we identified the distinguished coordinates as inertial coordinates. In flat spacetime, these are globally Lorentzian. In them, the metric takes on a constant value everywhere; its derivatives vanish. Hence, the pseudotensor is indeed zero. So, Pitts is right in his counter to Bauer: Adapting the coordinates to the symmetries of flat spacetime resolves Bauer's paradox.[18]

What does this reasoning imply for Schrödinger's objection, i.e. when applied to pseudotensors on *non-flat* spacetimes? From the definition of Einstein's pseudotensor, it is evident already that in inertial (i.e. Fermi or Riemann) coordinates it vanishes: Pitts' reply to Bauer thus *trivialises* the physical significance of the Einstein pseudotensor!

[18] The same argument rebuts Read's remark that, on the (standard) response equation interpretation (see §2.1) even in flat spacetime $\nabla_a T^a_b = 0$, implies non-conservation of energy for arbitrary coordinates (see Read, 2018). In adapted/inertial coordinates of flat spacetime -viz. (global) Lorentz coordinates- $\nabla_a T^a_b = 0$ reduces to $\partial_a T^a_b = 0$.

Pitts gainsays this conclusion. According to him, the vanishing of the Einstein pseudotensor in, say, the exterior of a Schwarzschild black hole, when adopting uni-modular quasi-Cartesian coordinates, would be worrisome *only if* it could be made to vanish in a neighbourhood (thereby spoiling quasi-locality), or if the Einstein pseudotensor indeed vanished for every coordinate system. Neither is the case. More specifically, Pitts avers that there exist *infinitely many components* of gravitational energy (§3.3). Hence, according to Pitts, one need not be disquieted by the fact that *some* components are zero. This retort hinges crucially on Pitts' own proposal for gravitational energy in GR. Reasons for scepticism about its adequacy will be presented in §3.3. Suppose here that the reader shares my scepticism. Then, the reasoning we reconstructed for Pitts' (perceptive!) above reply to Bauer undermines his reply to Schrödinger: The adapted local inertial coordinates for the Schwarzschild case are indeed uni-modular quasi-Cartesian. In these coordinates, the Einstein pseudotensor is zero. (Inertial coordinates are adapted to free-fall frames: In them, the metric takes on a constant numerical value.)

Let's dwell a little on the vicious coordinate-dependence. It can become virulent also in practice - generalising Schrödinger's point. For instance, the Landau-Lifshitz pseudotensor -an alternative to Einstein's (see below)- yields *negative* energy densities for Reissner-Nordström spacetimes, when calculated in quasi-Cartesian coordinates (Virbhadra, 1991). Negative energy densities violate the weak energy condition.[19] Therefore, they are usually (e.g. Malament, 2012, Ch. 2.5) considered unphysical. By contrast, calculations of Einstein's and other pseudotensors give reasonable and mutually consistent results for Kerr-Schild Cartesian coordinates. For cylindrical gravitational waves, the pseudotensor exhibits a similar coordinate-dependence: Their energy-momentum densities associated with the Einstein pseudotensor vanish in polar coordinates; Cartesian coordinates, by contrast, yield reasonable results (Rosen & Vibhadra, 1993). But what disqualifies quasi-Cartesian coordinates a priori? Kerr-Schild Cartesian coordinates are not adapted, *either*: They are not inertial coordinates. The same applies to the cylindrical gravitational wave: The global Cartesian coordinates employed there are not inertial.

The second problem of pseudotensors consists in their non-uniqueness (see Trautmann, 1962, esp. sect. 5-5; Anderson, 1967, Ch. 13 for details). An infinite number of possible alternative pseudo-tensors exists. None is a priori privileged over the other. We saw already that it does not affect the validity of a continuity equation, $\mathfrak{T}_a{}^b = \sqrt{|g|}(T_a^b + \mathfrak{t}_a^b)$, if we add an arbitrary superpotential of the form $-\partial_c \mathfrak{U}_a^{[bc]}$:

$$\partial_b\left(\mathfrak{T}_a{}^b - \partial_c\mathfrak{U}_a^{[bc]}\right) \equiv \partial_b\mathfrak{T}_a{}^b \tag{12}$$

Such an addition amounts to a re-distribution of total energy-momentum. Depending on how the metric falls off, this re-distribution is physically significant.

Via a choice of a superpotential and the Einstein Equations, one can define arbitrary pseudotensors:

$$\sqrt{|g|}t_a{}^b = \partial_c\mathfrak{U}_a^{[bc]} + \frac{1}{2\kappa}\sqrt{|g|}G_a^b \tag{13}$$

Different choices of superpotentials correspond to different pseudotensors. Einstein's, for instance, follows from von Freud's choice of the superpotential,

$$(F)\mathfrak{U}_a^{[bc]} = \frac{1}{2\kappa\sqrt{|g|}} g_{ad}\partial_e\left(|g| g^{b[d}g^{e]c}\right) \tag{14}$$

Is underdetermination the issue here? If so, wherein does the situation differ from the non-uniqueness of energy-momenta in other classical field theories? After all, they too are only defined up to a superpotential.

Consider the so-called "Bergmann form" of superpotentials:

$$(B)\mathfrak{U}^{[ab]} := (F)\mathfrak{U}_c^{[ab]}\xi^c \tag{15}$$

Here, $\xi^a$ generates a one-parameter group of coordinate transformations whose variations, $\delta x^b = \varepsilon\xi^b(x)$ leave the action (quasi-) invariant, with an infinitesimal $\varepsilon$. This one-parameter group forms a subgroup of the general continuous group of coordinate transformations. (In other words: We have a theory with local gauge symmetry that allows for a non-trivial global subgroup. In this case, we can combine Noether's First and Second Theorem. For details, see e.g. Brading and Brown, 2002, sect.5; Brading, 2002, 2005; Ohanian, 2013.) Then, in vacuo ($T_b^a = 0$) one has purely gravitational (axial-/pseudo-vectorial) energy-momentum 4-current along $\xi$ as

$$\sqrt{|g|}t^a[\xi] = \partial_b\left((F)\mathfrak{U}_c^{[ab]}\xi^c\right) \tag{16}$$

In the presence of matter (with the associated 4-current $T_b^a\xi^b \neq 0$), we have the *total* energy-momentum 4-current along $\xi$,

$$\tilde{j}^a_{(tot)}[\xi] := \sqrt{|g|}\left(t^a + T_b^a\xi^b\right) \tag{17}$$

It can be re-written via a superpotential,

$$\tilde{j}^a_{(tot)}[\xi] = \partial_c\left((F)\mathfrak{U}_b^{[ac]}\xi^b\right) \tag{18}$$

Due to the asymmetry in the superpotential's upper indices, the r.h.s satisfies an ordinary continuity equation in *all* coordinate systems $\partial_a\tilde{j}^a_{(tot)}[\xi] = 0$.

Consonant with our terminology of §2.1, the total-energy-momentum 4-current $j^a_{(tot)}$ possesses no sinks/sources.

Note that the quantities $\xi^c$ need *not* constitute a vector field (Trautmann, 1962). E.g. choosing them such that the components $\frac{\xi^b g_{ab}}{\sqrt{|g|}}$ are constants yields an alternative to Einstein's, widespread in astrophysical applications (cf. Poisson & Will, 2016) - the Landau-Lifshitz pseudo-tensor $(LL)\mathfrak{t}^{ab}$. (It has the merits of being index-symmetric, $(LL)\mathfrak{t}^{[ab]} = 0$ and built from only 1st derivatives of the metric (Landau & Lifshitz, 1975, §96): $(LL)\mathfrak{t}^{[ab]} = 0\mathfrak{t}^{ab} + \sqrt{|g|}T^{ab} = \partial_c(\sqrt{|g|}\,g^{ad}(F)\mathfrak{U}_d^{[bc]})$.).

All known pseudotensors can be derived from the von-Freud and Bergmann form, including Lorentz and Levi-Civita's proposal (for details, see Goldberg, 1958; Trautmann, 1962, p. 190; Horský & Novotný, 1969).

The freedom to choose any superpotential renders the local representation of gravitational energy *banefully* under-determined. There exist infinitely many superpotentials, one for each possible (not necessarily tensorial!) $\xi^b$. However, no such $\xi^b$ is inherently privileged in a generic spacetime (with exceptions to be discussed presently): Each corresponds to a possible "gauge" of pseudotensors. The gravitational energy-momentum 4-current $t^a[\xi] = \frac{1}{\sqrt{|g|}}\tilde{t}^a[\xi]$ *ad-hoc* privileges a direction. Equivalently, with each choice of a $t^a[\xi]$ one exalts -by an ad-hoc *stipulation* a one-

[19] That is: For a time-like vector field $\xi$ and the energy-momentum-tensor $T_{ab}$ the energy-density relative to $\xi$ is positive.$T_{ab}\xi^a\xi^b \geq 0$

parameter group of coordinates transformations, by itself failing to be privileged.[20]

This is not merely a sin against GR's spirit. Different energy-momentum complexes *can* yield different energy distributions for the same gravitational background.[21] E.g. the energy for the exterior of the Kerr-Newman black hole, determined via Møller's pseudotensor, equals twice the energy, obtained from Tolman's, Einstein's or Landau/Lifshitz's pseudotensor (Virbhadra, 1990).[22] The ambiguity thus threatens the well-definedness of gravitational energy-momentum.

By contrast, the freedom in the choice of superpotentials is in most cases benign in pre-GR theories. Firstly, due to the compactness of the support of matter fields and suitable fall-off conditions, it doesn't affect the values of the corresponding Noether charges. Secondly, the spacetime settings of pre-GR theories contain symmetries. Their associated Killing vectors then serve as such a compass for privileged directions. I turn to this now.

One can evade the charge of ad-hoc privileging *arbitrary* directions by attending to those directions that *are* inherently privileged. (Recall §2.) Consider first *symmetric* spacetimes. Here, the directions along Killing field $\xi^b$ are privileged.

For such spacetimes, Komar (1959) arrived at the following expression for the superpotential:

$$K^{1l[ab]}[\xi] = \frac{1}{2\kappa}\partial_b\left(\sqrt{|g|}\,\nabla^{[a}\xi^{b]}\right) \tag{19}$$

The resulting total 4-current (weight-1 density) reads:

$$\tilde{J}^a_{(tot)} = \sqrt{|g|}\left(t^a[\xi] + T^a_b\xi^b\right) = \partial_b\left(K^{[1lab]}\right) \tag{20}$$

Thanks to its anti-symmetry, Komar's superpotential can be re-written explicitly as a genuine tensor density of weight one:

$$\partial_b\left(\sqrt{|g|}\,\nabla^{[a}\xi^{b]}\right) = \sqrt{|g|}\nabla_b\left(\nabla^{[a}\xi^{b]}\right) \tag{21}$$

Consequently, $J^a_{(tot)} = \frac{1}{\sqrt{|g|}}\tilde{J}^a_{(tot)}$ is indeed a genuine vector. It is covariantly conserved:

$$0 = \partial_a\tilde{J}^a_{(tot)} = \sqrt{|g|}\nabla_a J^a_{(tot)} \tag{22}$$

Given that $\nabla_a(T^a_b\xi^b) = 0$, it follows that also the gravitational energy-momentum 4-current $t^a[\xi]$, too, is a genuine vector that is covariantly conserved, $\nabla_a t^a[\xi] = 0$.

In analogy to the ontological status of apparent forces, I argued earlier that for energy-momentum balances we should be realists only about those terms that survive in inertial frames *, i.e. upon switching to normal coordinates.

Evaluating the gravitational 4-current,

$$t^a[\xi] = \frac{1}{\sqrt{|g|}}\left(\partial_b\left(K^{1l[ab]}[\xi]\right) - \sqrt{|g|}\,T^a_b\xi^b\right) \tag{23}$$

in normal coordinates and harnessing the Killing property yields:

$$t^\mu[\xi]|_* = \frac{1}{2\kappa}\partial_\nu\partial^{[\nu}\xi^{\mu]} - T^\mu_\nu\xi^\nu = -\frac{1}{\kappa}\xi^\mu - T^\mu_\nu\xi^\nu \tag{24}$$

Here, $\Box = \eta_{\mu\nu}\partial^\mu\partial^\nu$ denotes the flat-space, Cartesian d'Alembert operator.

However, neither term making up this 4-current is suitably connected with gravitational degrees of freedom to represent local gravitational energy-momentum.

Consider first the second term, $T^\mu_\nu\xi^\nu$. It is the (conserved) matter energy-momentum flux along the direction of the Killing field. As such, it is unrelated to gravitational degrees of freedom.

The first term, $\xi^\mu = -\partial^\mu(\partial_\nu\xi^\nu)$, encodes how the "source density" $\partial_\nu\xi^\nu$ of the Killing field varies along inertial worldlines. It, by contrast, *is* related to gravitational degrees of freedom via the identity, holding for all Killing fields (e.g. Padmanabhan, 2010, p. 220)

$$\nabla_b\nabla_a\xi_c = R_{dbac}\xi^d \tag{25}$$

A simple calculation shows that

$$\Box\xi^\mu = R^\mu_\nu\xi^\nu \tag{26}$$

So, albeit indeed related to gravitational effects (taken here to be represented by curvature effects), the first term is too *coarsely* related to gravitational effects: In particular, it ascribes to *all* matter-free regions of any arbitrary spacetime the same value: zero. This seems counterintuitive.

In conclusion, for symmetric spacetimes, the pseudotensorial candidate for gravitational energy-momentum paradoxically turns out not to be related in the right way to gravitational degrees of freedom.

For *non-symmetric* spacetimes, the only inherently privileged $\xi$s are those describing inertial trajectories. In the adapted/comoving (i.e. normal) coordinates, the Bergmann form trivially vanishes. (Recall: $g|_* = const.$) Harking back to our thoughts in §2.1, we educe that resulting gravitational energy-momentum 4-current *worthy of realist commitment* is zero:

$$t^a[\xi]|_* = 0 \tag{27}$$

[20] A complementary argument can be obtained from the Hamiltonian perspective. Together with the Hamiltonian constraints and appropriate coordinate conditions, the Hamiltonian takes the form of the surface integral.

$$\mathcal{H} = -\frac{c^4}{16\pi G}\sum_{\alpha,\beta=1,2,3}\oint_{i^0} ds_\alpha\left(\partial_\beta h_{\alpha\beta} - \delta_{\alpha\beta}\sum_{\gamma=1,2,3}h_{\gamma\gamma}\right).$$

Here, $ds_\alpha$ denotes the surface element on spacelike infinity $i^0$ and $h_{\alpha\beta}$ is the spatial metric induced on the spacelike hypersurfaces of the 3+1-foliation (see e.g. Poisson, 2007, Ch. 4.2 for technical details). Using different Hamiltonian formalisms, this surface integral can be represented in different ways, as volume integrals with different integrands:

- The standard ADM form:

$$\mathcal{H}_{ADM} = -\frac{c^4}{16\pi G}\int d^3x\sum_{\alpha,\beta=1,2,3}\left(\partial_\alpha\partial_\beta h_{\alpha\beta} - \delta_{\alpha\beta}\sum_{\gamma=1,2,3}h_{\gamma\gamma}\right)$$

- Dirac's form:

$$\mathcal{H}_D = \frac{c^4}{16\pi G}\int d^3x\sum_{\alpha,\beta=1,2,3}\partial_\alpha\left(|\gamma|^{\frac{1}{2}}\partial_\beta\left(\gamma\gamma^{\alpha\beta}\right)\right)$$

with $\gamma^{\alpha\beta}$ as the inverse of $h_{\alpha\beta}$ and $\gamma = \det(\gamma^{\alpha\beta})$.

- Schwinger's form: $\mathcal{H}_S = \frac{c^4}{16\pi G}\int d^3x\sum_{\alpha,\beta=1,2,3}\partial_\alpha\partial_\beta(\gamma\gamma^{\alpha\beta})$

Although the resultant total energies all agree, $\mathcal{H}_{ADM} = \mathcal{H}_D = \mathcal{H}_S$, the integrands, i.e. gravitational energy densities, differ non-trivially. Schäfer, 2014, p. 17) concludes that "the notion of gravitational binding energy density has no physical or observational meaning.

[21] This does not seem to be the rule, though (Multamäki et al., 2008).

[22] The energy distributions of the Einstein and Møller pseudotensor differ also for the deSitter, the Schwarzschild solution, the charged regular metric, the stringy charged black hole and Gödel-type spacetimes (Gad, 2004, p. 2).

In summary, for both cases where the arbitrariness objection to pseudotensors could seemingly be averted, we wind up with the same conclusion as in §2.1–2: Gravitational energy-momentum is trivialised. With the problem of vicious coordinate-dependence still looming without remedy, it thus seems preferable to reject the pseudotensorial approach to local gravitational energy-momentum altogether.

I will therefore move on and inspect two heterodox alternative proposals from the more recent philosophical literature.

### 3.3. Pitts' object

Recently, Pitts (2010) made an astute suggestion: Take your pet pseudotensor, and declare the totality of its values in every possible coordinate system an infinite-component object sui generis, with each component corresponding to the value of the pseudotensor in some coordinate system. Since each component satisfies a continuity equation, so does the whole object (suitably defining derivatives for such objects).

Pitts' object provides thoughtful answers to the criticism of §2.2: By construction, it is coordinate-*independent*. Hence, it extricates gravitational energy-momentum from vicious coordinate-dependence. Pitts rightly extols this.

What about the ambiguity/arbitrariness problem? It persists. If one picks *one* pseudotensor of the Goldberg-Bergmann type and "Pittsifies" it, one obtains indeed a well-defined object. Yet, why prefer *this* pseudotensor over others? In terms of the Von Freud or Bergmann form, different choices for a preferred direction for a gravitational energy-momentum are still possible. Hence, one can construct again an infinite number of Pitts objects, one for each pseudotensor. Furthermore, why not Pittsify other, non-pseudotensorial expressions (involving e.g. background metrics or auxiliary connections, each in itself no less suitable a priori)? This exacerbates the ambiguity. (Normally, one could plausibly discard such objects as parasitic on auxiliary structure that GR simpliciter lacks. Pitts' strategy deprives one of this argument: The object Pittsified over, say, all possible auxiliary metrics no longer depends on this auxiliary structure; only each coordinate does. In consequence, in order to restrict Pittsification to pseudotensors, Pitts has to summon other arguments than he has presented so-far.)

He might parry by demonstrating that one particular Pitts object, say, the Pittsified Belinfante-Rosenfeld symmetrized Einstein pseudotensor, is indeed the best candidate. This is certainly *conceivable*: The list of attractive pseudotensors of the Bergmann form can be further whittled down by excluding e.g. the Landau-Lifshitz pseudotensors or the Møller pseudotensor (on account of its anomalous factor, diagnosed by Katz (1985)). To-date, though, such a comprehensive analysis is still pending.

Should our hopes for uniqueness be dashed, Pitts, 2017, (sect. 13.4) envisages a way to turn this vice into a virtue: As the action contains infinitely many symmetries, it may appear natural to allow for infinitely many gravitational energies. Perhaps, Pitts proposes, this proves an advantage in the context of black hole thermodynamics: After all, Nester and collaborators suggested that different gravitational energies correspond to different free energies and the like under different boundary conditions. However, to judge that such considerations buttress Pitts' proposal seems premature: At present, it is controversial (see e.g. Dougherty & Callender, 2016) whether the correspondence between black hole thermodynamics and thermodynamics is substantive, rather than a speculation based on partial and formal analogies.

So, let's assume that the (non-)uniqueness problem defies a satisfactory resolution. In that case, Pitts (2017, p. 270) rightly warns against double standards: One must not demand of gravitational energy, what *non*-gravitational energy does not satisfy, either. Pitts points out that non-uniqueness poses a problem even for scalar fields (Callan, Coleman, & Jackiw, 1970). GR's gravitational energy (Pitts-style) would then appear no worse off than other field theories. But this, I think, is misleading: Attempts to improve the (Belinfante-Rosenfeld-symmetrized) canonical energy-momentum tensor, "[…] are largely 'ad hoc' procedures focused on special models of field theory, often geared to the needs of quantum field theory and ungeometric in spirit" (Forger & Römer, 2003, p. 3). Requiring a certain "ultra-locality", Forger and Römer show in a geometric, systematic manner that uniqueness of the energy-momentum tensor *can* be restored: For non-gravitational matter, it coincides with the variationally defined energy-momentum tensor. (For its gravitational counterpart, the Einstein tensor, this leads us back to Lorenz and Levi-Civita's proposal of §3.1.)

Of course, Forger and Römer's result can scarcely lay claim to a proof from indubitable first principles. If thus one regards it as little more than a stipulation, it would be unfair to decry Pitts' proposal as wanting for its inability to solve the uniqueness problem: *Nobody* who champions some realism for gravitational energy in GR has solved it. The point of the critique of Pitts' proposal in this section then could therefore only be twofold: Firstly, to question some of Pitts' insinuations that his proposal provides any particular *advantage*. This might eventually turn out to be true, but at present, more work needs to be done. Secondly, I happily grant Pitts' proposal the status of the most promising avenues for realists about gravitational energy. Hence, if even *it* suffers from grave problems – so much the better for eliminativism.

I close with two considerations, both revolving around the physical significance of Pitts' object. First: Doesn't Pitts owe us an argument why his object should be considered physically *meaningful*? Analogously, we could Pittsify, say, the electromagnetic potentials of a system: Gather the totality of all its possible gauges into one formal object. Is this artificial seeming Pittsified 4-potential physically meaningful? I am not aware of any theoretical or practical context in which physicists ever calculate infinite numbers of energies.

Pitts could respond: *Wherever* gravitational pseudotensors are useful, his proposal affords a coherent interpretation of such pseudotensors. Let's consider three more concrete forms of this argument, related to an interpretation to the Noether Theorems, the equivalence with the Einstein Equations, and Anderson's framework of geometric objects, respectively.

The first can be cashed out as the ability of Pitts' strategy to provide an intelligible interpretation of Noether's Theorems for GR. It gives what seems the natural answer to the question of how many conserved energies there are in GR, namely: infinitely many – corresponding to infinitely many possible rigid time translation symmetries. But the argument is not cogent: Why should the Noether Theorems be in *need* of an interpretation? Whether one thinks they do, depends on one's willingness to regard the Lagrangian as physical. This is controversial. Arguably, it is actually more natural to regard the Lagrangian merely as a computational prop. The Noether machinery is only a tool to conveniently derive continuity equations. Their validity, however, does not presuppose anything from the Noetherian framework: They follow from the field equations alone.

As second elaboration of the argument focuses on the interpretation of these continuity equations. Pitts reminds us of Anderson's observation that the totality of continuity equations of all possible pseudotensors is equivalent to the Einstein Equations: Surely, one may be tempted to conclude, since the latter are physically meaningful, so is the equivalent totality of continuity equations, i.e. the local conservation law for the corresponding Pitts object. At least for the time being, I want to resist that temptation:

Should one infer from the equivalence of Feynman's path integral formalism with standard quantum mechanics that their paths equipped with their complex amplitudes provide a coherent interpretation of quantum mechanics? This would be too quick (Zeh, 2011). Likewise, one may challenge that a concept's computational utility on its own suffices to warrant the kind of realism that underlies Pitts' reasoning. Compare: It is at least controversial whether to include the quantum potential amongst the ontology of Bohmian Mechanics – despite its utility in many applications, such as the semi-classical approximation schemes (see e.g. Goldstein, 2017, sect. 5 for details).

The third and last worry about the significance of Pitts' object consists in some discomfort one may feel about the expressly non-geometric nature of Pitts' object: How does such a non-geometric and non-tensorially representable object fit into a geometric, local field theory? Pitts replies: If Anderson's standard formal framework of geometric objects cannot accommodate gravitational energy, but the latter is a good idea, seek an alternative framework that encompasses also non-geometric objects! To me it seems more cautious to resolve this conflict in favour of the framework of geometric objects: If forced to choose between a well-established, useful "global" ontological framework and my hunches about *one* quantity whose meaningfulness has conceptually already been called into question, I prefer to sacrifice the latter.

In summary: While interesting and promising, Pitts' object leaves questions crucial for its physical significance unanswered.

### 3.4. *The cosmological constant*

According to its standard interpretation, one construes the cosmological constant $\Lambda$ as vacuum energy. Based on this standard interpretation, Baker, 2005, (sect. 4.2) raised the following question: If $\Lambda$ is the energy contribution from empty space-time, shouldn't $T^{(\Lambda)}_{ab} = -\frac{\Lambda}{2\kappa}g_{ab}$ count as a natural candidate for gravitational energy-density? Not only does $T^{(\Lambda)}_{ab}$ play the functional role of a (negative) energy density of a perfect "cosmic fluid", composed purely of matter-free spacetime, but due to metric compatibility, it even satisfies a conservation law, $\nabla^b T^{(\Lambda)}_{ab} = 0$.

Baker's proposal is a non-starter, however, for three reasons. Firstly, the Einstein Equations with a cosmological constant, $G_{ab} + \Lambda g_{ab} = 2\kappa T_{ab}\kappa$, are a minimal *extension* of Einstein's original GR, obtained from adding a constant to the Einstein-Hilbert Lagrangian. Hence, for standard GR, in which the cosmological constant is absent/zero, the gravitational energy would identically vanish. This trivializes gravitational energy. Secondly, what is *really* meant by interpreting $\Lambda$ as vacuum energy? On one common view, this vacuum energy refers to the sum of the energy fluctuations of the quantum mechanical ground state.[23] $\Lambda$ thus is *not* energy of "empty space". Rather, it is the zero-point energy of an all-pervasive quantum field, i.e. attributable to *matter*. Lastly, although $T^{(\Lambda)}_{ab}$ plays the functional role of an energy density, nothing compels us ascribe it to the r.h.s. of the Einstein Equations, as a source: No less are we licenced to ascribe it to the l.h.s., where $\Lambda$ could simply serve as some parameter of the "gravitational field strength functional",[24] ${}_{\Lambda}G_{ab}[g_{ab}] = G_{ab} + \Lambda g_{ab}$ – a status comparable with parameters featuring in other non-linear theories, e.g. the soliton equation. Should one indeed interpret $\Lambda$ as a property of spacetime itself, then via a contraction of the Einstein Equations (in vacuum) it is easily seen to equal scalar *curvature* of spacetime, unperturbed by matter: $\Lambda = 4R$. This contradicts what one would expect of a suitable notion for gravitational energy: As a contraction of the Riemann tensor, it wouldn't yield any contribution from gravitational radiation, encoded in Weyl structure. Furthermore, the proposal does not reduce to the potential energy of Newtonian gravity.

In conclusion, at best, Baker's interpretation of $\Lambda$ as gravitational energy defies plausibility. At worst, it rests on a conflation of classical and quantum vacuum.

## 4. Summary and conclusion

We started the preceding analysis by investigating conservation of energy-momentum in generic, non-symmetric spacetimes. Considerations of the priority and explanatory distinction of inertial frames led us to restrict our realist commitments in energy-momentum balances to the terms retained after an evaluation in inertial frames and the adapted coordinates. We found that the matter energy-momentum 4-currents along the inherently preferred directions in such non-symmetric spacetimes (viz. along inertial trajectories) possess no sinks/sources. Whilst thus matter energy-momentum is conserved locally, *globally* it is not: The energy-momentum contained in an observer's spacelike hypersurface varies in time.

In symmetric spacetimes, the matter energy-momentum 4-current along their Killing fields is conserved both locally *and* globally: The associated "charges" are independent of the choice of space-like hypersurfaces.

Non-trivial local gravitational energy-momentum did not arise in these considerations: It turned out to be an idle wheel in the context of matter energy-momentum conservation. This inspired the working hypothesis that gravitational energy-momentum is eliminated in GR: Analogously to apparent forces in CM, it is reduced to representational artefacts forged by physically unprivileged descriptions.

Subsequently, we scrutinised proposals for local *gravitational* energy-momentum. Nontensorial expressions are inevitable. Exemplarily, the discussion focused on pseudotensors, as they emerge naturally from generalizations of Noether's Theorems, applied to GR's purely gravitational Lagrangian. Pseudotensors face two main problems: firstly, a mismatch between the spacetime symmetries and the symmetries that their preferred coordinates pick out, and secondly an ambiguity that threatens to introduce arbitrariness/ad-hocness. The latter worry can be allayed – but only at the price of trivializing the gravitational energy-momentum 4-current. These issues suggest that one abandon also the pseudotensorial route to local gravitational energy-momentum.

As a way out, we considered Pitts' strategy to define an infinitely many component object, via the totality of all possible pseudotensors. Whilst addressing the issue of vicious coordinate-dependence, Pitts' proposal provided no satisfactory answer to the problem of arbitrariness. Its physical significance remains doubtful.

Eventually, we examined and discarded Baker's proposal of the cosmological constant as candidate for local gravitational energy.

[23] This interpretation of $\Lambda$ displays a discrepancy between the quantum field theoretical predictions and cosmological observations by roughly 120 (!) orders of magnitude (see e.g. Carroll, 2000; Rugh & Zinkernagel, 2000).

[24] By that I mean the following. Construe both sides of the Einstein Equations as functionals of the metric (as well as, for the energy-momentum tensor, the matter fields), $G_{ab}[g_{ab}] = 2\kappa T_{ab}[g_{ab}; \Psi]$ (Recall that $T_{ab}$ depends on $g_{ab}$, see Lehmkuhl, 2010 for an analysis.). The Einstein Equations codify how the matter fields and the metric interdepend (Nerlich, 2013, Ch. 9): Together with initial data, they determine (some part of) the dynamics of the metric. (Recall that the Einstein Equations *constrain*, but don't *fix* the trace-free part of the Riemann tensor, the Weyl tensor.) The functional on the r.h.s., $T_{ab}[g_{ab}; \Psi]$, plays the role of a source density, analogously to the particle current, $J^a_A := \frac{\delta I_p}{\delta A^a_A}$, (say, fermions) of a Yang-Mills field $A^a_A$ and interaction Lagrangian $I_p$, (Szabados, 2012, sect. 3.1.2). The functional on the l.h.s., $G_{ab}[g_{ab}]$ plays the role of a field-strength functional, with the metric as the "field strength".

GR forces us to revise (i.a.) two notions, central to pre-GR hunches. Firstly, global energy-momentum conservation becomes a contingent fact, dependent on the contingent symmetries of the spacetime. Secondly, local gravitational energy-momentum is eliminated: It is no longer a meaningful physical quantity. In a sense, it has been geometrized (or rather: "inertialised") away.

This verdict is largely in agreement with the GR literature (e.g. Pauli, 1981, p. 177; Weyl, 1923, p. 273; Eddington, 1923, p. 137). Even Einstein (1918, in: Gorelik, 2002, p. 25), despite initially advocating his pseudotensorial approach, ultimately conceded that "(t)hus [...] we come to ascribe more reality to an integral than to its differentials." Others (e.g. Weyl, 1923, p. 273) concurred: Can we reserve a realist commitment only for the *global* (integral) notions of gravitational energy-momentum? But this is a question for another day.

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