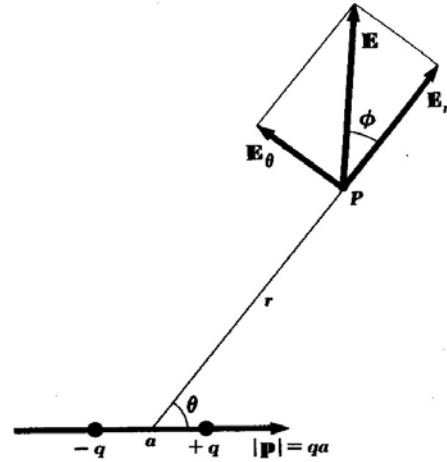# Chapter 6   ELECTRIC FIELDS IN DIELECTRICS

Figure 1   Electric field components at P caused by an electric dipole

INTRODUCTION

The discussion of electric fields has ignored the influence of matter on the E field except for the discussion of E fields in electric conductors. The behavior of matter in electric fields can greatly influence the E field as shown for the case of a good conductor. The response of matter to an electric field falls into two main categories, polarization of bound charges in atoms or molecules, and the flow of electric current due to the existence of unbound charges in matter as in conductors. A non-conducting material is called a dielectric. All the charges in a dielectric are bound and thus they cannot support an electric current. In spite of this, the electric field is changed greatly by the presence of a dielectric material. This chapter will discuss the properties of the electric dipole that underlies the behaviour of dielectrics. This knowledge then will be used to discuss the microscopic aspects of the influence of electric fields on atoms, which is needed to fully understand the macroscopic behavior of dielectrics on electric fields. This chapter finishes with a discussion of some important practical consequences.

THE ELECTRIC DIPOLE FIELD

Prior to discussion of a dielectric it is necessary to be able to consider the electric field of the electric dipole as well as consider the torque and translational force that can act on an electric dipole in an electric field. Remember that an electric dipole comprises two equal and opposite charges q separated by a distance d. The electric dipole moment of the dipole $\overrightarrow{\mathbf{p}}$ is defined as:

$$\overrightarrow{\mathbf{p}} = q\overrightarrow{\mathbf{d}}$$

where $\overrightarrow{\mathbf{d}}$ points from the negative to positive charge. The concept of electric potential is especially useful for calculating the electric field for the electric dipole.

The electric potential relative to infinity at the point P in figure 1 is given by:

$$V_P = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{\left[r - \left(\frac{d}{2}\right)\cos\theta_2\right]} - \frac{q}{\left[r + \left(\frac{d}{2}\right)\cos\theta_1\right]}\right)$$

where θ_2 is the angle for the positive charge and θ_1 for the negative charge. Let $r >> d$ then $\theta_1 \approx \theta_2 \approx \theta$, and the equation simplifies to:

$$V_P = \frac{q}{4\pi\varepsilon_o}\frac{d\cos\theta}{\left[r^2 - \left(\frac{d}{2}\right)^2\cos^2\theta\right]} \simeq \frac{q}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2}$$

The electric dipole moment is given by:

$$\overrightarrow{\mathbf{p}} = \widehat{\mathbf{i}}qd$$

that is, the electric dipole moment is a vector quantity as opposed to the monopole moment q.

Thus the electric potential relative to infinity can be written as:

$$V_P = \frac{\overrightarrow{\mathbf{p}}\cdot\widehat{\mathbf{r}}}{4\pi\varepsilon_o r^2} \qquad r >> d$$

since $\widehat{\mathbf{i}}\cdot\widehat{\mathbf{r}} = \cos\theta$ where $\widehat{\mathbf{r}}$ point from the electric dipole towards the point P.
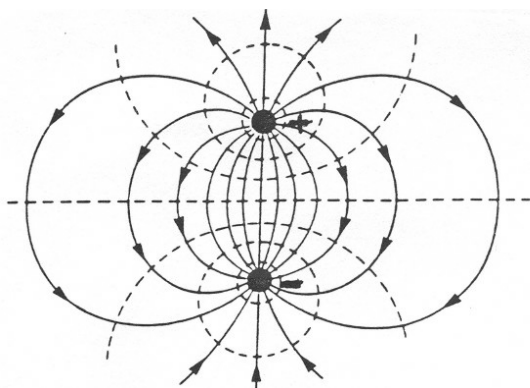
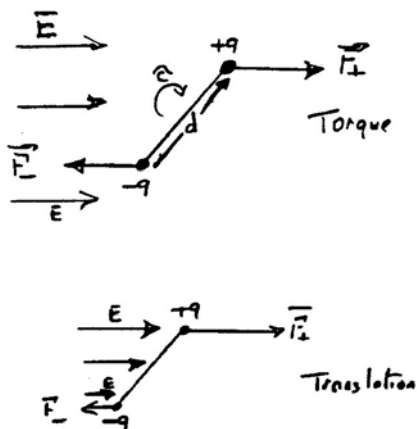Figure 2   The electric potential and field for an electric dipole.

Figure 3   Torque and translational forces for an electric dipole in an electric field.

The electric field for the electric dipole, in the far-field approximation, can be derived from the above potential since $\overrightarrow{\mathbf{E}} = -\overrightarrow{\mathbf{\nabla}}V$ as was discussed previously. The electric field lines and equipotentials are shown in figure 2.

FORCES ON AN ELECTRIC DIPOLE

When an electric dipole is in an electric field a torque is generated by the opposing forces acting on the two dipole charges which have opposite signs. As shown, in figure 3, the torque τ is given by:

$$\overrightarrow{\tau} = 2(\frac{d}{2}qE\sin\theta) = \overrightarrow{\mathbf{p}} \times \overrightarrow{\mathbf{E}}$$

Remember that the cross product implies that the torque is a vector perpendicular to $\overrightarrow{\mathbf{p}}$ and $\overrightarrow{\mathbf{E}}$. The torque causes the electric dipole to align with the electric field just like a compass needle. Grass seeds, which became small electric dipoles in an E field, were shown to align with the E field in the earlier demonstration.

If the electric field is non-uniform then the net translational force of the two charges of an electric dipole are not equal and opposite leading to a translational force. It can be shown that the net force is given by the gradient of the scalar product $\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}}$. That is

$$\overrightarrow{\mathbf{F}_{net}} = \overrightarrow{\mathbf{\nabla}} \left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}} \right)$$

When the dipole and electric field are aligned then this simplifies to

$$\overrightarrow{\mathbf{F}_{net}} = qd\frac{dE}{dx}$$

that is, the gradient of the electric field times qd just equals the difference in in the opposing forces on the two electric poles. This force will be used later when discussing the electrostatic precipitator.

DIELECTRIC POLARIZATION

The polarization of a dielectric can be explained on a microscopic scale by considering the polarization of atoms and molecules.

Atomic polarization

The spatial distribution of atomic electrons is centered on the positively charged nucleus when no external electric field acts. Thus the center of the negative and positive charge distributions are the same. However, in the presence of an external electric field, the forces on the positively-charge nucleus and negatively-charged electrons induces a small relative displacement of the two charge distributions leading to a non-zero dipole moment. It turns out that the displacement is proportional to the applied electric field. Thus the net electric dipole moment $\overrightarrow{\mathbf{p}} = \alpha\overrightarrow{\mathbf{E}}$ where α, the constant of proportionality, is called the atomic polarizability. Typically the displacement of the two charge distributions is very small, that is, the order of $10^{-15}m$, which is comparable to the size of the nucleus.

Molecular polarization

Non-polar molecules are those without a permanent electric dipole moment. As with an atom, when these non-polar molecules are in an electric field they acquire an induced dipole moment. Linear molecules, such as CO_2 are easier to polarize along the axis of the dipole than perpendicular to axis of the molecule leading to 20 to 30 times larger polarization than in the atom. That is, one is dealing with relative displacement of the charges along the molecular axis of about $10^{-14}m$. In this case, it is possible to have $\overrightarrow{\mathbf{p}} = \alpha\overrightarrow{\mathbf{E}}$ where $\overrightarrow{\mathbf{p}}$ and $\overrightarrow{\mathbf{E}}$ are not parallel, that is, for some materials, α may not be a simple scalar number.
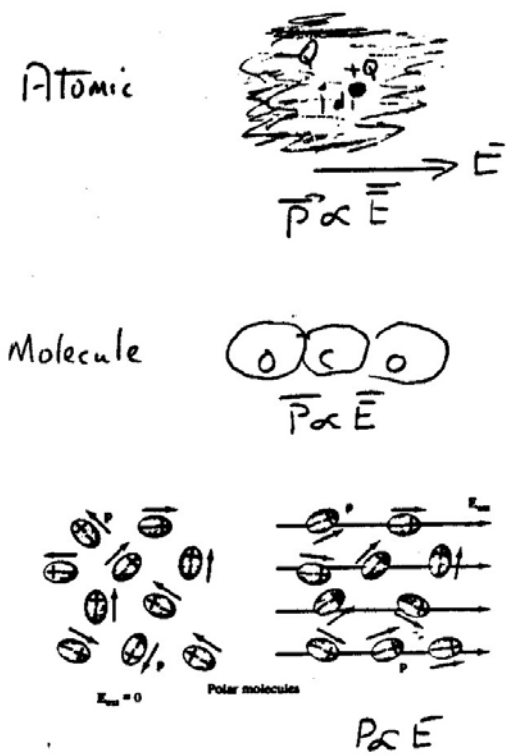
Figure 4   Atomic, molecular and polar molecule contributions to polarization in dielectrics.

Align polar molecules

Polar molecules, such as HCl or H_2O, have a permanent electric dipole moment since atomic binding causes electrons to be preferentially bonded to one of the atoms. Thus one is dealing with equal and opposite electric charge displaced the order of $10^{-10}m$, the size of the molecule, which is 10^5 times larger that atomic polarization. These polar molecules tend to be randomly oriented in the absence of an external electric field. An external electric field applies a torque tending to align these dipoles along the electric field. This alignment torque, which is proportional to E, is opposed either by the random thermal motion destroying alignment or the elastic forces in the solid lattice. The polarization in most materials leads to a net average electric dipole moment $\overrightarrow{\mathbf{p}} = \alpha \overrightarrow{\mathbf{E}}$ where the coefficient α is very much larger than for atomic polarization. The polarizability of polar molecules is orders of magnitude larger than the other two mechanisms. Note that thermal motion tends to destroy alignment of polar molecules. Thus the net dipole moment increases as the temperature of the material is decreased. Also note that the polarization is frequency dependent. At low frequencies the polar molecules contribute a large polarization. However, at high frequencies, the net polarization decreases because the polar molecules cannot follow the rapidly-changing E field because of the large moment of inertia of the polar molecules.

Most dielectrics are linear dielectrics, that is, where all three mechanisms lead to the average electric dipole moment being proportional to the electric field E, that is,

$$\overrightarrow{\mathbf{p}} = \alpha \overrightarrow{\mathbf{E}} \qquad \text{linear dielectric}$$

This has important consequences that appreciably simplifies treatment of electric fields in matter as will be discussed.

A nice example of $\overrightarrow{\mathbf{p}} = \alpha \overrightarrow{\mathbf{E}}$ is the electrostatic precipitator which uses a highly non-uniform electric field near the high voltage comb of points to both polarize the dielectric in smoke and then attract it in the non-linear electric field near the points. For a linear dielectric the translational force on a dipole in a non-linear electric field is

$$\overrightarrow{\mathbf{F}_{net}} = \overrightarrow{\nabla} \left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}} \right) = \overrightarrow{\nabla} \left(\alpha E^2 \right)$$

This force attracts the dielectric into the region with the highest gradient of E^2. Thus any dielectric is attracted to the sharp points. High-voltage equipment becomes dirty near any high-field region for the same reason.

ELECTRIC FIELD IN A DIELECTRIC

The response of a dielectric can be treated in macroscopic terms. The prior discussion showed that microscopic polarization leads to an effective net separation of the positive and negative charge distributions leaving a surface charge distribution on the two surfaces of the dielectric due to the fact that the displaced charge distributions do not cancel at these surfaces.

For simplicity consider the case of a parallel-plate capacitor shown in figure 5, the negative charge from the induced dipole polarization is adjacent to the positive capacitor electrode while the positive induced charge is adjacent to the negative electrode. The induced polarization charge density σ_{pol} is real charge but these are bound charges as opposed to the free charge density σ_{free} applied to the conducting electrode plates. In the dielectric, the induced (bound) charge distribution causes an induced electric field E_{pol} that is opposite to the external electric field. Thus the net field in the dielectric is $E_{net} = E_{ext} - E_{pol}$. The induced field is zero outside of the dielectric.

The polarization, and thus the induced surface charge density σ_{pol}, of the dielectric in the parallel-plate capacitor, depends on E_{net} in the dielectric, not E_{ext}. For linear dielectrics the proportionality can be written as;

$$\sigma_{pol} = \chi_e \varepsilon_0 E_{net}$$

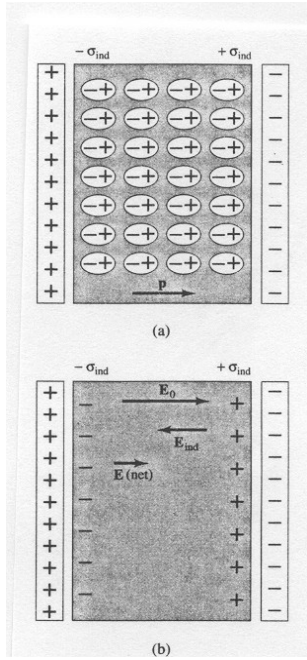where the factor χ_e does not depend on E_{net}.

Figure 5 a) Induced charges for a dielectric in a parallel-plate capacitor. b) The induced and external electric fields.

Figure 6   Dielectric constant and dielectric strengths for various materials.

Dielectric Constants and Strengths of Various Materials		
Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

Gauss's law can be used to relate the surface charge distributions to the electric fields. Taking an infinitessimal pillbox shaped Gaussian surface enclosing an element of the capacitor plate, gives that ;

$$E_{ext} = \frac{\sigma_{free}}{\varepsilon_0}$$

Similarly a infinitessimal pillbox-shaped Gaussian surface enclosing an element of the surface of the dielectric gives:

$$E_{pol} = \frac{\sigma_{pol}}{\varepsilon_0}$$

The net field in the dielectric is:

$$E_{net} = E_{ext} - E_{pol} = \frac{1}{\varepsilon_0}(\sigma_{free} - \sigma_{pol})$$

The problem with this dependence is that it is necessary to know E_{net} to compute σ_{pol} in order to calculate E_{net}. This problem can be resolved by rewriting this equation using the dependence of σ_{pol} on E_{net}:

$$E_{net}(1 + \chi_e) = \frac{\sigma_{free}}{\varepsilon_0} = E_{ext}$$

Define the *dielectric constant*

$$\kappa_e \equiv 1 + \chi_e$$

then we get:

$$E_{net} = \frac{\sigma_{free}}{\kappa_e \varepsilon_0} = \frac{E_{ext}}{\kappa_e}$$

Thus E_{net} in the dielectric is a factor κ_e *weaker* than when there is no dielectric. Knowing E_{net} and the known applied charge distribution σ_{free} gives the induced charge density σ_{pol}:

$$\sigma_{pol} = \frac{\kappa_e - 1}{\kappa_e}\sigma_{free}$$

As shown in figure 6, the dielectric constant κ_e for materials ranges from 1.00 for vacuum or air, to between 2 and 6 for plastics, and to 80 for water at low frequencies. That is, the net field in the dielectric can be substantially smaller than the external applied electric field and the induced surface charge distribution can be comparable to the applied free charge distribution. These dielectrics are close to linear up to the dielectric strength of over $MV/meter$.

The above discussion considered the simple case of a parallel-plate capacitor for which only surface distributions of bound charge occur. In general, polarization leads to volume bound charge distributions which will not be discussed. However the macroscopic consequences are adequately described using the simple case discussed here. Note that one can consider a good conductor as having an infinite dielectric constant resulting in the electric field in the conductor being zero.

E FIELD AT DIELECTRIC BOUNDARIES

In general, the polarization surface charge density σ_{pol} leads to a polarization field $E_{pol} = \frac{\sigma_{pol}}{\varepsilon_0}$ perpendicular to the surface inside the dielectric. This weakens the component of the external electric field perpendicular to the surface inside the dielectric. From above, it was shown that the normal component to the surface in the air and in the dielectric are related by

$$E_\perp^{air} = \kappa_e E_\perp^{dielectric}$$

However, the parallel component of the electric field at the surface is unchanged by the polarization of the dielectric since the polarization charge produces a field perpendicular to the surface. Thus

$$E_\parallel^{air} = E_\parallel^{dielectric}$$

The boundary requirements for the normal and tangential electric field vectors at the interface between a dielectric and air are summarized in figure 7. As a result, as shown in figure 8, the electric field is refracted, that is bent at an air-dielectric boundary. This is important, as will be shown later; this is the origin of the refraction of light at an interface between a dielectric such as glass and air.
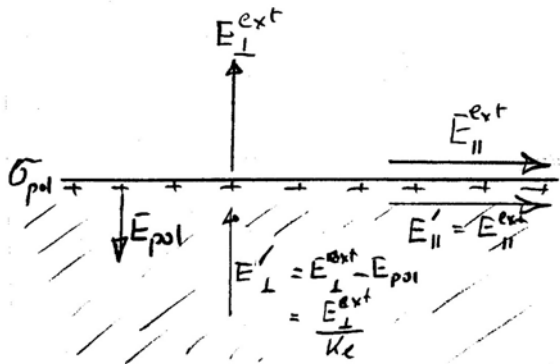

Figure 7 Perpendicular and parallel components of the electric field at an interface between a dielectric and vacuum.
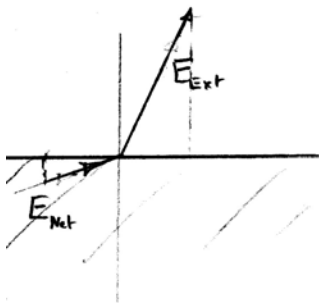

Figure 8 Refraction of electric field at boundary of dielectric.

CAPACITANCE WITH DIELECTRICS

Consider a parallel-plate capacitor, spacing t, partially filled with a dielectric slab of thickness d.

The surface charge distribution σ_{free} on the surface of the capacitor plate is related to $E_\perp^{air}$ by Gauss's law, that is

$$E_\perp^{air} = \frac{\sigma_{free}}{\varepsilon_o}$$

Inside the dielectric this field is reduced by the factor κ_e, that is

$$E_\perp^{dielectric} = \frac{\sigma_{free}}{\kappa_e \varepsilon_o}$$

Thus the potential difference between the capacitor plates is given by the line integral:

$$\Delta V = -\int \overrightarrow{\mathbf{E}} \cdot \overrightarrow{d\mathbf{l}} = (t-d)E_\perp^{air} + d.E_\perp^{dielectric}$$

$$\Delta V = (t-d)E_\perp^{air} + \frac{d}{\kappa_e}E_\perp^{air}$$

$$\Delta V = \frac{\sigma_{free}}{\varepsilon_0}\left[t - (\frac{\kappa_e - 1}{\kappa_e})d\right]$$

Since $\sigma_{free} = \frac{Q}{A}$ we get that the net capacitance of the partially-filled parallel-plate capacitor is:

$$C = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{\left[t - (\frac{\kappa_e - 1}{\kappa_e})d\right]}$$

A useful special case is when the space between the plates is filled completely with dielectric, then $d = t$ and:

$$C = \frac{A\varepsilon_0 \kappa_e}{d}$$

The capacitance of a capacitor filled with dielectric of constant κ_e, is a factor κ_e times larger than when the region between the capacitor plates is a vacuum. That is;

$$C_{dielectric} = \kappa_e C_{vacuum}$$

This is the reason that the spacer in capacitors usually is made of an insulator having a high dielectric constant.
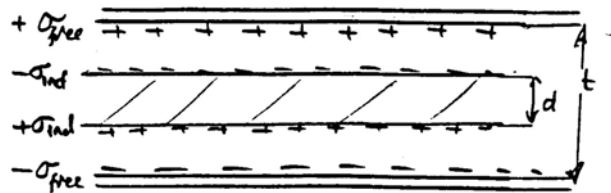

Figure 9 Parallel plate capacitor partially filled with a uniform thickness dielectric slab.

ENERGY DENSITY IN E FIELDS

An important question is where in the capacitor is the electrostatic energy stored and how is the electromagnetic energy is transmitted through space from the sun to earth. When discussing electromagnetic phenomena, such as these, it is useful to use the concept of energy stored in the electric field. It can be shown that the energy density, energy per unit volume, in an electric field is given by

$$\eta = \frac{1}{2}\kappa_e\varepsilon_0 E^2 \ joules/m^3$$

Thus the total stored energy can be expressed by integrating this energy density over all space;

$$U = \int_{\substack{all \\ space}} \frac{1}{2}\kappa_e\varepsilon_0 E^2 d\tau$$

The above definition of the energy density in the electric field gives the correct answer $U = \frac{1}{2}CV^2$ for the special case of a parallel-plate capacitor of area A, spacing d assuming the volume is filled with dielectric with dielectric constant κ_e. The voltage difference across the capacitor

$$V = Ed$$

Integrating over the volume Ad gives

$$U = \frac{1}{2}\varepsilon_0\kappa_e Ad\left(\frac{V}{d}\right)^2 = \frac{1}{2}CV^2$$

since $C = \frac{\kappa_e\varepsilon_0 A}{d}$. That is, the energy density in an electric field gives the same answer for the energy stored in a capacitor. For a capacitor filled with a dielectric, the additional energy is stored in the polarization of the dielectric. This is demonstrated by the dissectible capacitor. The capacitor can be disassembled and then reassembled while retaining the stored energy in the dielectric.

Stored energy for a uniformly-charged sphere

An important example of use of the energy density is the stored electric energy for a uniformly-charged sphere of charge. Consider a sphere of radius R and charge Q, that is, uniform charge density ρ where $Q = \frac{4}{3}\pi R^3\rho$. The stored energy for a uniformly charge sphere of charge is given by integrating

$$U = \int_{\substack{all \\ space}} \frac{1}{2}\varepsilon_0 E^2 d\tau$$

Note that this has to be integrated over all space, which includes both inside the charged sphere and outside, since the electric field in non zero both inside and outside the sphere. Using Gauss's law it was shown that:

$$\overrightarrow{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_o r^2}\widehat{\mathbf{r}} \qquad\qquad r > R$$

$$\overrightarrow{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_o R^3}\overrightarrow{\mathbf{r}} \qquad\qquad r < R$$

The integral thus becomes:

$$U = \frac{\varepsilon_0}{2}\frac{Q^2}{(4\pi\varepsilon_o)^2}\left[\int_0^R \frac{r^2 4\pi r^2 dr}{R^6} + \int_R^\infty \frac{4\pi r^2 dr}{r^4}\right]$$

That is:

$$U = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_o R}$$

This potential energy is called the Coulomb energy.

This potential energy of a uniformly charged sphere can be applied to compute the electrostatic energy stored in a nucleus. The nuclear charge is $Q = Ze$, and the nuclear radius $R = 1.2A^{\frac{1}{3}} \times 10^{-15}m$. Substitution into the formula derived above gives,

$$U = 0.72\frac{Z^2}{A^{\frac{1}{3}}}MeV$$

For a ^{235}U uranium nucleus the Coulomb energy is $987MeV$ which is enormous. The uranium nucleus can split into two halves each having about half of the mass. Each of these two pieces will have Coulomb energies of $309MeV$, that is, the sum of the Coulomb energies of the two daughter nuclei is $370MeV$ lower than the parent ^{235}U nucleus. About half of this missing energy is liberated as kinetic energy and the remainder in excitation energy of the daughter nuclei and kinetic energy of the emitted neutrons. Nuclei heavier that uranium do not occur naturally because it is energetically more favourable for these nuclei to fission into pairs of lighter nuclei.

SUMMARY

This lecture has focussed on the influence of matter on electric fields. It was discussed earlier that the electrostatic electric field E is zero inside good conductors. As a result electric conductors are equipotentials. Dielectric materials also supress the electric field inside the dielectric due to alignment of electric dipoles. An electric dipole has an electric dipole moment given by $\overrightarrow{\mathbf{p}} = q\overline{\mathbf{d}}$. The torque on such an electric dipole in an electric field is given by,

$$\overrightarrow{\tau} = \overrightarrow{\mathbf{p}} \times \overrightarrow{\mathbf{E}}$$

For linear dielectrics the alignment of the electric dipole moments is proportional to the electric field

$$\overrightarrow{\mathbf{p}} = \alpha\overrightarrow{\mathbf{E}}$$

The electric field inside a dielectric, $\overrightarrow{\mathbf{E}_{net}}$ for a dielectric constant κ_e, is related to the applied field $\overrightarrow{\mathbf{E}_{free}}$ by

$$\overrightarrow{\mathbf{E}_{net}} = \frac{\overrightarrow{\mathbf{E}_{free}}}{\kappa_e}$$

that is, the actual electric field inside the dielectric is reduced by the factor κ_E.

The electric field at the boundary of a dielectric is refracted since the normal component of the electric field at the surface is reduced by the factor κ_e in the dielectric.

The concept of capacitance was discussed where

$$C = \frac{\Delta Q}{\Delta V}$$

The capacitance of a system of conductors usually is a constant except at very high elecric fields. The capacitance of a capacitor filled with a dielectric is increased by a factor κ_e relative to the same capacitor without the dielectric.

$$C_{dielectric} = \kappa_e C_{vacuum}$$

The energy density in E fields was shown to equal

$$\eta = \frac{1}{2}\kappa_e \varepsilon_0 E^2 \; joules/m^3$$

Thus the total stored energy can be expressed by integrating this energy density over all space;

$$U = \int_{all \atop space} \frac{1}{2}\kappa_e \varepsilon_0 E^2 d\tau$$

This relation will occur frequently when discussing the energy transmitted by an electromagnetic field.

Reading assignment: Giancoli; Chapter 24