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# New Journal of Physics

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## **Dephasing-assisted transport: quantum networks and biomolecules**

### **M B Plenio**<sup>1,2,4</sup> and S F Huelga<sup>3</sup>

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**Abstract.** Transport phenomena are fundamental in physics. They allow for information and energy to be exchanged between individual constituents of communication systems, networks or even biological entities. Environmental noise will generally hinder the efficiency of the transport process. However, and contrary to intuition, there are situations in classical systems where thermal fluctuations are actually instrumental in assisting transport phenomena. Here we show that, even at zero temperature, transport of excitations across dissipative quantum networks can be enhanced by local dephasing noise. We explain the underlying physical mechanisms behind this phenomenon and propose possible experimental demonstrations in quantum optics. Our results suggest that the presence of entanglement does not play an essential role for energy transport and may even hinder it. We argue that Nature may be routinely exploiting dephasing noise and show that the transport of excitations in simplified models of light harvesting molecules does benefit from such noise assisted processes. These results point toward the possibility for designing optimized structures for transport, for example in artificial nanostructures, assisted by noise.

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#### **Contents**



#### **1. Introduction**

Noise is an inevitable feature of any physical system, be it natural or artificial. Typically, the presence of noise is associated with the deterioration of performance for fundamental processes such as information processing and storage, sensing or transport, in systems ranging from proteins to computing devices.

However, the presence of noise does not always hinder the efficiency of an information process and biological systems provide a paradigm of efficient performance assisted by a noisy environment [\[1\]](#page-13-0). A vivid illustration of the counterintuitive role that noise may play is provided by the phenomenon of stochastic resonance (SR) [\[2\]](#page-13-0). Here thermal noise may enhance the response of the system to a weak coherent signal, optimizing its response at an intermediate noise level [\[3\]](#page-13-0). Some experimental evidence suggests that biological systems employ SR-like strategies to enhance transport and sensing [\[4,](#page-13-0) [5\]](#page-13-0). Noise in the form of thermal fluctuations may also lead to directed transport in ratchets and play a helpful role in Brownian motors [\[6\]](#page-13-0)–[\[8\]](#page-13-0). It therefore seems natural to try and draw analogies with complex classical networks so that the physical mechanisms that underpin their functioning when subject to noise can be perhaps mirrored and eventually used to optimize the performance of complex quantum networks. Recently, tentative first steps toward the exploration of the concept of SR in quantum many-body systems  $[9]$ – $[11]$  and quantum communication channels  $[12]$ – $[14]$  have been undertaken, whereas other studies have focused on analyzing the persistence of coherence effects in biological systems. In particular, detecting the presence of quantum entanglement has been the object of considerable attention  $[15]$ <sup>5</sup>. It was noted, however, that even if found, it would be unclear whether such entanglement has any functional importance or is simply the unavoidable by-product of coherent quantum dynamics in such systems [\[17\]](#page-13-0).

Here, we show that dephasing noise, which leads to the destruction of quantum coherence and entanglement as a result of phase randomization, may nevertheless be an essential resource to enhance the transport of excitations when combined with coherent dynamics. Indeed, we show that a dissipative quantum network subject to dephasing can exhibit an enhanced capacity for transmission of classical information when seen as a communication channel, even though

 $5$  See [\[16\]](#page-13-0) for recent experimental results.

<span id="page-3-0"></span>

**Figure 1.** Sites (blue spheres), modeled here as spin-1/2 particles or qubits, are interacting with each other (dashed line) to form a network. The particles may suffer dissipative losses as well as dephasing. The red arrow indicates an irreversible transfer of excitations from the network to a sink that acts as a receiver.

its quantum capacity and quantum coherence are diminished by the presence of noise. This suggests that it is the constructive interplay between dephasing noise and coherent dynamics, rather than the mere presence or absence of entanglement, that is responsible for the improved transport of excitations. Recently, this enhancement of quantum transport due to the interplay between coherence and the environment has been suggested for chromophoric complexes [\[18\]](#page-13-0) (see [\[19\]](#page-13-0)–[\[22\]](#page-13-0) and note added). Chromophoric complexes are molecules that absorb light to create an exciton which is then transported to a reaction center where it is used to trigger further processes to bind its energy in a chemical form.

In addition to the clarifying nature of these results, it is intriguing to speculate that Nature appears to exploit noise-assisted processes to maximize the system's performance and it will be worthwhile to explore how similar processes may be useful for the design of improved transport in nanostructures and perhaps even quantum information processors.

#### **2. The basic setting**

We consider a network of N sites that may support excitations which can be exchanged between lattice sites by hopping (see figure 1). The Hamiltonian that describes this situation is then given by

$$
H = \sum_{k=1}^{N} \hbar \omega_k \sigma_k^+ \sigma_k^- + \sum_{k \neq l} \hbar v_{k,l} (\sigma_k^- \sigma_l^+ + \sigma_k^+ \sigma_l^-), \qquad (1)
$$

where  $\sigma_k^+$  ( $\sigma_k^$  $h_k$ <sup>-</sup>) are the raising and lowering operators for site *k*,  $\hbar \omega_k$  is the local site excitation energies and  $v_{k,l}$  denotes the hopping rate of an excitation between the sites  $k$  and  $l$ . It should be noted that the dynamics in this system preserves the total excitation number in the system. This is not an essential feature but makes the system amenable to efficient numerical analysis.

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<span id="page-4-0"></span>We will assume that the system is susceptible simultaneously to two distinct types of noise processes, a dissipative process that reduces the number of excitations in the system at rate  $\Gamma_k$ and a dephasing process that randomizes the phase of local excitations at rate  $\gamma_k$ .

Initially, we will assume that we can describe both processes by using a Markovian master equation with local dephasing and dissipation terms. It is important to note, however, that the effects found here persist when taking account of the system–environment interaction in a more detailed manner (see section [6\)](#page-10-0). Dissipative processes, which lead to energy loss, are then described by the Lindblad super-operator

$$
\mathcal{L}_{\text{diss}}(\rho) = \sum_{k=1}^{N} \Gamma_k[-\{\sigma_k^+\sigma_k^-, \rho\} + 2\sigma_k^-\rho\sigma_k^+],\tag{2}
$$

while energy-conserving dephasing processes are described by the operator

$$
\mathcal{L}_{\text{deph}}(\rho) = \sum_{k=1}^{N} \gamma_k \left[ -\{\sigma_k^+ \sigma_k^{(-)}, \rho\} + 2\sigma_k^+ \sigma_k^- \rho \sigma_k^+ \sigma_k^- \right]. \tag{3}
$$

Finally, in order to be able to measure the total transfer of excitation, we designate an additional site, numbered  $N+1$ , which is populated by an irreversible decay process from a chosen level *k* as described by the Lindblad operator

$$
\mathcal{L}_{\text{sink}}(\rho) = \Gamma_{N+1}[-\{\sigma_k^+ \sigma_{N+1}^- \sigma_{N+1}^+ \sigma_k^-,\rho\} + 2\sigma_{N+1}^+ \sigma_k^- \rho \sigma_k^+ \sigma_{N+1}^-].
$$
\n(4)

The subindex 'sink' emphasizes that no population can escape of site  $N+1$ . For definitiveness and simplicity, the initial state of the network at  $t = 0$  will be assumed to be a single excitation in site 1 unless stated otherwise.

The key question that we will pose and answer is the following: *In a given time T*, *how much of the initial population in site* 1 *will have been transferred to the sink at site N* + 1 *and how is this transfer affected by the presence of dephasing and dissipative noise.*

In the remainder of this paper, we will demonstrate that, in certain settings, the presence of dephasing noise can assist the transfer of population from site 1 to the sink at site  $N+1$ considerably. It is an intriguing observation that this noise enhanced transfer does not occur for all possible Hamiltonians of the type given by equation [\(1\)](#page-3-0) and may depend also on properties of the noise such as its spatial dependence. These noise rates can be optimized numerically, and in very simple cases analytically, to yield the strongest possible effect. One may suspect that natural, biological systems, have actually made use of such an optimization.

#### **3. The case of a linear chain**

We begin with a brief analysis of the uniform linear chain with only nearest-neighbor interactions so that in equation [\(1\)](#page-3-0) the coupling strengths satisfy  $v_{l,k} = v_{k,l} = v \delta_{l,k+1}$  for  $k = 1, \ldots, N - 1$  and  $\omega_k = \omega$ , while  $\Gamma_k = \Gamma$  for  $k = 1, \ldots, N$ . Extensive numerical searches show that, for arbitrary choices of  $\Gamma_{N+1}$ ,  $\Gamma$  and  $\omega$ , arbitrary transmission times *T* and chains of the length  $N = 2, \ldots, 12$ , the optimal choice of dephasing noise rates vanish. We have used a directed random walk algorithm with multiple initial states which has never exceeded the values for the noise-free chain and approached them to within at least 10<sup>−</sup><sup>8</sup> . We were able to derive formulae for the case  $T = \infty$  and short chains which demonstrate this behavior analytically.

For  $N = 2$ , with  $\omega_1 = \omega_2 = \omega$  and arbitrary  $v_{1,2}$ ,  $\gamma_i$  and  $\Gamma_i$ , we find, with the abbreviation  $\gamma = \gamma_1 + \gamma_2$  and  $x = 2\Gamma_1^3 + \Gamma_1\Gamma_3(3\Gamma_1 + \Gamma_3)$ , that the population of the sink is given by

$$
p_{\text{sink}} = \frac{\Gamma_3 v_{1,2}^2}{x + \Gamma_1 (\Gamma_1 + \Gamma_3) \gamma + (\Gamma_3 + 2\Gamma_1) v_{1,2}^2},\tag{5}
$$

which is evidently maximized for  $\gamma = 0$ . One may also obtain the analytical expressions for  $N = 3$  described by equations [\(1\)](#page-3-0)–[\(4\)](#page-4-0) for the choice and  $\Gamma_k = \Gamma$  for  $k = 1, 2, 3, 4$  and demonstrate that the optimal dephasing level is  $\gamma = 0$ . We find

$$
p_{\text{sink}} = \frac{(4\Gamma + \gamma_1 + \gamma_3)v^2}{36\Gamma^5 + 6a\Gamma^4 + 2\Gamma^3e + \Gamma^2(2c + dv^2) + \Gamma v^2f + 4(\gamma_1 + \gamma_3)v^4},
$$

where

$$
a = (5\gamma_1 + 5\gamma_2 + 4\gamma_3),\tag{6}
$$

$$
b = \gamma_1 \gamma_2 + \gamma_1 \gamma_3 + \gamma_2 \gamma_3,\tag{7}
$$

$$
c = \gamma_1(\gamma_2^2 + \gamma_3^2) + \gamma_2(\gamma_1^2 + \gamma_3^2) + \gamma_3(\gamma_1^2 + \gamma_2^2) + 2\gamma_1\gamma_2\gamma_3,
$$
\n(8)

$$
d = 32\gamma_3 + 25\gamma_2 + 29\gamma_1,\tag{9}
$$

$$
e = (3\gamma_1^2 + 3\gamma_2^2 + 8b + 2\gamma_3^2 + 32v^2),\tag{10}
$$

$$
f = (3\gamma_1^2 + 7b + 4\gamma_3^2 + 15v^2). \tag{11}
$$

Then one first observes that the optimal choice is  $\gamma_2 = 0$  as it only occurs in the denominator with positive coefficients. In the remaining expression one then substitutes  $\gamma_k = \tilde{\gamma}_k^2$  allowing also for negative  $\tilde{\gamma}_k$ . Then differentiation w.r.t these  $\tilde{\gamma}_k$  shows that the gradient only vanishes for  $\tilde{\gamma}_1 = \tilde{\gamma}_2 = 0.$ 

This approach, though more tedious, may be taken to higher values of *N* as well. Extensive numerical searches lend further support to the observation that dephasing does not improve excitation transfer for uniform chains but a general proof has remained elusive.

So far, the findings are consistent with the expectation that noise does not enhance the transport of excitations. However, for non-uniform chains we encounter the different and perhaps surprising situation where noise can significantly enhance the transfer rate of excitations.

As an illustrative example, we may keep the nearest-neighbor coupling uniform but allow for one site to have a different site energy  $\omega$ . If we choose  $N = 3$ ,  $\omega_1 = \omega_3 = 1$ ,  $\Gamma_1 = \Gamma_2 =$  $\Gamma_3 = 1/100$ ,  $v_{1,2} = v_{2,3} = 1/10$ ,  $\Gamma_{N+1} = 1/5$  and  $T = \infty$ , then we obtain the results depicted in figure [2.](#page-6-0) One observes that dephasing assists the transmission only when site 2 is sufficiently detuned from the neighboring sites.

This example suggests a simple picture to explain the reason for the dephasing enhanced population transfer through the chain. Site 2 is strongly detuned from its neighboring sites and

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<span id="page-6-0"></span>

**Figure 2.** The optimal improvement of the transfer efficiency is plotted versus the site frequency  $\omega_2$  in a chain of length  $N = 3$  and system parameters  $\omega_1 = \omega_3 = 1$ ,  $\Gamma_1 = \Gamma_2 = \Gamma_3 = 1/100$ ,  $v_{1,2} = v_{2,3} = 1/10$ ,  $\Gamma_{N+1} = 1/5$  and  $T = \infty$ . One observes that dephasing only assists the transmission probability in some frequency intervals.

the coupling v to its neighbors is comparatively weak, i.e.  $v \ll \delta \omega$  with  $\delta \omega = \min[ |\omega_2 - \omega_1|,$  $|\omega_3 - \omega_2|$ ]. Hence, the transport rate is limited by a quantity of order  $v^2/\delta\omega$ , as it is a secondorder process, also known as superexchange [\[23\]](#page-13-0) in solid state physics, due to the lack of resonant modes between neighboring sites. Introducing dephasing noise leads to a broadening of the energy level at each site *k* and a linewidth proportional to the dephasing rate  $\gamma_k$ . Then, with increasing dephasing rate, the broadened lines of neighboring sites begin to overlap and the population transfer will be enhanced as resonant modes are now available. Enhancing the dephasing rate further will eventually lead to a weakening of the transfer as the modes are distributed over a very large interval and resonant modes have a small weight. Dissipation does not lead to the same enhancement as, crucially, the gain to the broadening of the line is overcompensated by the irreversible loss of excitation. This is corroborated by numerical studies where increasing dissipation does not assist the transport. The physical picture outlined above is confirmed in figure [3.](#page-7-0) We chose a chain of length 3 which suffers dephasing only in site 2 and uniform dissipation with rates  $\Gamma_k = 1/100$  along the chain while  $\omega_1 = \omega_2/4 = \omega_3 = 1$ and  $v_{1,2} = v_{2,3} = 1/10$ . The close relationship of this model to Raman transitions in quantum optics will be exploited to propose a realizable experiment in a highly controlled environment to verify these effects (see section [7](#page-10-0) on experimental realizations). In the examples above, the improvement of excitation transfer due to the dephasing is small. One can easily show, however, that this improvement may be made arbitrarily large in the sense that without noise

<span id="page-7-0"></span>

**Figure 3.** The difference between transfer efficiency and the efficiency without dephasing is plotted versus the dephasing rate  $\gamma_2$  in a chain of length  $N = 3$ and  $\omega_1 = \omega_2/4 = \omega_3 = 1$ ,  $v_{1,2} = v_{2,3} = 1/10$ ,  $\gamma_1 = \gamma_3 = 0$ ,  $\Gamma_k = 1/100$  for  $k =$ 1, ..., *N*,  $\Gamma_{N+1} = 1/5$  and  $T = \infty$ . Initially increasing dephasing assists the transfer of excitation while very strong dephasing suppresses the transport.

the transfer rate approaches zero while it approaches unity arbitrarily closely for optimal noise levels. As an example, for  $N = 3$ ,  $\omega_1 = \omega_3 = 1$ ;  $\omega_2 = 100$ ,  $v_{1,2} = v_{2,3} = v$ ,  $\gamma_1 = \gamma_3 = 0$ and  $\Gamma_1 = \Gamma_2 = \Gamma_3 = v^2/f$  and  $\Gamma_4 = 10^5 v$  we find for  $\Delta p = p_{\text{sink}}(\gamma_{2,\text{opt}}) - p_{\text{sink}}(\gamma_2 = 0)$  that

$$
\lim_{\nu \to 0} \Delta p = \frac{f^2 \gamma_2^2}{f^2 \gamma_2^2 + 3f \gamma_2 ((\omega_2 - 1)^2 + \gamma_2^2) + ((\omega_2 - 1)^2 + \gamma_2^2)^2}.
$$
\n(12)

This is maximized for  $\gamma_2 = \omega_2 - 1$  when it takes the value  $\Delta p = f^2/(f^2 + 6f(\omega_2 - 1) + 4(\omega (2-1)^2$ ). In the limit  $f \rightarrow \infty$  this approaches 1, that is, without noise the excitation transfer vanishes, whereas with noise it achieves unit efficiency! It should be noted that being a system of fixed finite size, the effect may not be directly attributed to Anderson localization [\[24\]](#page-13-0) which, in addition, does not occur in systems attached to a sink, as is assumed here [\[25\]](#page-13-0). However, dephasing noise may play a constructive role because by destroying destructive interference it can open additional transport channels [\[26\]](#page-13-0).

#### **4. Entanglement and coherence in the channel**

We have seen that the transport of excitations in the system may be assisted considerably by local dephasing. Now we would like to discuss briefly the quantum coherence properties during transmission by studying the presence of entanglement and the ability of the chain to transmit quantum information. To this end, we consider how entanglement is transported along the chain when it is used to propagate one half of a maximally entangled state to obtain an insight into how

<span id="page-8-0"></span>

**Figure 4.** The time evolution of the entanglement between a decoupled site and the sites in the chain of length  $N = 4$  and system parameters  $\omega_1 = \omega_2 = \omega_4 = 10$ ,  $\omega = 14$ ,  $v_{1,2} = v_{2,3} = v_{3,4} = 1$ ,  $\Gamma_k = 1/10$  for  $k = 1, ..., N$  and  $\Gamma_{N+1} = 1$ . The initial state is a maximally entangled state between the decoupled site and the first site of the chain. Dephasing destroys entanglement along the chain and has no beneficial effect.

the quantum capacity of this channel is affected by dephasing. To illustrate this, we consider a chain of  $N = 4$  sites (see figure 4). We choose the same parameters as in figure [2](#page-6-0) and fix  $\omega_3 = 14$ . Comparison of the entanglement between an uncoupled site and the various sites in the chain for vanishing dephasing and the optimal choice of the dephasing for excitation transfer show that, while entanglement propagates through the system, the amount of entanglement decreases with increasing dephasing. In fact, the dephasing rate that optimizes the ability of the channel to transmit quantum information vanishes, in contrast to the situation for excitation transfer. Therefore, although dephasing may enhance the propagation of excitations, it also destroys quantum coherence and in the present setting it leaves an overall detrimental effect.

#### **5. Complex networks and light-harvesting molecules**

So far, we have demonstrated that in linear chains local dephasing noise may enhance the transfer of excitations. Going beyond this, we will now consider fully connected networks subject to Markovian loss and dephasing. These will serve as simplified, Markovian, models for the transfer of excitons in the Fenna–Matthews–Olson (FMO) complex of *Prosthecochloris aestuarii*, which is a pigment–protein complex that consists of seven bacteriochlorophyll-a (BChla) molecules (see [\[20\]](#page-13-0)–[\[22\]](#page-13-0) and note added for closely related work). This complex is

able to absorb light to create an exciton. This exciton then propagates through the complex until it reaches the *reaction center* where its energy is then used to trigger further processes that bind the energy in chemical form [\[18,](#page-13-0) [27\]](#page-13-0). The Hamiltonian of this complex may be approximated by equation [\(1\)](#page-3-0), where the site energies and coupling constants may be taken from tables 2 and 4 of  $[18]$ . We then find, in matrix form

$$
H = \begin{pmatrix} 215 & -104.1 & 5.1 & -4.3 & 4.7 & -15.1 & -7.8 \\ -104.1 & 220.0 & 32.6 & 7.1 & 5.4 & 8.3 & 0.8 \\ 5.1 & 32.6 & 0.0 & -46.8 & 1.0 & -8.1 & 5.1 \\ -4.3 & 7.1 & -46.8 & 125.0 & -70.7 & -14.7 & -61.5 \\ 4.7 & 5.4 & 1.0 & -70.7 & 450.0 & 89.7 & -2.5 \\ -15.1 & 8.3 & -8.1 & -14.7 & 89.7 & 330.0 & 32.7 \\ -7.8 & 0.8 & 5.1 & -61.5 & -2.5 & 32.7 & 280.0 \end{pmatrix},
$$
(13)

where we have shifted the zero of energy by 12 230 (all numbers are given in the units of 1.988 865 × 10<sup>-23</sup> nm = 1.2414 × 10<sup>-4</sup> eV) for all sites corresponding to a wavelength of  $\cong$ 800 nm. Recent work [\[18\]](#page-13-0) suggests that it is this site 3 that couples to the reaction center at site 8. For this rate, somewhat arbitrarily, we chose  $\Gamma_{3,8} = 10/1.88$  corresponding to about 1 ps<sup>-1</sup> (value in the literature range from 0.25 ps<sup>-1</sup> [\[18\]](#page-13-0) and 1 ps<sup>-1</sup> [\[20\]](#page-13-0) to 4 ps<sup>-1</sup> [\[15\]](#page-13-0)). Again, we will assume the presence of both dissipative noise (loss of excitons) and dephasing noise (due to the presence of a phonon bath consisting of vibrational modes of the molecule). Note that we assume a Markovian master equation to describe these processes. While this may represent a rather simplified model of the FMO complex it allows us to exemplify dephasing enhanced transport in complex networks in a simple setting that, we believe, reflects the essential dynamics of FMO complexes. More discussions concerning the Markovian approximation may be found in section [6.](#page-10-0) The measured lifetime of excitons is of the order of 1 ns which determines a dissipative decay rate of  $2\Gamma_k = 1/188$  and that we assume to be the same for each site  $[18]$ . If we neglect the presence of any form of dephasing and we start with a single excitation on site 1, then we observe that the excitation is transferred to the reaction centre (site 8). For a time  $T = 5$ , we find that the amount of excitation that is transferred is  $p_{\text{sink}} = 0.551926$ . Optimal dephasing rates that maximize the transfer rate of the initial excitation in site 1 considerably improve on that. For  $T = 5$ , we find the optimal dephasing rates  $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7) = (469.34, 5.36, 99.13, 5.55, 114.86, 1.88, 291.08)$ and the much improved value  $p_{\text{sink}} = 0.988526$ . For  $T = \infty$ , we find the dephasing free transfer probability of  $p_{\text{sink}} = 0.81425$ , whereas for the optimal dephasing rates  $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7) = (27.40, 26.84, 1.22, 87.12, 99.59, 232.76, 88.35)$  we find  $p_{\text{sink}} =$ 0.999 11. It should be noted that these dephasing rates are comparable to the inter-site coupling rates which suggests that a more accurate treatment will need to go beyond the use of Markovian master equations (see section [6](#page-10-0) below for a brief discussion).

We conclude that dephasing may lead to a very strong enhancement of the transfer rate of excitations in a realistic network. In fact, in models obtained from spectroscopic data measured on the FMO complex it is indeed observed that almost complete transport should take place within time  $T = 5$  [\[18\]](#page-13-0). It is remarkable that such a rapid transfer cannot be explained from a purely coherent dynamics following equation (13) and the results above suggest that the

<span id="page-10-0"></span>underlying reason for the speed up is the presence of dephasing which may even be local. Needless to say, these suggestions will have to be verified in more accurate, non-Markovian, models of the dynamics of the FMO complex [\[28\]](#page-13-0).

#### **6. Beyond Markovian master equations**

So far, we have demonstrated the existence of dephasing enhanced excitation transfer in fully connected networks employing a master equation description. The optimized dephasing rates that have been obtained, in particular those in the context of our simplified model of the FMO complex, can be comparable to the coherent interaction strengths and may be similar to the spectral width of the bath responsible for the dephasing [\[18\]](#page-13-0). This may not be fully compatible with the Markovian master equation approach employed so far as its derivation relies on several assumptions including the weak coupling hypothesis and the requirement for the bath to be memoryless [\[29\]](#page-13-0). The derivation is further complicated for systems with several constituents where the local coupling of its constituents is not compatible with non-local structure of the eigenmodes of the systems. This is especially so when the coherent inter subsystem coupling is of comparable strength to the system–environment coupling. The situation is made more difficult due to spatial as well as temporal correlations in the environmental noise (which is to be expected in particular for the FMO complex but also many other realizations of coupled chains in contact with an environment). Bloch–Redfield equations and other effective descriptions are sometimes used but still represent approximations to the correct dynamics [\[29\]](#page-13-0), where the errors are often difficult to estimate precisely.

Therefore, we demonstrate briefly that dephasing assisted transfer of excitation can also be observed when one uses a microscopic model of an environment that may, in addition, exhibit non-Markovian behavior. To this end we study the effect of an environment which is modeled by brief interactions between two-level systems and individual subsystems of the chain in which excitation transport is taking place. The strength and nature of the interactions can be chosen to implement dephasing (elastic collisions) and dissipation (inelastic collisions). Non-Markovian effects can be included in the model depending on the spatial and temporal memory of the environment particles. Interaction strengths are determined for a single site system to obtain the dissipation rate  $\Gamma$  and dephasing rate  $\gamma$ . This simplified model allows us to study the effect of more realistic environments outside the master equation picture and results are summarized in figure [5.](#page-11-0) A more detailed simulation of excitation transfer taking account of the full-environment is beyond the scope of the present work and will be presented elsewhere [\[28\]](#page-13-0).

#### **7. Experimental realizations**

The FMO complex provides a fascinating setting for the observation of dephasing enhanced transport but it is also a very challenging environment to verify the effect precisely. Here we present several physical systems in which the dephasing enhanced excitation transfer may be observed and which are at the same time highly controllable. Perhaps the simplest such setting is found in atomic physics (see figure [6\)](#page-12-0) where the behavior of a chain of three sites may be simulated using detuned Raman transitions in ions such as  $Ca^+$ ,  $Sr^+$  or  $Ba^+$ . The master equation of this system simulates exactly that of a chain with a single excitation as has been described throughout this paper. Atomic populations may be measured with very high accuracy using quantum jump detection [\[31,](#page-13-0) [32\]](#page-14-0).

<span id="page-11-0"></span>

**Figure 5.** Here we show how the transfer in the presence of dephasing into a bath that is modeled by a collisional model where local sites briefly interact with a single particle. The interaction strength is chosen such that in an uncoupled systems the sites suffer the optimal decoherence rates  $\gamma_{\text{opt}}$  as presented in the previous section multiplied with factors 0, 0.0064, 0.16 and 1. The dynamics is similar to that observed for the master equation approach and shows only minor deviations. Increased dephasing rates do improve the excitation transfer also in this model.

A variety of other natural implementations of dephasing-assisted excitation transport can be conceived and will be studied in detail elsewhere. Firstly, the oscillations of ions in a linear ion trap transversal to the trap axis realizes a harmonic chain [\[33\]](#page-14-0) that allows for the implementation of a variety of operations such as preparation of Fock states and is capable of supporting nearest-neighbor coupling between neighboring ion oscillators [\[34\]](#page-14-0) and allowing high efficiency readout by quantum jump detection [\[31\]](#page-13-0). When restricting to the single excitation space, the dynamics of the system is described by master equations that become equivalent to those presented in this paper.

Furthermore, harmonic chains are also realized in coupled arrays of cavities which have recently received considerable attention in the context of quantum simulators [\[35\]](#page-14-0). Ultra-cold atoms in optical lattices which have previously been used to study thermal-assisted transport in Brownian ratchets [\[36\]](#page-14-0) present another scenario in which to study such dephasing-assisted processes. Chains of superconducting qubits or superconducting stripline cavities [\[37\]](#page-14-0) may also provide possible settings for the observation of the effects described above.

<span id="page-12-0"></span>

**Figure 6.** An atomic system with Raman transitions provides a transparent illustration of dephasing-assisted transport. The required level structure may be realized in  $Ca^+$ ,  $Sr^+$  or  $Ba^+$ . Each atomic level represents a site in the chain which may be populated. Starting with all the population in level 1, one may then irradiate the system with classical laser fields of Rabi-frequency  $\Omega$  on the  $1 \leftrightarrow 2$ and the  $3 \leftrightarrow 2$  transition [\[30\]](#page-13-0). Level 3 in turn is assumed to decay spontaneously into an additional level  $|r\rangle$  that plays the role of the recipient. Spontaneous decay of the chain as a whole is modeled by spontaneous decay into level  $|0\rangle$  from which no population can enter the levels  $|1\rangle$ ,  $|2\rangle$ ,  $|3\rangle$  and  $|r\rangle$  anymore. Dephasing noise may now enter the system affecting level 2, for example, through magnetic field fluctuations.

#### **8. Conclusions**

The results presented here demonstrate that while dephasing noise destroys quantum correlations, it may at the same time enhance the transport of excitations. In fact, the efficient transport observed in certain biological systems has been shown to be incompatible with a fully coherent evolution while it can be explained if the system is subject to local dephasing. Hence, in this context, the presence of quantum coherence, and therefore entanglement, in the system, does not seem to be enhancing excitation transfer. This suggests that entanglement that may be present in bio-molecules, though interesting, may not be a universal functional resource. It is a timely question to confirm these results within a more elaborate decoherence model for the dynamics of biomolecules where the assumptions of markovianity and weak coupling can be relaxed and we have presented a preliminary study in this direction using a collisional model for the environment.

Importantly, the results presented here suggest that it may be possible to design and optimize the performance of nano-fabricated transmission lines in naturally noisy environments to achieve strongly enhanced transfer efficiencies employing the concept of noise-assisted transport.

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*Notes added.* While finalizing this work, we became aware of independently obtained but closely related results presented in [20]–[22]. There it was shown that quantum transport can be enhanced by an interplay between coherent dynamics and environment effects with particular emphasis on excitonic energy transfer in light harvesting complexes [20]. The roles of the different physical processes that contribute to the energy transfer efficiency have been studied in [21] and the enhancement of quantum transport due to a pure dephasing environment within the Haaken–Strobl model was demonstrated in [22].

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