Fine Tuning, Sequestering, and the Swampland

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We conjecture and present evidence that any effective field theory coupled to gravity in flat space admits at most a finite number of fine tunings, depending on the amount of supersymmetry and spacetime dimension. In particular, this means that there are infinitely many non-trivial correlations between the allowed deformations of a given effective field theory in the gravitational context. Fine tuning of parameters allows us to obtain some consistent CFTs in the IR limit of gravitational theories. Related to finiteness of fine tunings, we conjecture that except for a finite number of CFTs, the rest cannot be consistently coupled to gravity and belong to the swampland. Moreover, we argue that even though matter sectors coupled to gravity may sometimes be partially sequestered, there is an irreducible level of mixing between them, correlating and coupling infinitely many operators between these sectors.

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Introduction

The standard lore of effective field theory organizes physics according to energy scales. Once the ultraviolet (UV) boundary conditions of an effective field theory are specified, the consequences for low energy physics in principle follow from an analysis of renormalization group (RG) flow. It is natural to ask whether the parameters of the UV completion are completely arbitrary "order one" constants, or whether additional structure is required for consistency with quantum gravity. This has direct bearing on a number of hierarchy problems in particle physics, including the radiative stability of the Higgs mass relative to the Planck scale and the impact of extra sectors on the Standard Models of particle physics and cosmology.

In this note we argue that for a given effective field theory in flat space coupled to quantum gravity in the same spacetime dimension,[1] *only a finite number of parameters can ever be tuned*. More precisely, in an effective field theory with cutoff the Planck scale M_{pl} and Lagrangian:

$$L_{\text{eff}} = \sum_i \frac{g_i}{M_{pl}^{\nu_i}} \mathcal{O}_i, \tag{1}$$

we argue that the infinite set of couplings g_i are all determined by a *finite* list of parameters in the effective field theory. Moreover, we conjecture this number only depends on the number of supersymmetries and the macroscopic dimension of spacetime.

The primary evidence for our conjecture is that there appear to be a finite number of Minkowski vacua arising in string theory compactifications. For example, it is widely believed that there are a finite number of Calabi-Yau threefolds, and tadpole cancellation conditions enforce sharp upper bounds on the number of flux quanta / discrete parameters. These statements have natural generalizations in the context of M- and F-theory compactifications as well.

Even so, the consequences for low energy physics of these simple observations appear to have not been completely spelled out. Finiteness in the number of compactification geometries means there are also at most a finite number of tunable parameters in any given effective field theory. This greatly limits the structure of low energy effective field theories which can emerge in the infrared (IR) of a consistent string compactification. For example, assuming that there are only a finite number of conformal field theories (CFTs) for a fixed total number of relevant deformations, it implies there are a finite number of conformal fixed points which can be realized in quantum gravity. Additionally, distinct field theory sectors which are decoupled in the $M_{pl} \to \infty$ limit are necessarily always coupled when gravity is not switched off. While this latter statement may not at first appear surprising, finiteness in the number of allowed compactifications and their moduli means that there is an infinite amount of correlated mixing between extra sectors.

The plan in the rest of this note will be to expand on the above points. First, we discuss finiteness of the number of CFTs which can consistently couple to gravity, also illustrating with examples in different dimensions and amounts of supersymmetry. We then discuss some additional phenomenological consequences of having a finite number of tunable couplings.

CFTs and Quantum Gravity

In this section we consider the coupling of a conformal field theory (CFT) in flat space to gravity. CFTs are central to many aspects of quantum field theory (QFT),

[1] Note that a stack of D3-branes in 10D flat space would not satisfy this condition.

because most QFTs can be viewed as moving away from, or towards a conformal fixed point. More precisely, by "coupling a CFT to gravity" we mean there is a gravitational theory with an IR limit which includes the corresponding CFT.

The passage towards and away from a fixed point is dictated by its space of possible deformations. In general terms, such deformations can arise either through varying the couplings of the CFT or through operator VEVs. We refer to these as the "parameters" of the CFT.

If we consider the coupling of such a theory to gravity, the parameters controlling such deformations will correspond to dynamical fields, as in general any quantum gravitational theory is conjectured to have no free parameters [1]. Indeed, in the context of string compactification, one should expect that *all* of the parameters appearing in a putative CFT originate from vevs of fields which are light relative to the Planck scale. Such fields could originate from operators within the CFT itself, or from fields outside of the CFT coupled to gravity. But such parameters cannot be tuned arbitrarily. For quantum gravity in Minkowski space, this follows directly from the conjectured finiteness of such vacua in string theory [1].[2] Combining these statements, we learn that of the a priori infinitely many possible deformations of a CFT, only a *finite* number of them can be fine tuned, and the rest are determined in terms of the fine tuned ones!

This in particular implies that if we want to obtain a CFT in the infrared limit of a gravitational theory, it had better be the case that the total number of relevant deformations and VEVs that takes us away from the conformal point are not too large. Otherwise, we may not have enough light modes in the gravity theory to fine tune in order to realize the CFT point. Assuming that for a fixed number of relevant deformations there are only a finite number of CFTs, which seems quite reasonable, we thus conclude that only a finite number of CFTs can be realized as IR limits of a gravitational theory. In other words *nearly all CFTs belong to the swampland!*

Of course it is not a priori clear that we can get *any* CFT in the infrared limit of a gravitational theory. As we shortly explain, however, these are abundant in string constructions, and arise by tuning all relevant deformations to zero in a given string compactification. Moreover, we find that this fine tuning is not limited only to relevant deformations and in some cases some of the irrelevant operators can also be tuned to zero. But we will find that almost all irrelevant parameters are necessarily switched on in quantum gravity.

Having spelled out the general reason to expect such conjectures to be true, we now turn to explicit examples illustrating the main points.

Examples

We organize our discussion of examples according to the number of spacetime dimensions, as well as the number of supersymmetries.

Consider first the case of 6D SCFTs. Such theories admit either $\mathcal{N} = (2,0)$ or $\mathcal{N} = (1,0)$ supersymmetry, namely sixteen or eight real supercharges, respectively. In both cases, there is a conjectural classification of the resulting theories which can result from string / F-theory compactification. An interesting feature of these examples is that there are no supersymmetric relevant or marginal deformations of these theories [3, 4]. Instead, all the parameters are obtained from operator VEVs.

For interacting 6D SCFTs with $(2,0)$ supersymmetry there is a famous ADE classification which is obtained from compactifications of type IIB on hyperkahler geometries with an ADE singularity [5]. Such geometries all have the local presentation $\mathbb{C}^2/\Gamma_{ADE}$ with $\Gamma_{ADE} \subset SU(2)$ a finite order subgroup.[3] All other $(2,0)$ SCFTs are then obtained by taking tensor products of these basic building blocks. Such theories are then classified by semi-simple Lie algebras $\mathfrak{g}_1 \times ... \times \mathfrak{g}_n$ with each $\mathfrak{g}_i$ a simple ADE Lie algebra. In all these cases, the space of possible deformations is controlled by the number of tensor multiplets which is given by the rank of the corresponding semi-simple Lie algebra. This leads to an infinite list of such SCFTs.

Even so, nearly all of these theories belong to the 6D swampland since they cannot be coupled to 6D $(2,0)$ supergravity. In this case, the total rank of the semi-simple Lie algebra $\mathfrak{g}_1 \times ... \times \mathfrak{g}_n$ is bounded above by:

$$\mathrm{rank}(\mathfrak{g}_1 \times ... \times \mathfrak{g}_n) \leq 21 \tag{2}$$

This follows from anomaly cancellation for $(2,0)$ supergravity coupled to matter fields, which in this case fixes the total number of tensor multiplets to be 21 (see reference [7]). Thus, any non-trivial CFTs that can emerge must fit within this constraint. This is also consistent with the fact that to reach a $(2,0)$ supergravity theory from compactification of type IIB strings in the first place, we need to use a K3 surface, and there is only one manifold of this topology.

In fact, one can in principle also classify all possible 6D SCFTs which can be obtained from singular limits

[2] For AdS vacua [2] there is a refined version of this conjecture which appears to hold. The reason for the caveat is that even though fluxes allow one to tune the size of the AdS space, this is correlated with the size of the extra dimensions.

[3] There is also some choice in the topological data of these theories associated with the spectrum of various defect operators (see e.g. [6]). This will not play an important role in our considerations.

in the moduli space of IIB compactified on a K3 surface (see e.g. [5, 8]):[4]

$$\mathcal{M}_{IIB} \simeq O(\Gamma_{5,21})\backslash O(5,21)/(O(5)\times O(21)). \quad (3)$$

To get a 6D SCFT, we have the necessary condition that the root system for the corresponding semi-simple Lie algebra embeds in $\Gamma_{5,21}$. As two examples in this class, note that we can get a single group corresponding to the D_{21} theory. As another example, note that we can get three decoupled SCFTs, $E_8 \times E_8 \times A_5$ coupled to $(2,0)$ supergravity in six dimensions. Both examples saturate the upper bound of 21 tensor multiplets, and the latter example also shows we can have multiple decoupled CFTs in the deep infrared.

Consider next 6D theories with minimal $\mathcal{N} = (1,0)$ supersymmetry. Anomaly cancellation, along with physical constraints such as the requirement that all kinetic energy terms remain positive definite imposes stringent conditions on admissible low energy theories. Embedding such theories in string theory imposes additional limitations, cutting this to a finite list of possibilities [9]. The broadest class of UV completions is obtained via compactifications of F-theory (see e.g. [9–11] as well as the review [12]). In F-theory, all such vacua are realized by a choice of an elliptically fibered Calabi-Yau threefold with base B a Kähler surface. The dimension of the tensor branch moduli space is $h^{1,1}(B) - 1$ while the number of hypermultiplets of the Higgs branch moduli space is counted by $h^{2,1}$ of the Calabi-Yau threefold. 6D SCFT sectors arise from simultaneously collapsing $\mathbb{P}^1$'s in the base B, and a classification of the infinite list of possibilities was completed in references [13, 14] (see [15] for a review). It is also possible to couple some of these 6D CFTs to gravity (see e.g. [16–18]). An example of this sort is to take the Calabi-Yau threefold $(T^2\times T^2\times T^2)/\mathbb{Z}_3$, where we tune the complex structure of each T^2 to admit a $\mathbb{Z}_3$ symmetry. We will revisit this example in lower dimensions as obtained by compactification on further circles.

Nevertheless, almost all of the 6D $(1,0)$ CFTs, with the exception of a finite number of them, require a non-compact Calabi-Yau threefold. This means they cannot be dynamically coupled to gravity in six dimensions. Let us briefly review this finiteness property. There are general finiteness results on the existence of *elliptically fibered* Calabi-Yau threefolds [20, 21] which in turn imply upper bounds on the number of tensor, vector and hyper multiplets [9]. While the exact upper bound is unknown, in practice, the largest dimension for the tensor branch moduli space is 193, as found in [22, 23], which fits with general statements available for F-theory models with a toric base [11]. Indeed, there is a rigorous upper bound on the Hodge numbers $h^{1,1}$ and $h^{2,1}$ of elliptic threefolds with a toric base [19], and there is a general expectation that moving beyond the toric case will not greatly affect these bounds. For our present purposes, this of course means that the number of 6D SCFTs which can be consistently coupled to 6D supergravity is also finite. Despite this finiteness in the allowed realizations of $(1,0)$ SCFTs coupled to gravity arising in string theory, anomaly cancellation constraints allow in principle infinitely many possible SCFTs coupling to gravity [9]. However, at least some of these infinite families can be ruled out by additional consistency conditions [24], and it is natural to conjecture that when all constraints are imposed only a finite number of them will survive.

In the case of 5D SCFTs, the superconformal algebra allows for $\mathcal{N} = 1$ supersymmetry, namely eight real supercharges. One way to generate a large class of examples is to consider M-theory compactified on a Calabi-Yau threefold with at least one divisor which collapses to a point at finite distance in moduli space. Such singularities are expected to generate most, if not all of the possible 5D SCFTs. The general point is that these canonical singularities are localized at isolated patches of the Calabi-Yau, but are coupled to one another by effects inherited from 11D supergravity.

One way to generate a large class of examples of this sort is to take a 6D $(1,0)$ SCFT and compactify it on a circle [25, 26]. An illustrative example of this type is given by the Calabi-Yau threefold $(T^2 \times T^2 \times T^2)/\mathbb{Z}_3$, where we tune the complex structure of each T^2 to admit a $\mathbb{Z}_3$ symmetry. There are precisely 27 orbifold fixed points, each of which is locally characterized by the geometry $\mathbb{C}^3/\mathbb{Z}_3$ which in the resolved phase is described by a $\mathbb{P}^2$ collapsing to zero size. In the limit where the Calabi-Yau threefold decompactifies, we have 27 5D SCFTs decoupled from 5D supergravity, each of which is described by a 5D SCFT [27, 28]. The Coulomb branch is given by 27 5D $\mathcal{N} = 1$ vector multiplets, each with a real scalar. The Coulomb branch parameter dictates the size of the $\mathbb{P}^2$, and there are BPS states obtained from M2-branes wrapped on curves in each $\mathbb{P}^2$. In particular, even in the limit where we couple to quantum gravity, all 27 Coulomb branch parameters can be tuned to zero independently.

But it is also true that when gravity is switched on, these different sectors cannot be completely decoupled. To see this, suppose we go away from the CFT point by moving onto the Coulomb branch, giving finite size to each $\mathbb{P}^2$. In this case, some of the BPS states which were previously massless at the CFT point (corresponding to M2-branes wrapped on curves inside each $\mathbb{P}^2$) now acquire a mass proportional to $m_i \sim \phi_i$ the Coulomb branch parameter. Here, we have normalized the scalar fields to have mass dimension one and fermions to have dimension two. If we denote by L the characteristic length scale of the T^2's, there is a Kaluza-Klein (KK)

[4] Observe also that compactifying on a further S^1 leads to a dual description in terms of heterotic strings on T^5.

mass scale $M_{KK} \sim 1/L$. Integrating out the whole tower of KK states from compactification of the 11D supergravity model will induce various higher-dimension operators in the 5D effective field theory. This includes higher derivative interactions as well as four-fermion interactions. Indeed, letting ψ_i denote one such fermionic field describing 5D excitations of an M2-brane wrapped on a curve, it is not difficult to see that exchange of the KK tower of 11D supergravity modes induces interactions between fermions in previously decoupled sectors of the form:[5]

$$L_{mix} \supset \frac{m_i m_j}{M_{pl}^3 M_{KK}^2} \overline{\psi}_i \psi_i \overline{\psi}_j \psi_j, \tag{4}$$

with M_{pl} the 5D Planck scale. Since we also have $m_i \sim \phi_i$, we learn that there is a mixing term of the form:

$$L_{mix} \supset C_{ij} \mathcal{O}_i \mathcal{O}_j \tag{5}$$

where $\mathcal{O}_i = \phi_i \overline{\psi}_i \psi_i, \quad \mathcal{O}_j = \phi_j \overline{\psi}_j \psi_j$, and

$$C_{ij} \sim \frac{1}{M_{pl}^5} \left(\frac{M_{pl}}{M_{KK}} \right)^2. \tag{6}$$

Observe that although naive dimensional analysis might suggest $C_{ij} \sim 1/M_{pl}^5$, this is really a lower bound on the strength of this interaction. The 5D Planck scale is related to the 11D Planck scale M_{11D} and the Kaluza-Klein scale M_{KK} via:

$$\left(\frac{M_{KK}}{M_{11D}} \right)^6 \left(\frac{M_{pl}}{M_{11D}} \right)^3 \sim 1. \tag{7}$$

Since we have assumed L is large relative to the 11D Planck length anyway, this also means:

$$M_{pl} > M_{11D} > M_{KK} \tag{8}$$

So returning to line (6), we learn that $C_{ij} \gtrsim 1/M_{pl}^5$. In other words the strength of the interaction between two CFTs is at the very least dictated by the Planck scale, but there can be additional enhancement when there is a hierarchy between the Kaluza-Klein scale and the 5D Planck scale. This example also illustrates that although we can sometimes tune the relevant deformations to zero (reaching a fixed point), irrelevant deformations and interactions between different CFTs cannot be tuned away.

Let us now turn to 4D examples. Here, we consider stringy examples with $\mathcal{N} = 4, 2, 1$ supersymmetries, and briefly comment on the non-supersymmetric case.

Consider first 4D $\mathcal{N} = 4$ theories. These SCFTs are labelled by a choice of semi-simple gauge group and can all be viewed as descending from compactification of the 6D $(2,0)$ theories on a suitable T^2 (possibly with twists). Coupling such sectors to supergravity has been studied in the literature (see e.g. [30]) but does not appear to impose any significant constraint on the actual matter content of the theory.

By contrast, the stringy list of possibilities are quite limited. Much as in our discussion of 6D $(2,0)$ theories coupled to gravity and the bound of line (2), the total rank is bounded above by:

$$r \leq 22. \tag{9}$$

One way to get the maximal rank is by compactification of type II strings on $K3 \times T^2$ or equivalently heterotic strings on T^6, leading to the Narain charge lattice $\Gamma_{6,22}$ with a gauge group of total rank 22. The 6×22 deformations of the Narain lattice correspond to deforming the $\mathcal{N} = 4$ matter sectors with the Coulomb branch VEVs in the Cartan of the group. In this case there are no relevant deformations of $\mathcal{N} = 4$ SCFTs preserving $\mathcal{N} = 4$ SUSY. There are also other compactifications known where we can also get reduced ranks, as in CHL strings [29] by including automorphism twists of heterotic strings after compactification on torii. Thus it is natural to conjecture that $r = 22$ is the maximum rank of $\mathcal{N} = 4$ supersymmetric theories that can be coupled to $\mathcal{N} = 4$ supergravity in flat space.

Let us now proceed to 4D $\mathcal{N} = 2$ SCFTs coupled to $\mathcal{N} = 2$ supergravity. There are a number of ways to generate examples of this sort. The largest known class of examples is obtained from compactification of type II strings on Calabi-Yau threefolds. Let us in particular consider type IIB on Calabi-Yau threefolds. The non-compact versions of these manifolds near singularities are known to lead in the IR to 4D $\mathcal{N} = 2$ SCFTs [32] (see also [33–35]). For example, if we consider hypersurface singularities of the type:

$$uv + f(z_1, z_2) = 0 \tag{10}$$

where $u, v, z_1, z_2 \in \mathbb{C}$ and $f(z_1, z_2)$ is a quasihomogeneous polynomial, we get an SCFT in the IR. Let us assign weights q_1, q_2 to z_1, z_2 such that

$$f(\lambda^{q_1} z_1, \lambda^{q_2} z_2) = \lambda f(z_1, z_2). \tag{11}$$

Deformations away from the fixed point correspond to adding monomials in $z_1^a z_2^b$ to $f(z_1, z_2)$. Such a deformation has weight $aq_1 + bq_2$. We can organize the various types of deformation away from the fixed point using the quantity $w = (1 - q_1 - q_2)$. In particular, we have:

$$\text{Operator VEVs}: \ 0 \leq aq_1 + bq_2 < w \tag{12}$$

$$\text{Relevant Deformation}: \ w \leq aq_1 + bq_2 < 1 \tag{13}$$

$$\text{Marginal Deformation}: \ aq_1 + bq_2 = 1 \tag{14}$$

$$\text{Irrelevant Deformation}: \ 1 < aq_1 + bq_2 \leq 2w. \tag{15}$$

[5] One way to derive this coupling is to consider the force between wrapped M2-branes and anti-M2-branes.

There are, of course many additional deformations which are automatically set to zero in the deformation ring. These are all irrelevant operator deformations of the SCFT.

To illustrate, consider the special case:

$$f(z_1, z_2) = z_1^n + z_2^n \tag{16}$$

which is known as the (A_{n-1}, A_{n-1}) Argyres-Douglas theory (see e.g. [31]). In this case $q_1 = q_2 = 1/n$ and monomials in the singularity deformation ring are given by $z_1^a z_2^b$ which has weight $(a+b)/n$. The ring of deformations is generated by those monomials with $0 \leq a, b \leq n-2$. In the physical theory, these sort according to the following inequalities:

$$\text{Operator VEVs}: \; 0 \leq a+b < n-2 \tag{17}$$

$$\text{Relevant Deformation}: \; n-2 \leq a+b < n \tag{18}$$

$$\text{Marginal Deformation}: \; a+b = 1 \tag{19}$$

$$\text{Irrelevant Deformation}: \; n < a+b \leq 2n-4. \tag{20}$$

We can now ask whether this and related examples of $\mathcal{N}=2$ SCFTs can be coupled to quantum gravity. From the perspective of string compactification, this is equivalent to asking whether we can find such singularities in *compact* Calabi-Yau threefolds.[6] First of all, since it is widely believed that there are only a finite number of Calabi-Yau threefolds, this would immediately imply only a finite number of such models can appear in quantum gravity. Far more non-trivial is that this set is non-empty: some of these examples *consistently embed* in compact examples.

To illustrate, consider type IIB on a Calabi-Yau threefold given by an elliptic fibration over a base $\mathbb{P}^1 \times \mathbb{P}^1$. The minimal Weierstrass model for this geometry is:

$$y^2 = x^3 + x f_{8,8} + g_{12,12} \tag{21}$$

where $f_{8,8}$ is homogeneous of bidegree $(8,8)$, i.e., it is a polynomial of degree 8 in homogeneous coordinates of each $\mathbb{P}^1$, and $g_{12,12}$ has bidegree $(12,12)$. Consider, in the affine patch near the origin the following choices:

$$f_{8,8} = -\frac{3}{4} + \beta z_1^8 z_2^8 \tag{22}$$

$$g_{12,12} = \frac{1}{4} + z_1^{12} + z_2^{12}. \tag{23}$$

The higher order term in line (22) corresponds to an irrelevant deformation of the local singularity structure near $z_1 = z_2 = 0$. The local geometry after a suitable coordinate shift in x and y, results in the (A_{11}, A_{11}) Argyres-Douglas theory:

$$uv + z_1^{12} + z_2^{12} = 0. \tag{24}$$

As can be seen from this example in a quantum theory of gravity we can fine tune not only all the relevant operator VEVs and deformations but also some of the irrelevant operators that could appear. For example, keeping the higher order terms we see that some of the irrelevant deformations are also set to zero and the first irrelevant term that appears in $f_{8,8}$ is $z_1^8 z_2^8$.

Descending to theories with even less supersymmetry, there are several known constructions of 4D $\mathcal{N}=1$ SCFTs in limits where 4D gravity is decoupled. This includes compactifications of $(1,0)$ theories from 6D, various singular limits in local geometries, as well as brane probes of singular geometries and intersecting branes.

Coupling to gravity is more challenging because in addition to specifying a background geometry, it is also necessary to include the effects of fluxes and non-perturbative contributions to the low energy effective theory including possible generation of superpotentials. In a globally complete model the number of flux quanta and branes which can be introduced is also strongly constrained by tadpole cancellation considerations [37]. This again imposes a finiteness condition on possible SCFTs coupled to gravity. Finally, while it would of course be interesting to discuss stringy examples with completely broken supersymmetry in flat space, there are at present no completely controlled examples of this sort. We leave this topic for future work.

Phenomenological Considerations

Our main conjecture is that embedding an effective field theory in quantum gravity only allows a finite number of tunings. From this perspective, it is natural to ask about the potential consequences for particle physics and cosmology. Here, we discuss two such aspects, one connected with the possibility of having additional decoupled extra sectors, and also the extent to which any effective field theory in quantum gravity can be fine tuned.

In the context of stringy particle physics models, there can be many extra sectors beyond the Standard Model. Such sectors provide dark matter candidates, and have also been considered in the model building literature as a possible means to source supersymmetry breaking effects. Sometimes the dynamics of these extra sectors can lead to problematic effects such as large flavor changing neutral currents so it is also common to posit that potentially problematic higher dimension operators can be suppressed, that is, "sequestered" (see e.g. [38]). Some aspects of sequestering in string theory have been studied in [39, 40]. It was found in reference [41] that there is

[6] Note that the non-renormalization theorems of type IIB strings imply that the complex structures, which are part of vector multiplets, do not get deformed by quantum corrections since the string coupling is in a hypermultiplet. In other words, these statements are exact quantum mechanically as well.

often some mixing for different field theory sectors, even when placed in different warped throats. Our conjecture on the appearance of only a finite number of fine tunings means there is an irreducible amount of mixing between any two QFT sectors in flat space which will happen with at least the strengths given by naive expectations of gravitational effects related to inverse powers of M_{pl}. On the constructive side, we note that we can have partially sequestered sectors where relevant operators are tuned to zero.

This circle of ideas is also of relevance in the specific context of stringy cosmological quintessence models, particularly as motivated by general proposed swampland constraints on dark energy [42, 43]. As discussed there, to be consistent with observations, a quintessence field can couple strongly only to the dark sector, and it is natural to view it as part of the dark sector. The present considerations allow for some level of sequestering, and suggest a general avenue for exploring this class of dark matter / quintessence models which are very weakly coupled to the visible sector.

Finally, the fact that we can only tune a finite number of parameters in our low energy effective field theory also has bearing on the stability of the electroweak scale in the Standard Model, namely that the mass of the Higgs is far smaller than the Planck scale. One may naturally ask if quantum gravity has any bearing on this question. Naively it may appear that it does not; however there have already been suggestions that the small value of neutrino masses may be related to a refined version of the weak gravity conjecture [44, 47]. Similarly, it has been suggested that the resolution of the hierarchy problem may have a swampland explanation [48]. Such a fine tuning can arise in a quantum theory of gravity, so at least the present conjectures are not inconsistent with large hierarchies of scale. Needless to say, it is reassuring that the conditions we are proposing here are perfectly compatible with our observed Universe!

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