Axiomathes (2011) 21:303–313
DOI 10.1007/s10516-010-9137-5

ORIGINAL PAPER

Distance and Similarity Measures in Generalised Quantum Theory

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Received: 13 September 2010 / Accepted: 9 November 2010 / Published online: 25 November 2010

Abstract A summary of recent experimental results shows that entanglement can be generated more easily than before, and that there are improved chances for its persistence. An eminent finding of Generalised Quantum Theory is the insight that the notion of entanglement can be extended, such that, e.g., psychological or psychophysical problem areas can be included, too. First, a general condition for entanglement to occur is given by the term 'common prearranged context'. A formalised treatment requires a quantitative definition of the similarity or dissimilarity between two complex structures which takes their internal structures into account. After some specific remarks on distance, metrics, and semi-metrics in mathematics, a procedure is described for setting up a similarity function with the required properties. This procedure is in analogy with the two-step character of measurement and with the well-known properties of perspective notions. A general methodology can be derived for handling perspective notions. Finally, these concepts supply heuristic clues towards a formalised treatment of the notions of 'meaning' and 'interpretation'.

Keywords Distance function · Entanglement · Generalised Quantum Theory · Macro-entanglement · Metric · Perspective notions · Semi-metric · Similarity measure

1 The Role of Entanglement in Generalised Quantum Theory

Already the first publication on Generalised Quantum Theory (GQT) documents the fundamental role of nonlocality and entanglement (Atmanspacher et al. 2002). There the notion of entanglement is extended beyond the concept of nonlocal

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correlations between non-commuting properties of quantum systems. For instance, it can be utilised for exploration in psychological or psychophysical problem areas, as exemplified by detailed studies on the placebo effect and on the transference and countertransference phenomena in psychotherapy.

Nonlocality and entanglement belong to the most intriguing discoveries of modern quantum theory. After the experiments by Aspect et al. (1982a, b) these theoretical concepts can be regarded as empirically confirmed beyond any reasonable doubt. More recently, in an experiment by Anton Zeilinger and his team photons remained entangled over a distance of 144 km (Perdigues Armengol et al. 2008). The observed correlations between measurements performed on spatially separated objects cannot be explained by any theory which is constrained to local variables. ‘Spatially separated’ means that, within a given time limit, one object cannot reach the other object by signalling (taking into account the velocity of light as the ultimate bound).

From a phenomenological perspective, entanglement can be understood as a concrete, *observable* correlatedness between two or more spatially separated entities. This correlatedness cannot be interpreted as causal interaction, nor can nonlocality and entanglement be used for ‘superluminal communication’.

The next step will be a selective list of rather recent experimental results, showing that the conditions for entanglement to occur can be loosened and that the chances for its persistence gradually improve (Sect. 2). These results may be suggestive for the assumption that entanglement in the broader sense as addressed before, as a nonlocal coupling also including macroscopic entities, can claim some plausibility.

After this trip through the laboratories the focus will be laid again on entanglement in the broader sense as outlined in the beginning. In a first intuitive approach it is shown that conditions for entanglement to occur are closely related to the concept of similarity between the entities involved (Sect. 3). Next, the mathematics of distance and similarity measures will be summed up in a selective manner (Sect. 4). The present topic requires a detailed study of similarity measures defined on sets of complex structures (Sect. 5), which will also lead to some insight on fundamental questions related to perspective notions.

2 Experimental Results on Entanglement

Recent experiments show that entanglement is not only a matter of a few particles or waves—rather, two “entire beams of light” (Boyer et al. 2008), two laser beams (Wagner et al. 2008), or two separate samples of atoms containing 10^{12} atoms each (Julsgaard et al. 2001) can be entangled. Furthermore, entanglement can have an influence on macroscopic quantities, like heat capacity or magnetic properties (Vedral 2003; Brukner et al. 2006).

There is quite a variety of experimental results which support the prediction that the conditions for entanglement to occur and to persevere—in spite of adverse conditions—will still become less restrictive, and techniques of its observation will advance:

1. Entanglement is no longer an exclusive matter of microphysics. “Now, however, entanglement is recognized to be ubiquitous and robust.” (Vedral 2008, p. 1004) There is work in progress on “entanglement in macroscopic systems” (Müller-Ebhardt et al. 2008).
2. In special laboratory settings, entanglement can be possible at high temperatures, or persist in spite of increasing temperature (Ferreira et al. 2006; Amico et al. 2008; Briegel and Popescu 2008; Markham et al. 2008); it can occur “even at room temperature, without the need for any manipulation” (Vedral 2008, p. 1006).
3. Again in special settings, some systems properties due to entanglement are accessible to external macroscopic observation (Kofler and Brukner 2006; Vedral 2008).
4. Entanglement between two photons can even persist in spite of massive obstacles placed in their paths. Altewischer et al. (2002) placed optically thick metal films perforated with a periodic array of subwavelength holes in the path of two entangled photons. It was found that the photons were converted into surface-plasmon waves, which tunnel through the holes before reradiating as photons at the far side, and entanglement survives this conversion process.
5. Multi-particle entanglement: Chances are growing to entangle more then just two or three entities (Eisert and Gross 2006). Entanglement of two separated mechanical oscillators has been demonstrated (Jost et al. 2009).

Entanglement contributes to the ‘avian compass’, the ability of certain migratory birds to sense very subtle variations of Earth’s magnetic field. Pairs of spatially separated, but entangled electrons are influenced by external magnetic fields, and by specific mechanisms this effect is registered and further processed (Gauger et al. 2009; Cai et al. 2010).

3 Entanglement and Similarity

The enormous variety of phenomena of macroscopic entanglement, to be covered by any future theory, can be highlighted by the following tiny selection of effects, which at a first glance appear disparate.

Synchronicity is the occurrence of *acausal meaningful* coincidences (e.g., a sudden encounter with an old friend, unseen for many years). This concept due C.G. Jung is an offspring of Jung’s cooperation with the physicist Wolfgang Pauli (Jung 1955; for more recent sources see Atmanspacher et al. 2001, von Lucadou et al. 2007).

A *pathogenetic trial* is a test of a specific substance (expected to be of medical significance) by registering the symptoms experienced by healthy volunteers. Some recent studies used a meticulous experimental design (double-blind etc.). The interesting outcome is the fact that typical symptoms were also reported by those provers who had only received an undistinguishable fake (Walach et al. 2004).

Elitzur and Dolev (2003) describe an experiment in which two atoms are entangled by a “future interaction”. “Unlike the ordinary EPR, where the two particles interacted earlier, here their common event lies in their *future*” (p. 301).

Now a ‘generic term’ is required which includes both microscopic and macroscopic nonlocal correlatedness, and which is suited to cover the variety of its manifestations. Here the term *common prearranged context* is proposed. Two persons, objects, or processes can be comprised by such a common context, which may have been generated by nature or by human persons or institutions. The principal appearances can be characterized as follows:

1. *Common historical context*: e.g., two photons generated by the same process, as in the standard EPR experiments,
2. *Common future:* e.g., two atoms entangled by future interaction,
3. *Common biographic context:* e.g., two persons familiar with each other for some time,
4. *Common organizational context:* both partners are involved in the same organized process, e.g., the same action, project, experiment, test, etc., both have a role or a function in it; this also holds for relations between persons and objects (e.g., patient and remedy in the case of extreme dilutions).

These four characteristic patterns have some essential features in common.

The common context can be given by nature, as in the classical pair-formation. In experimental situations, the common prearranged context corresponds to the decision of the initiator. Anyway, a prearrangement of context always implies an assignment of meaning. If, e.g., a person joins an organised activity, this is not a merely physical process, but it has a meaning for all participants. As will be discussed later, the term ‘meaning’ is admissible also in the case of non-living objects—the lock-key principle illustrates that lock and key have a meaning for each other.

The requirement of a common prearranged context shows the necessity of a preceding step, which may possibly be provided by nature, and which may be seen in analogy with *preparation* in quantum theory. This necessary preceding step will later find its mirror image in the property that every measurement is a two-step process (Sect. 5.1).

Finally, the four conditions for entanglement have in common that there is always a relationship between entities (persons, objects, or processes) which can be circumscribed by the intuitive term ‘affinity’.

Here a reservation is necessary. The proposed term “common prearranged context” only addresses *conditions* for entanglement *to be initiated*. It is does not characterise entanglement as such, nor the course of an ensuing process once entanglement between defined entities was started (nor a possible decay of entanglement). Exactly the positive conditions in the ‘start phase’ can be associated with intuitive notions like ‘connection’, ‘correspondence’, or ‘affinity’.

It is just this intuitive concept of affinity that requires a mathematical analysis. Therefore, the next logical step is to set up a mathematical formalism destined to supply an exact version of ‘affinity’. The concept of *similarity*, together with its one of the suitable definitions (Sect. 5), fulfils this requirement. Furthermore, it will turn out that the other two features just handled, the role of meaning and the two-step character of any measurement, have their equivalents in this new concept. But before, some mathematical fundamentals should be studied.

4 Mathematics of Distance and Similarity

Metric spaces were introduced in 1914 by Felix Hausdorff. In a metric space a distance function $d(x,y)$ is defined (for all pairs (x,y)) such that:

M1: $d(x,y) \geq 0$ (non-negativity)
M2: if $x = y$ then $d(x,y) = 0$
M3: if $d(x,y) = 0$ then $x = y$
M4: $d(x,y) = d(y,x)$ (symmetry)
M5: $d(x,y) \leq d(x,z) + d(z,y)$ (triangle inequality)

The structure thus defined is called a *metric*. This standard definition has been generalised in many directions (Deza and Deza 2009). An important variant is identical with a proposal due to Maurice Fréchet in 1906: a *semi-metric* (also called *pseudo-metric*) is obtained by simply dropping **M3**. This means that there can be pairs (x,y) with $x \neq y$, representing pairs of different entities–which nevertheless are ascribed a 'distance' equal to zero. In view of the mathematical formulation for entanglement, with its inseparable description, it may be worth a speculation asking whether semi-metrics can be meaningful here.

The intended mathematical formalisation of similarity (Sect. 3) can be performed by means of a dissimilarity function: given a finite set of elements, to each pair (x,y) a function $d(x,y)$ must be defined such that $d(x,y)$ has the usual properties of a metric. Then a great dissimilarity corresponds to low similarity, and vice versa (the transformation is trivial). So we have a function with the required properties—dissimilarity is easier for mathematical handling, whereas similarity comes nearer to human intuition.

Similarity measures are no recent development. Under headlines like "similarity search" or "similarity retrieval" they are a tool in modern information retrieval, with applications to chemistry and image data analysis (Kowalski 1997, pp. 152–157). The underlying material has the shape of numerical lists and tables. In the present context, however, an advanced definition is needed, which makes a consequent use of structural information and enables extensions to different kinds of complex structures (e.g., hierarchical structures).

5 Similarity Between Complex Structures

5.1 Measurement as a Two-Stage Process

Any act of measurement (or observation) is an ordered sequence of several consecutive phases. In an abridged version, the following two phases can be distinguished:

1. The initiator decides what is to be measured and how this shall be done, simultaneously defining the context, e.g., the purpose of the measurement, the required accuracy, etc.
2. The measurement as such is performed, including registration and documentation.

This inevitable two-step character of any measurement is often camouflaged by the phenomenon of 'tacit context'. If somebody in a laboratory is told to 'measure the temperature', then the context is self-evident: location and time are clear, as well a the required precision, the available instruments, etc. So we find the illusion of a 'single-step measurement': it is broadly assumed that only the name of a variable must be said, and then everything is clear.

5.2 Perspective Notions

Perspective notions are terms which–beyond the well-known context-dependence of any meaning of words–require an explicit statement of the context. A simple example is the term 'classification': e.g., the chemical elements can be classified according to their atomic weights, specific weights, electrochemical or radioactive properties, etc. The tasks to classify a single object or to subdivide a given set can be accomplished only after the purpose of the classification or the relevant criteria have been disclosed. The phenomenon of tacit context (Sect. 5.1) can hide this fact and bring about an additional complication.

Similarity and *dissimilarity* are perspective notions. Within a given set, two objects can be more or less similar depending on their size, shape, weight, appearance, structure, function, etc. Similarity between structures is never a property of the structures themselves; rather, it is defined by an observer, and different similarity functions mirror the different individual views of different observers.

5.3 Possible Mathematical Tools

5.3.1 General Requirements

Any mathematical tool for handling entanglement must fulfil the criteria of *nonlocality* and *selectivity*. Entanglement is immune against bounds imposed by distance or maximal signal velocity, but conversely, a third party, geometrically very near to one of two entangled partners, even need not be aware of the fact that something is going on.

As a consequence, the distance function which has to be formulated here has nothing in common with the ordinary (geometric) distance between two points in space. Rather, we have to quantify the distance between two complex objects, both equipped with an internal structure.

5.3.2 Hierarchical Matrices

Sets of objects, particularly with a hierarchical internal structure, can be represented by hierarchical matrices, which were originally introduced for numerical mathematics (Bebendorf 2008). Selected entries of an ordinary matrix are replaced by matrices, and this is repeated recursively (over a finite number of steps).

This tool has a competitor: the graph grammars to be treated in the next sections. Under mathematical aspects, both descriptive tools may be considered equivalent.

The difference lies in human cognition (ease of learning, problem-solving heuristics, etc.). It can well be imagined that one or another reader, due to individual prior experience and propensity, may prefer a representation by hierarchical matrices, which are outlined here briefly for the sake of fairness and completeness.

5.3.3 Graph Grammars

The mathematical tool primarily proposed here is based upon *graph grammars*. (These were defined in 1969 and since have proved their usefulness in a great variety of application fields (Drewes 2007); for details see Gernert (1997), with diagrams and references.) In the simplest case, each object is represented by a finite connected graph, possibly with vertex labels. These vertex labels may be allowed to have the form of graphs themselves, and by recursion we arrive at the concept of *hierarchical graphs*, which enable a handling of objects with a hierarchical internal structure.

In the easiest case, a finite set $\mathbf{G} = \{G_1, G_2, \ldots, G_n\}$ of finite connected graphs is presupposed, and a function $d(G_i,G_k)$ is to be defined. A *graph grammar* is given by a startgraph and a finite number of production rules. Each production rule permits the generation of a new graph from one of the already existing graphs. A production rule is a triple $\{S_l, S_r, R\}$, where S_l is the subgraph to be replaced (left-hand side), S_r is the subgraph to be substituted for it (right-hand side), and R denotes the embedding rule (which governs the way how S_r has to be inserted).

An example (Fig. 1) shows the startgraph and a subset of those graphs which can be generated in at most two transformation steps. Here only one production rule exists: the addition of a new edge such that it has exactly one vertex in common with the previous graph. Generally, the 'continuation' is not unique, because at a certain spot one from several admitted production rules must be chosen, or one production rule can operate with respect to different subgraphs. Hence we will find 'branching', as indicated by the arrows in Fig. 1.

For the present application a graph grammar Γ is required which generates *at least* all graphs in the given set $\mathbf{G}$. If Γ has been fixed, then the requested distance function can be defined by

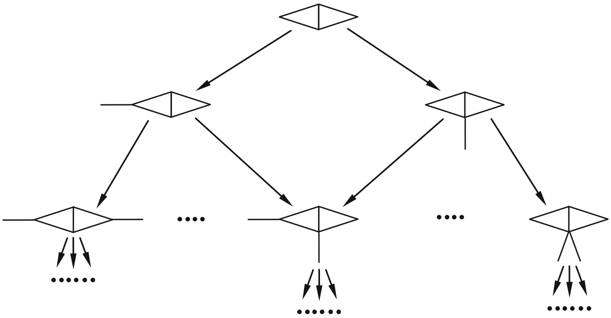

Fig. 1 Example of a graph grammar—some of the graphs generated in the first steps

$$d(\mathrm{G_i}, \mathrm{G_k}) = \min L(\mathrm{G_i}, \mathrm{G_k}),$$

where $L(\mathrm{G_i},\mathrm{G_k})$ is length of a 'path' that leads from $\mathrm{G_i}$ to $\mathrm{G_k}$ by applying production rules from Γ and the inverse transformations; each such step contributes 1 to the length L (L corresponds to the number of steps 'upward' and 'downward' in the tree-like diagram representing Γ). The measurement of similarity by means of graph grammars can be extended to graphs with vertex- and/or edge-labels, and finally to hierarchical graphs as mentioned above.

For any given finite set $\mathbf{G}$ of graphs it is always possible to set up a graph grammar Γ such that at least all graphs in $\mathbf{G}$ are generated by Γ. But, apart from trivial cases, a graph grammar specified in this way cannot be unique. Rather, there is a multitude of graph grammars which are all suited to represent $\mathbf{G}$. The reason behind this lies in the fact that 'similarity' is a perspective notion (Sect. 5.2); different graph grammars, all with the required properties, mirror the different preconceptions of different authors.

5.4 More about the Role of Graph Grammars in Generalised Quantum Theory

For the moment, there is a loose connection between graph grammars and GQT. Primary facts are the observable events, from 'classical' EPR processes to the psychological effects as mentioned in the beginning (Sect. 1). Here graph grammars take over a servant role, as a mathematical tool in order to replace intuitive terms like 'affinity' (Sect. 3) by a similarity measure with the usual mathematical properties and fitting the user's predisposition.

In the present context, all further applications—or interpretations—of graph grammars must be placed under the reservation that they still can only be plausible speculations.

For a better understanding of the following it should be recalled that also macroscopic entanglement (Sect. 3) is to be included. Only simple finite graphs will be considered (an extension to hierarchical graphs is possible, yet with some mathematical legwork).

As before, let there be a finite set $\mathbf{G} = \{\mathrm{G_1}, \mathrm{G_2}, \ldots, \mathrm{G_n}\}$ of finite connected graphs (no two isomorphic). Different styles of defining a distance function $d(\mathrm{G_i},\mathrm{G_k})$ will be regarded, but now under a novel aspect.

In a first step, take the graph $\mathrm{K_1}$ (consisting of exactly one vertex) as the startgraph, and define the production rule $\mathrm{P_m}$ by the transition from $\mathrm{K_1}$ to $\mathrm{G_m}$ ($\mathrm{m} = 1, \ldots, \mathrm{n}$). This leads to an equivalent of the Boolean metric (with $d(\mathrm{G_i},\mathrm{G_k}) = 2$ if $\mathrm{i} \neq \mathrm{k}$, and else zero), which fulfils the mathematical postulates, but is vacuous.

By way of contrast, a graph grammar is presupposed in which some pairs of graphs have a low dissimilarity, whereas other pairs show a marked disparity. Two of the given graphs, $\mathrm{G_i}$ and $\mathrm{G_k}$, with a small distance $d(\mathrm{G_i},\mathrm{G_k})$, are selected. From the latter property it follows that (in nontrivial cases) there exist many pairs of isomorphic subgraphs with one partner in $\mathrm{G_i}$ and the other in $\mathrm{G_k}$. As a consequence, there is not only the presupposed similarity between the entire graphs $\mathrm{G_i}$ and $\mathrm{G_k}$, but we also have a lot of 'correspondences' on a lower level. (For the moment, these ‘correspondences’ are defined by isomorphism between subgraphs—extensions and a quantitative version are beyond the scope of this paper).

Now effects can happen which are scarcely possible in pairs of quite dissimilar graphs. There may be correspondences between subgraphs, sub-subgraphs,…, etc., and finally a ‘structure in layers’. This permits us to formulate an issue of GQT at least as a question: If there are two or more such layers, will then macro-entanglement be initialised top-down or bottom-up, or in an interplay of both modes?

Indeed, processes of stepwise (recursive) structure adaptation can be empirically found. The classical example is text interpretation: in order to understand a text, one must grasp the words, but word meaning depends on the total text. Nevertheless, an adequate text understanding can be attained through a recursive process of stepwise improvement.

An experimental test can be based, e.g., on reactions between macromolecules, where an unusual outcome (e.g., a reaction rate above expectation) would require a non-classical interpretation, and variations of parameters may reveal the character of processes.

6 Concluding Remarks and Outlook

The term ‘meaning’ can no longer be banned from scientific language (nor restricted to part of the scientific disciplines). A scrutiny of the historical development displays that the adverse attitude against this term arose from motives which can be understood from the history and sociology of science, but not from scientific logic.

In GQT meaning plays a central role. It even occurs in the characterizations of fundamental terms, e.g.: “An observable is any (more or less) meaningful property of the system Σ which can be investigated in a given context. Non-trivial observables must exist, whenever Σ has enough internal structure to be a possible object for a meaningful study” (Atmanspacher et al. 2002 p. 389). Meaning is also related with synchronicity and with “common prearranged context” as proposed here.

Meaning is never a property of an object itself. Rather, it is created by an act of assignment of meaning, or *interpretation*. This is in analogy with the facts that measurement is a two-step process (Sect. 5.1) and that perspective notions, like similarity, require a two-step procedure (Sect. 5.3).

The role of meaning provokes the question whether this term can be formally defined—in the context of physics—beyond the level of verbal characterisation. The known facts of entanglement may even trigger a speculation about ‘hidden organising structures’ which would be responsible for steering a process in a certain direction. Possible answers are immediately constrained by known results: there is no explanation for quantum indeterminism by hidden variables, and local hidden variables are excluded; following Leggett (2003), nonlocal hidden variable theories of a special class (called ‘crypto-nonlocal theories’) are ruled out.

A simple model for a procedure which selects a substructure (considered relevant under some aspects) is given by selector matrices (Gernert 2000). Let a graph G be given by its adjacency matrix A, and define a matrix S with the entry 1 at some places of the main diagonal (and all the rest equal to 0). Then the multiplication AS (with AS = SA) exactly selects the subgraph of G defined by the non-zero entries in S. Extensions and modifications are possible, as well as functions which represent the varying degree of coupling between two processes depending on parameters or conditions.

Terms like 'emergence of meaning' and 'self-creation of meaning' are established in the context of self-organisation (Haken 2006, p. 23), where this aspect cannot be circumvented. Wheeler (1984/1989) even coined the term 'meaning physics', which he distinguished from earlier physics that can do without this notion. One of the next developments within GQT will be a concept for a formal characterisation of meaning and of its way of interacting within physical processes; there are reasons for expecting that, for a future mathematical formulation, the methodology outlined here can become essential.

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