Victor L. Berdichevsky

Variational Principles of Continuum Mechanics

I. Fundamentals

ISBN 978-3-540-88466-8

With 79 Figures

Springer

Victor L. Berdichevsky

Variational Principles of Continuum Mechanics

II. Applications

ISBN 978-3-540-88468-2

With 33 Figures

Springer

IMM is promoted under the auspices of ISIMM (International Society for the Interaction of Mechanics and Mathematics).

Author

V.L. Berdichevsky
Professor of Mechanics
Department of Mechanical Engineering
Wayne State University
Detroit, MI 48202
USA
vberd@eng.wayne.edu

ISSN 1860-6245 e-ISSN 1860-6253
ISBN 978-3-540-88466-8 e-ISBN 978-3-540-88467-5
DOI 10.1007/978-3-540-88467-5
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2008942378

Cover design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Our mind is frail as our senses are; it would lose itself in the complexity of the world if that complexity were not harmonious; like the short-sighted, it would only see details, and would be obliged to forget each of these details before examining the next because it would be incapable of taking in the whole. The only facts worthy of our attention are those which introduce order into this complexity and so make it accessible to us.

H. Poincaré, Science and Method

Preface

There are about 500 books on variational principles. They are concerned mostly with the mathematical aspects of the topic. The major goal of this book is to discuss the physical origin of the variational principles and the intrinsic interrelations between them. For example, the Gibbs principles appear not as the first principles of the theory of thermodynamic equilibrium but as a consequence of the Einstein formula for thermodynamic fluctuations. The mathematical issues are considered as long as they shed light on the physical outcomes and/or provide a useful technique for direct study of variational problems.

The book is a completely rewritten version of the author's monograph *Variational Principles of Continuum Mechanics* which appeared in Russian in 1983. I have been postponing the English translation because I wished to include the variational principles of irreversible processes in the new edition. Reaching an understanding of this subject took longer than I expected. In its final form, this book covers all aspects of the story. The part concerned with irreversible processes is tiny, but it determines the accents put on all the results presented. The other new issues included in the book are: entropy of microstructure, variational principles of vortex line dynamics, variational principles and integration in functional spaces, some stochastic variational problems, variational principle for probability densities of local fields in composites with random structure, variational theory of turbulence; these topics have not been covered previously in monographic literature. Other than that, the scope of the book is the same though the text differs considerably due to many detailed explanations added to make the level of the book suitable for graduate students.

Grass Lake, Michigan *V.L. Berdichevsky*

Acknowledgements

This book has been affected by many influences. The most essential one was that of my teacher, L.I. Sedov, who set up the standards of scholarly work. My late friend P.P. Mosolov, the most knowledgeable expert in variational calculus I knew, explained to me the notion of "feeling the constraints" (Sect. 5.5). He insisted that the first 30 pages of any good scientific book must be written at a level undergraduate students can comprehend. His point influenced the way the beginning of the book is written; it seems that the limit suggested is exceeded. One short chat with V.I. Arnold in the 1980s advanced considerably my understanding of thermodynamics for ergodic Hamiltonian systems. Further work on this topic has resulted in a series of my papers on the subject; an overview of the "derivation of thermodynamics from mechanics" is given in Chap. 2. I learned the asymptotics in homogenization of periodic structures from N.S. Bakhvalov (Sect. 17.2), its generalization to random structures from S. Kozlov (Sect. 17.4), and conditions of elastic phase equilibrium from M. Grinfeld (Sect. 7.4).

The last but not the least: two decisive contributions to the book were made by my daughter Jenia and my former student Partha Kempanna. Jenia translated the Russian edition into English, and Partha prepared the Tex file and shaped the outlook of the book. At the final phase of the work I received help from Maria Lipkovich. The advice of Vlad Soutyrine at each stage of this work was, as always, very helpful. Dewey Hodges, Le Khanh Chau and Lev Truskinovsky read the manuscript and made valuable comments. I had many discussions of the issues considered here with Boris Shoykhet.

The usual words of gratitude can hardly convey my special feelings toward everyone mentioned.

Contents - I. Fundamentals

Contents - II. Applications

Part III Some Applications of Variational Methods to Development of Continuum Mechanics Models

Introduction

A variational principle is an assertion stating that some quantity defined for all possible processes reaches its minimum (or maximum, or stationary) value for the real process. Variational principles yield the equations governing the real processes. The equations following from a variational principle possess a very special structure. The major feature of this structure is the reciprocity of physical interactions: action of one field on another creates an opposite and, in some sense, symmetric reaction. All equations of microphysics possess such a structure. Perhaps this is the most fundamental law of Nature revealed up to now.

Macrophysics operates with the averaged characteristics of microfields. The variational structure of microequations affects the structure of macroequations. In particular, for equilibrium processes, the variational structure of microequations brings up the classical equilibrium thermodynamics. In the case of non-equilibrium reversible processes the variational structure of microequations yields a variational structure of macroequations. The governing equations of irreversible processes also possess a special structure. This structure, however, is not purely variational.

The above-mentioned explains the fundamental role of the variational principles in modeling physical phenomena. If the interactions between various fields are absent or simple enough, then one does not need the variational approach to construct the governing equations. However, if the interactions in the system are not trivial (e.g. nonlinear and/or involving high derivatives, kinematical constraints, etc.) the variational approach becomes the only method to obtain physically sensible governing equations.

Another important use of the variational principles is the direct qualitative and quantitative analysis of real processes which is based solely on the variational formulation and does not employ the governing equations. Such analysis is very advanced for solids while for fluids the major developments are still ahead.

The book aims to review the two above-mentioned sides of the variational approach: the variational approach both as a universal tool to describe physical phenomena and as a source for qualitative and quantitative methods of studying particular problems. In addition, a thorough account of the variational principles discovered in various branches of continuum mechanics is given, and some gaps are filled in.[1]

The book consists of three parts. Part I presents basic knowledge in the area, including variational principles for systems with a finite number of degrees of freedom, "the derivation of thermodynamics from mechanics," a review of basic concepts of continuum mechanics and general setting of variational principles of continuum mechanics. Part I also contains an exposition of the direct methods of calculus of variations. The major goal here is to prepare the reader to understand and to speak the "energy language," i.e. to be able to withdraw the necessary information directly from energy without using the corresponding differential equations. An important component of the energy language is the ability to work with energy depending on a small parameter. A way to do that (variational-asymptotic method) is discussed in detail. Another important component, duality theory, is also covered in detail. The variational-asymptotic method and duality theory are widely used throughout the book. Part II gives an account of variational principles for solids and fluids. Part III is concerned with applications of variational methods to shell and plate theory, beam theory, homogenization of periodic and random structures, shallow water theory, granular media theory and turbulence theory. The consideration of random structures is preceded by a review of stochastic variational problems. Some interesting variational principles that are beyond the main scope of the book are placed in Appendices. The details of some derivations that can be skipped without detriment for understanding of further material are also put in the appendices. By publisher's suggestion, the book is published in two volumes with volume 1 containing the first two parts of the book.

It is assumed that the reader knows the basics of calculus and tensor analysis. The latter, though, is not absolutely necessary as all tensor notations used are briefly outlined. Part I was used by the author as notes for the course Fundamentals of Mechanics, some chapters of Parts II and III were used in courses on elasticity theory and advanced fluid mechanics. Every effort was made to unify the notation for the broad range of the subjects considered. The notation is summarized at the end of each volume.

[1] Following the tradition, variational principles are named after their author; the references are given in Bibliographical Comments at the end of the book. Most of the variational principles with no name attached appeared first in the previous edition of this book.